

On the super edge local antimagic total labeling of related ladder graph

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Abstract. A graph G is an ordered pair of sets $G = (V, E)$, where V is a set of vertices and E is a set of edges. The concept of labeling graphs has recently gained a lot of popularity in the area of graph theory. Antimagic labeling is the interest topic to studied. The study of antimagic labeling motivated by Hartsfield and Ringel. Arumugam *et.al* developed the concept of antimagic labeling that is local antimagic coloring. Thus, we study the concept of local antimagic namely edge local antimagic total labeling. By an edge local antimagic total labeling, a bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, 3, \dots, |V(G)| + |E(G)|\}$ satisfying that for any two adjacent edges e_1 and e_2 , $w_t(e_1) \neq w_t(e_2)$, where for $e = ab \in G$, $w_t(e) = f(a) + f(b) + f(ab)$. Thus, any edge local antimagic total labeling induces a proper edge coloring of G if each edge e is assigned the color $w_t(e)$. It is considered to be a super edge local antimagic total coloring, if the smallest labels appear in the vertices. The chromatic number of super edge local antimagic total, denoted by $\gamma_{leat}(G)$, is the minimum number of colors taken over all colorings induced by super edge local antimagic total labelings of G . In this paper we study edge local antimagic total labeling and determined the chromatic number of graphs as follow grid graph, prism graph and mobious ladder graph.

1. Introduction

A graph G is an ordered pair of sets $G = (V, E)$. The elements of the set V usually called vertices and the elements of the set E are usually called edges. The cardinality of vertex and edge we denote by $|V(G)|$ and $|E(G)|$. In this study, we will be interested only in finite graphs. We can see the detail definition and notation of graph in [1, 2]. The concept of labeling graphs has recently gained a lot of popularity in the area of graph theory. Informally, by a graph labeling we will mean an assignment of integers to elements of a graph, such as vertices, edges, or both. We call these labelings a vertex labeling, edge labeling, or total labeling. In this study, we use the total labeling of



graphs. The labeling is called edge antimagic if all the edge weights of graph G have the distinct value. Hartsfield and Ringel [3] introduced the concept of an antimagic labeling. Dafik *et. al* has studied antimagic labeling. Some results can be found in [4, 5].

In 2017, Arumugam *et.al* [6] developed the concept of antimagic labeling that is local antimagic coloring. They gave a lower bound and an exact value of local antimagic coloring. Thus, we study the concept of local antimagic namely edge local antimagic total labeling. The different type of local antimagic coloring is developed by Agustin *et. al.* [7], namely local antimagic edge coloring. Agustin *et.al* [8] has study the concept edge local antimagic total labeling of graph using EAVL technique. By an edge local antimagic total labeling, a bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, 3, \dots, |V(G)| + |E(G)|\}$ satisfying that for any two adjacent edges e_1 and e_2 , $w_t(e_1) \neq w_t(e_2)$, where for $e = ab \in G$, $w_t(e) = f(a) + f(b) + f(ab)$. Thus, any edge local antimagic total labeling induces a proper edge coloring of G if each edge e is assigned the color $w_t(e)$. It is considered to be a super edge local antimagic total coloring, if the smallest labels appear in the vertices. The chromatic number of super edge local antimagic total, denoted by $\gamma_{leat}(G)$, is the minimum number of colors taken over all colorings induced by super edge local antimagic total labelings of G .

The other results about local antimagic of graphs can be seen in [8, 9, 10, 11, 12]. In this paper we study the concept of super edge local antimagic total labeling and determined the chromatic number of graphs as follow grid graph $G_{n,2}$, prism graph Pr_n and mobious ladder graph M_n .

2. Previous Result

To show our results, we take some definition from the previous results in the following.

Definition 2.1. [6] Let $G = (V, E)$ be a connected graph with $|V| = n$ and $|E| = m$. A bijection $f : E \rightarrow \{1, 2, \dots, m\}$ is called a local antimagic labeling if for any two adjacent vertices u and v , $w(u) \neq w(v)$, where $w(u) = \sum_{e \in E(u)} f(e)$, and $E(u)$ is the set of edges incident to u .

Definition 2.2. [8] Let $G(V, E)$ be a graph of vertex set V and edge set E . A bijection $f : V(G) \cup E(G) \rightarrow \{1, 2, 3, \dots, p + q\}$ where $p = |V(G)|$ and $q = |E(G)|$ such that for every two adjacent edges e_1 and e_2 for $e = ab \in G$ and $w_t(e) = f(a) + f(b) + f(ab)$, $w_t(e_1) \neq w_t(e_2)$. It is considered to be a super edge local antimagic total labeling, if the smallest labels appear in the vertices.

Lemma 2.1. [8] If $\Delta(G)$ is maximum degrees of G , then we have $\gamma_{leat}(G) \geq \Delta(G)$.

Theorem 2.1. [13] Vizing Theorem implies that $\Delta(G) \leq \chi(G) \leq \Delta(G) + 1$, where $\Delta(G)$ is the maximum degree of a vertex in G and $\chi(G)$ is the edge chromatic number.

3. Main Result

In this paper we have study the concept of super edge local antimagic total labeling and determined the chromatic number of grid graph $G_{n,2}$, prism graph Pr_n and mobious ladder graph M_n . We present our result as follows.

Theorem 3.1. Let $G_{n,2}$ be grid graph, