On resolving domination number of friendship graph and its operation

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Abstract. Let $G = (V, E)$ be a simple, finite, and connected graph of order n. A dominating set $D \subseteq V(G)$ such every vertex not in D is adjacent to at least one member of D. A dominating set of smallest size is called a minimum dominating set and it is known as the domination number. The domination number is the minimum cardinality of a dominating set and denoted by $\gamma(G)$. The other hand, for an ordered set $W = \{w_1, w_2, w_3, ..., w_k\}$ of vertices and a vertex v in a connected graph G, the (metric) representation of v with respect to W is the $k - vector$ $r(v|W) = (d(v, w_1), d(v, w_2), d(v, w_3), ..., d(v, w_k))$, where $d(x, y)$ represents the distance between the vertices x and y. The set W is a resolving set for G if distinct vertices of G have distinct representations with respect to W. A resolving set of minimum cardinality is called a minimum resolving set or a basis and the cardinality of a basis for G is its metric dimension $dim(G)$. A Set of vertices of a graph G that is both resolving and dominating is a resolving dominating set. The minimum cardinality of a resolving dominating set is called resolving domination number $\gamma_r(G)$. In this paper, we discussed the resolving domination number of friendship graphs and its operation.

1. Introduction

In this paper, all graphs are finite, simple and connected, for a more complete definition of the graph, see [8]. A graph G consists of a set of objects $V(G) = \{v_1, v_2, v_3, ...\}$ called *vertices* and other set $E(G) = \{e_1, e_2, e_3, ...\}$ whose elements are called *edges* . Graph is generally denoted by $G = (V(G, E(G))$ [4]. In a graph must have at least a vertex, even if there are no edges [2, 7]. The number of vertices in G can be denoted by $|V(G)|$, while the number of edges is represented by $|E(G)|$. $\Delta(G)$ is the maximum degree of graph G and $\delta(G)$ is its the minimum degree [14].

We discussed about the resolving domination number of friendship graphs and its operation (line, middle, total and central). Domination number is the smallest of a dominating set is called a minimum dominating set and its size is called as the domination number. The domination number is the minimum cardinality of a dominating set and denoted by $\gamma(G)$ [10].

The dimension of metric is one of the subjects in theory of graph. The issue of the release of the metric dimension was first introduced by Slater in 1975. He introduced the license and used the location set and location number for the set resolution and metric dimensions. He discussed these concepts in working with the US Sonar and Coast Guard stations [6]. In other studies, Harry and Melter [9] proposed the similar rules in their paper 'On the Metric Dimension of a Graph' Independently, they observed the rules of the number location as good as possible which did not violate the rules and mentioned it by the dimension of metric.

We must have at least a vertex which dominated all of vertices in graph G, and its vertex namely D, $D \subseteq V(G)$. The minimum dominating set that well known domination number is the smallest size of dominating set. The domination number is the minimum cardinality of a dominating set and denoted by $\gamma(G)$. The value of the domination number is always $\gamma(G)$ $\cup V(G)$. [10].

According to Harry and Melter, the subset W is a resolving set if $r(v|W)$ for every two vertices of G have distinct representations. The minimum resolving set or a basis for G is the resolving set from minimum cardinality of graph G. Afterwards in making it easier to mention the metric dimension of G called by $dim(G)$ [9][3].

The concept of metric dimension has proved to be useful in a variety of fields. Chartr *et al* [5] applied the resolving set of metric dimension in chemistry to classify the chemical compound. Khuller *et al* [12] also applied in robotic navigation. Furthermore, Sebo et al [11] applied in combinatorial search and optimization.

The resolving domination number is denoted by $\gamma_r(G)$, first introduced by [15]. The resolving dominating set S of vertices of G not only dominates all vertices G but also has the added feature that distinct vertices G have distinct representations with respect to S. It is the minimum cardinality of the resolving dominating set.

The friendship graph, denoted by F_n is several n triangle with an ordinary vertex. Friendship might also be formed by combining C_3 graphs using one common vertex as the center. The Friendship graphs in general can also be referred to as a flower [1]. Srinivasa and Murali [13] divined about the line, middle and total graph as follows. Let G be a graph with $V(G)$ is the vertex set and $E(G)$ is edge set. The line graph from the graph G, denoted by $L(G)$, is a new one from the graph G, but there is a change where the edge becomes a vertex, and if $uv \in E(G)$ then $u, v \in E(L(G))$. Futher more, the middle graph of G can be denoted by $M(G)$, is defined as the following conditions. When the vertex set of $M(G)$ is combined $V(G) \cup E(G)$. If $a, b \in V(M(G)$ and $xy \in E(M(G)$ then in case one of the following holds:

1) a, b are adjacent in G, when a, b are in $E(G)$
2) a, b are incident in G, when a is in $V(G)$, b is in $E(G)$ [13].

The total graph of G can be denoted by $T(G)$, is defined as the following conditions. When the vertex set of $T(G)$ is combined $V(G) \cup E(G)$. Two vertices $a, b \in T(G)$ and $ab \in E(T(G))$ in case one of the following holds:

1) a is adjacent to b in G, when a, b are in $V(G)$.
2) a, b are adjacent in G, when a, b are in $E(G)$.
3) a, b are incident in G, when a is in $V(G)$, b is in $E(G)$ [13].

While according to Kavita [11], Let G be a graph with vertex set $V(G)$ and edge set $E(G)$. The central graph of G can be denoted by $C(G)$. $C(G)$ is achieved by joining all the non-adjancent vertices of G, if $a, b \in V(G)$ and $ab \in E(G)$, then ab will be a vertex in $V(T(G))$.

2. Result

For the result, we determined the resolving domination number $\gamma r_(G)$ of friendship graph F_n and it's Line graph $L(F_n)$, Middle graph $M(F_n)$, Central graph $C(F_n)$ and Total graph $T(F_n)$. The theorem regarding of the dominating set to be used in this study is as follows:

Theorem 1. *Let G be a friendship graph F_n For all integer $n \geq 2$. The resolving domination number of F_n is $\gamma_r(F_n) = n$*

Table 1. The general form of $v \in V(F_n)$ respect to S

v	$r(v\|S)$	condition
x_{11}	$(0, \underbrace{2,\ldots,2}_{n-1})$	$n \geq 2$
x_{12}	$(1, \underbrace{2,\ldots,2}_{n-1})$	$n \geq 2$
x_{21}	$(2{,}0{,}2\ \underbrace{2,\ldots,2}_{n-1})$	$n \geq 2$
x_{22}	$(2{,}1{,}2\ \underbrace{2,\ldots,2}_{n-1})$	$n \geq 2$
x_{i1}	$(\underbrace{2,\ldots,2}_{i-1},0,\underbrace{2,\ldots,2}_{n-i+2})$	$n \geq 2$
x_{i2}	$(\underbrace{2,\ldots,2}_{i-1},1,\underbrace{2,\ldots,2}_{n-i+2})$	$n \geq 2$
Z	$(\underbrace{1,\ldots,1}_{n-1})$	$n \geq 2$

Proof. The cardinality of friendship graph F_n is $V(F_n) = \{A\} \cup \{x_{ij}; 1 \leq i \leq n; 1 \leq j \leq 2\}$. $E(F_n) = \{Ax_{i1}, Ax_{i2}; 1 \leq i \leq n\} \cup \{x_{i1}x_{i2}; 1 \leq i \leq n\}$; $|V(F_n)| = 2n + 1$ and $|E| = 3n$. The maximum degree of F_n is $\triangle(G) = 2n$ and $\delta(G) = 2$. For proofing the resolving domination number $F_n = n$, we have to prove the upper bound $\gamma_r(F_n) \leq n$ and the lower bound $\gamma_r(F_n) \geq n$.

In step one, we should prove the upper bound of resolving domination number F_n that is $\gamma_r(F_n) \leq n$. choose $S = \{x_i; 1 \leq i \leq n\}$ so the representation of vertex in will be different. It can be shown in the following table 1. And also S is a dominating set because the vertex in S dominates each vertex in F_n, so S is resolving dominating set. It can be shown in the following table 1. Consequently, we can conclude that $\gamma_r(F_n) \leq n$.

In Addition, for proving the lower bound of resolving domination number of $(F_n) \geq n$ for $\gamma_r(F_n) \geq n$. We assume that resolving domination number of $(F_n) < n$. When we take $|S| = n - 1$,then the friendship graph F_n will have one vertex as a center vertex . Because the assumption is $n - 1$ vertex are in S hence there are some conditions of graph F_n as the following:

a) If we put $n - 1$ vertex in the subdivided of (F_n), we will get one vertex in the subdivided that is not dominated by $n - 1$ vertex in S. This is caused by the vertex in the in the subdivided is not adjacent,and contradicts the definition of dominating set.

b) If we put $n-2$ vertex in the sub divided of F_n and 1 vertex in the center vertex, we will get two vertices in the subdivided of graph that have the same representation. This is caused by 2 vertices in the subdivided of graph is will have the same distance to several vertices in F_n. This contradicts the definition of resolving set.

Based on cases (a) and (b) above, we can see that S is not resolving dominating set. Because, the statement above contradicts with $F_n < n$ so that it must $F_n \geq n$. After proofing $\gamma_r(F_n) \leq n$ and $\gamma_r(F_n) \geq n$, we can get the conclusion is $\gamma_r(F_n) = n$. The example of resolving domination number of friendship graph (F_n) is shown in Figure 1.

Theorem 2. *Let G be a line friendship graph $L(F_n)$, for all integer $n \geq 4, n \equiv 0 mod 2$. The resolving domination number of $L(F_n)$ is $\gamma_r L(F_n) = n$*

Proof. The cardinality of line friendship graph $L(F_n)$ is $V(L(F_n)) = \{x_i; 1 \leq i \leq n\} \cup \{y_j; 1 \leq j \leq \frac{n}{2}\}$. $E(F_n) = \{x_i x_{i+1} \cup \{x_n x1\} \cup \{x_i y_j; i \equiv 1 mod2; 1 \leq j \leq \frac{n}{2}\} \cup \{x_{i+1} y_j; 1 \equiv 1 mod2; 1 \leq j \leq$

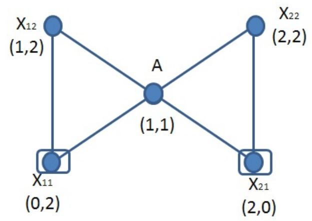

Figure 1. Friendship Graph (F_2)

Table 2. The general form of $v \in V(L(F_n))$ respect to S

v	$r(v\|S)$	condition
x_i	$(\underbrace{1,\ldots,1}_{i-1},\ 0,\ \underbrace{1,\ldots,1}_{n-i})$	$n \geq 4, n \equiv 0 mod 2$
y_j	$(\underbrace{2,\ldots,2}_{i-1},\ \underbrace{1,1}_{i,i+1}),\ \underbrace{2,\ldots,2}_{n-i-1})$	$n \geq 4, n \equiv 0 mod 2$

$\frac{n}{2}\} \cup \{x_ix_i; 1 \leq i \leq n\}$; $|V(L(F_n))| = \frac{3n}{2}$ and $|E(L(F_n))| = \frac{n(n-1)}{2} + n$. The maximum degree of F_n is $\triangle(G) = n$ and $\delta(G) = 2$. For proofing the resolving domination number $L(F_n) = n$, we have to prove the upper bound $\gamma_r(L(F_n)) \leq n$ and the lower bound $\gamma_r(L(F_n)) \geq 2n$.

In step one, we should prove the upper bound of resolving domination number $L(F_n)$ that is $\gamma_r(L(F_n)) \leq n$. choose $S = \{x_i; 1 \leq i \leq n\}$ so the representation of vertex in will be different. It can be shown in the following table 2. And also S is a dominating set because the vertex in S dominates each vertex in $L(F_n)$, so S is resolving dominating set. Consequently, we can conclude that $\gamma_r(L(F_n)) \leq n$.

In Addition, we proving the lower bound of resolving domination number of $L(F_n) \geq n$ is $\gamma_r(L(F_n)) \geq n$. We assume that resolving domination number of $L(F_n) < n$. When we take $|S| = n - 1$, then the line friendship graph $L(F_n)$ will have some conditions as the following:

a) If we put $n - 1$ vertex in the $L(F_n)$, we will get two vertices in the subdivided of graph that have the same representation. This is caused by 2 vertices in the subdivided of graph is will have the same distance to several vertices in F_n. This contradicts the definition of resolving set.

b) If we put $n - 2$ vertex in the $L(F_n)$ we will get one vertex in the subdivided that is not dominated by $n - 3$ vertex in S. This is caused by the vertex in the in the subdivided is not adjacent,and contradicts the definition of dominating set.

Based on cases (a) and (b) above, we can see that S is not resolving dominating set. The statement contradicts with $L(F_n) < n$ so that it must $F_n \geq n$. After proofing $\gamma_r(F_n) \leq n$ and $\gamma_r(L(F_n)) \geq n$, we can get the conclusion is $\gamma_r(L(F_n)) = n$. The example of resolving domination number of friendship graph $L(F_n)$ is shown in Figure 2.

Theorem 3. *Let G be a middle friendship graph $M(F_n)$, for all integer $n \geq 2$. The resolving domination number of $M(F_n)$ is $\gamma_r M(F_n) = n$*

Proof. The cardinality of middle friendship graph $M(F_n)$ is $V(M(F_n)) = \{A\} \cup \{x_i; 1 \leq i \leq n\} \cup \{y_j; 1 \leq j \leq \frac{n}{2}\} \cup \{Z_i; 1 \leq i \leq n\}$. $E(M(F_n)) = \{x_ix_{i+1}; 1 \leq i \leq n-1\} \cup \{x_nx_1\} \cup \{x_iy_j; 1 \leq i \leq n; 1 \leq j \leq \frac{n}{2}\} \cup \{x_{i+1}y_j; 1 \leq i \leq n-1; 1 \leq j \leq \frac{n}{2}\} \cup \{x_i; 1 \leq i \leq n\} \cup \{x_iZ_i; 1 \leq i \leq n\} \cup \{Z_iy_j; i \equiv 1 mod 2; 1 \leq j \leq \frac{n}{2}\} \cup \{Z_{i+1}y_j; i \equiv 1 mod 2; 1 \leq j \leq \frac{n}{2}\} \cup \{x_i, x_i; 1 \leq i \leq n\}$; $|V(M(F_n))| = 2n + \frac{n}{2} + 1$ and $|E(M(F_n))| = 5n + \frac{n(n-1)}{2}$. The maximum degree of $M(F_n)$ is

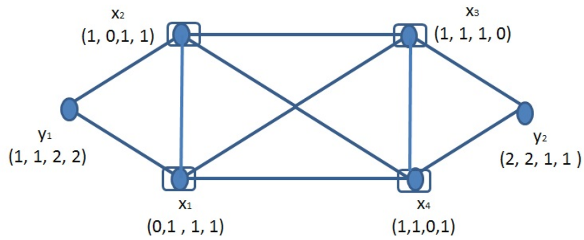

Figure 2. Line Friendship Graph $L(F_2)$

Table 3. The general form of $v \in V((M(F_n))$ respect to S

v	$r(v\|S)$	condition
x_i	$(\underbrace{1,\ldots,1}_{i-1}, 0, \underbrace{1,\ldots,1}_{n-i})$	$n \geq 4, n \equiv 0 mod 2$
y_j	$(\underbrace{2,\ldots,2}_{i-1}, \underbrace{1,1}_{i,i+1}), \underbrace{2,\ldots,2}_{n-i-1})$	$n \geq 4, n \equiv 0 mod 2$
Z_i	$(\underbrace{2,\ldots,2}_{i-1}, 1, \underbrace{2,\ldots,2}_{n-i})$	$n \geq 4, n \equiv 0 mod 2$
A	$(\underbrace{1,\ldots,1}_{n})$	$n \geq 2$

$\triangle = 2n$ and $\delta = 2$. For proofing the resolving domination number $M(F_n)$, we have to prove the upper bound $\gamma_r(M(F_n)) \leq n$ and the lower bound $\gamma_r(M(F_n)) \geq n$.

Step one, we should prove the upper bound of resolving domination number $M(F_n)$ that is $\gamma_r(M(F_n)) \leq n$. choose $S = \{x_i; 1 \leq i \leq n\}$ so the representation of vertex in $M(F_n)$ will be different. It can be shown in the following table 3. And also S is a dominating set because the vertex in S dominates each vertex in $M(F_n)$, so S is resolving dominating set. Consequently, we can conclude that $\gamma_r(F_n) \leq n$.

In Addition, for proving the lower bound of resolving domination number of $M(F_n) \geq n$ for $\gamma_r M(F_n) \geq n$. We assume that resolving domination number of $M(F_n) < n$. When we take $|S| = n - 1$, so there are some conditions of graph $M(F_n)$ as the following:

a) If we put $n-1$ vertex in the $C(F_n)$, we will get two vertices in the subdivided of graph that have the same representation. This is caused by 2 vertices in the subdivided of graph is will have the same distance to several vertices in $C(F_n)$. It contradicts the definition of resolving set.

b) If we put $n-2$ vertex in the $C(F_n)$ we will get one vertex in the subdivided that is not dominated by $n-2$ vertex in S. This is caused by the vertex in the in the subdivided is not adjacent, and contradicts the definition of dominating set.

Based on cases above, we can see that S is not resolving dominating set. The statement above contradicts with $M(F_n) < n$ so that it must $F_n \geq n$. After proofing $\gamma_r(M(F_n)) \leq n$ and $\gamma_r(M(F_n)) \geq n$, we can get the conclusion that $\gamma_r M(F_n) = n$. The example of resolving domination number of friendship graph $M(F_n)$ is shown in Figure 4.

Theorem 4. *Let G be a total friendship graph $T(F_n)$, for all integer $n \geq 2$. The resolving domination number of $T(F_n)$ is $\gamma_r T(F_n) = n$*

Proof. The cardinality of total friendship graph $T(F_n)$ is $V(T(F_n)) = \{A\} \cup \{x_i; 1 \leq i \leq n\} \cup \{y_j; 1 \leq j \leq \frac{n}{2}\} \cup \{Z_i; 1 \leq i \leq n\}$. $E(T(F_n)) = \{x_i x_{i+1}; 1 \leq i \leq n-1\} \cup \{x_n x_1\} \cup \{x_i y_j; 1 \leq$

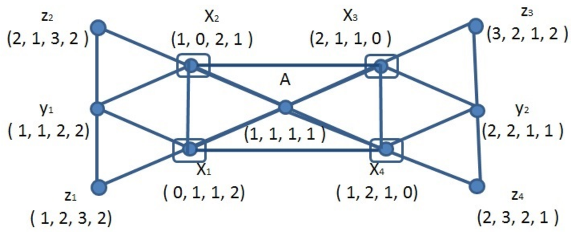

Figure 3. Middle Friendship Graph $M(F_2)$

$i \leq n; 1 \leq j \leq \frac{n}{2}\} \cup \{x_{i+1}y_j; 1 \leq i \leq n-1; 1 \leq j \leq \frac{n}{2}\} \cup \{x_i; 1 \leq i \leq n\} \cup \{x_iZ_i; 1 \leq i \leq n\} \cup \{Z_iy_j; i \equiv 1mod2; 1 \leq j \leq \frac{n}{2}\} \cup \{Z_{i+1}y_j; i \equiv 1mod2; 1 \leq j \leq \frac{n}{2}\} \cup \{x_i, x_i; 1 \leq i \leq n\} \cup \{z_iz_{i+1}; i \equiv 1mod2\} \cup \{Az_i; 1 \leq i \leq n\}$; $|V(T(F_n))| = 2n + \frac{n}{2} + 1$ and $|E(T(F_n))| = 6n + \frac{n}{2} + \frac{n(n-1)}{2}$. The maximum degree of $T(F_n)$ is $\triangle = 4n$ and $\delta = 4$. For proofing the resolving domination number $T(F_n)$, we have to prove the upper bound $\gamma_r(T(F_n)) \leq 2n$ and the lower bound $\gamma_r(T(F_n)) \geq 2n$.

Step one, we should prove the upper bound of resolving domination number $T(F_n)$ that is $\gamma_r T(F_n) \leq n$. choose $S = \{x_i; 1 \leq i \leq n\}$ so the representation of vertex in will be different. It can be shown in the following table 1. And also S is a dominating set because the vertex in S dominates each vertex in $T(F_n)$, so S is resolving dominating set. Consequently, we can conclude that $\gamma_r T(F_n) \leq n$.

In Addition, for proving the lower bound of resolving domination number of $T(F_n) \geq n$ for $\gamma_r(T(F_n)) \geq n$. We assume that resolving domination number of $(F_n) < n$. When we take $|S| = n-1$, then the central friendship graph $T(F_n)$ will have one vertex in A and each vertex in y_j . This causes the total friendship graph $T(F_n)$ will have some conditions of graph $T(F_n)$ as the following:

a) If we put $n-1$ vertex in the subdivided of $T(F_n)$, we will get one vertex in the subdivided that is not dominated by $n-1$ vertex in S. This is caused by the vertex in the in the subdivided is not adjacent,and contradicts the definition of dominating set.

b) If we put $n-2$ vertex in the sub divided of $T(F_n)$ and 1 vertex in the center vertex, we will get two vertices in the subdivided of graph that have the same representation. This is caused by 2 vertices in the subdivided of graph is will have the same distance to several vertices in $T(F_n)$. This contradicts the definition of resolving set.

Based on cases (a) and (b) above, we can see that S is not resolving dominating set. Because, the statement above contradicts with $T(F_n) < n$ so that it must $F_n \geq n$. After proofing $\gamma_r(T(F_n)) \leq n$ and $\gamma_r(T(F_n)) \geq n$, we can get the conclusion is $\gamma_r(T(F_n)) = n$. The example of resolving domination number of friendship graph $T(F_n)$ is shown in Figure 4.

Theorem 5. *Let G be a central friendship graph $C(F_n)$, for all integer $n \geq 2$. The resolving domination number of $C(F_n)$ is $\gamma_r C(F_n) = n+1$*

Proof. The cardinality of central friendship graph $C(F_n)$ is $V(C(F_n)) = \{A\} \cup \{x_i; 1 \leq i \leq n\} \cup \{y_j; 1 \leq j \leq \frac{n}{2}\} \cup \{Z_i; 1 \leq i \leq n\}$. $E(C(F_n)) = \{Ax_i; \leq i \leq n\} \cup \{x - iz_i; 1 \leq i \leq n\} \cup \{z_iy_j; i \equiv 1mod2; 1 \leq ji\frac{n}{2}\} \cup \{z_{i+1}y_j; 1mod2; 1 \leq j\frac{n}{2}\} \cup \{z_iz_{i+1}; 1 \leq 1 \equiv 0mod2n-1\} \cup \{z_nz_1\} \cup \{z_iz_i; 1 \leq i \leq n\}$; $|V(C(F_n))| = 2n + \frac{n}{2}$ and $|E(C(F_n))| = 2n + \frac{n}{2} + \frac{n(n-1)}{2}$. The maximum degree of $C(F_n)$ is $\triangle(G) = n$ and $\delta(G) = 2$. For proofing the resolving domination number $C(F_n)$, we have to prove the upper bound $\gamma_r C(F_n) \leq n+1$ and the lower bound $\gamma_r C(F_n) \geq n+1$.

In step one, we should prove the upper bound of resolving domination number $C(F_n)$ that is $\gamma_r(F_n) \leq n+1$. choose $S = \{x_i; 1 \leq i \leq n+1\}$ so the representation of vertex in will be different. It can be shown in the following table 1. And also S is a dominating set because the

Table 4. The general form of $v \in V(T(F_n))$ respect to S

v	$r(v\|S)$	condition
x_i	$(\underbrace{1,\ldots,1}_{i-1},\ 0,\ \underbrace{1,\ldots,1}_{n-i})$	$n \geq 4, n \equiv 0 mod 2$
y_j	$(\underbrace{2,\ldots,2}_{i-1},\ \underbrace{1,1}_{i,i+1}),\ \underbrace{2,\ldots,2}_{n-i-1})$	$n \geq 4, n \equiv 0 mod 2$
z_i	$(\underbrace{2,\ldots,2}_{i-1},\ 1,\ \underbrace{2,\ldots,2}_{n-i})$	$n \geq 4, n \equiv 0 mod 2$
A	$(\underbrace{1,\ldots,1}_{n})$	$n \geq 4, n \equiv 0 mod 2$

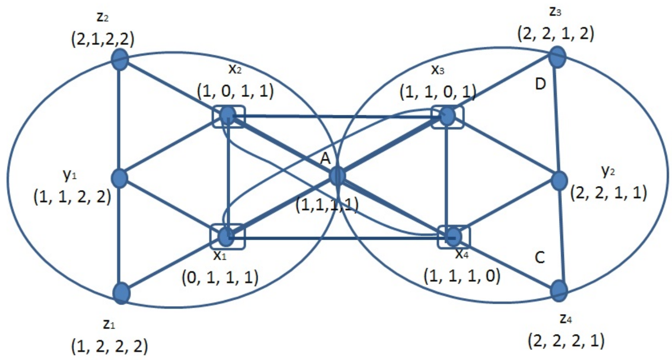

Figure 4. Total Friendship Graph $T(F_2)$

vertex in S dominates each vertex in $C(F_n)$, so S is resolving dominating set. Consequently, we can conclude that $\gamma_r C(F_n) \leq n+1$.

In Addition, for proving the lower bound of resolving domination number of $C(F_n) \geq n+1$ for $\gamma_r C(F_n) \geq n+1$. We assume that resolving domination number of $C(F_n) < n$. When we take $|S| = n$, then the central friendship graph $C(F_n)$ will have one vertex in A and each vertex in y_j . Then, there are some conditions of graph $C(F_n)$ as the following:

a) If we put n vertex in the $C(F_n)$, we will get two vertices in the subdivided of graph that have the same representation. This is caused by 2 vertices in the subdivided of graph is will have the same distance to several vertices in $C(F_n)$. It contradicts the definition of resolving set.

b) If we put $n-1$ vertex in the $C(F_n)$ we will get one vertex in the subdivided that is not dominated by $n-1$ vertex in S. This is caused by the vertex in the in the subdivided is not adjacent,and contradicts the definition of dominating set.

Based on cases (a) and (b) above, we can see that S is not resolving dominating set. The statement above contradicts with $C(F_n) < n+1$ so that it must $C(F_n) \geq n+1$. After proofing $\gamma_r(C(F_n) \leq n+1$ and $\gamma_r(C(F_n)) \geq n+1$, we can get the conclusion is $\gamma_r(C(F_n)) = n+1$. The example of resolving domination number of friendship graph $C(F_n)$ is shown in Figure 5.

3. Conclusion

We study the resolving domination number of Friendship $\gamma_r(F_n) = n$ for $n \geq 2$. The resolving domination number of line, middle, total and central Friendship graph is $\gamma_r(F_n) = \gamma_r(L(F_n)) = \gamma_r(M(F_n)) = \gamma_r(T(F_n)) = n$, and $\gamma_r(C(F_n)) = n+1$. The open problem of this research is

Table 5. The general form of $v \in V(C(F_n))$ respect to S

v	$r(v\|D_r)$	condition
x_i	$(\underbrace{3,\ldots,3}_{i-1}, 1, \underbrace{3,\ldots,3}_{n-i}, 1)$	$n \geq 4, n \equiv 0 mod 2$
y_j	$(\underbrace{2,\ldots,2}_{i-1}, \underbrace{1,1}_{i,i+1}), \underbrace{2,\ldots,2}_{n-i-1})$	$n \geq 4, n \equiv 0 mod 2$
z_i	$(\underbrace{1,\ldots,1}_{i-1}, \underbrace{0}_{i}, \underbrace{2}_{i+1}, \underbrace{1,\ldots,1}_{n-i-2}, 2)$	$n \geq 4, n \equiv 1 mod 2$
z_i	$(\underbrace{1,\ldots,1}_{i-1}, \underbrace{2}_{i}, \underbrace{0}_{i+1}, \underbrace{1,\ldots,1}_{n-i-2}, 2)$	$n \geq 4, n \equiv 0 mod 2$
A	$(\underbrace{2,\ldots,2}_{n}, 0)$	$n \geq 4, i \equiv 0 mod 2$

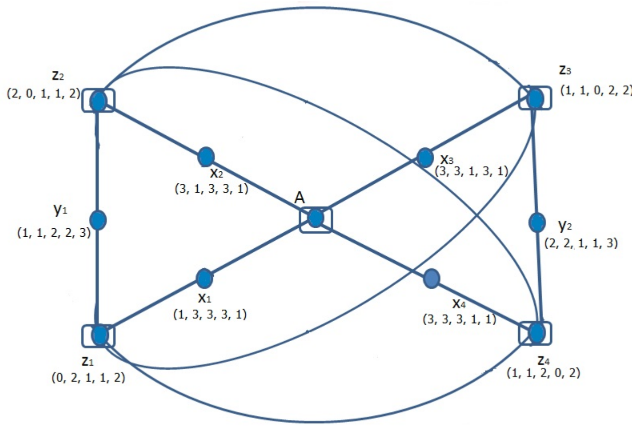

Figure 5. Central Friendship Graph $C(F_2)$

Open Problem 1. *Let G be a friendship graph, determine the resolving domination number of subdivided of G with m vertices on each edge.*

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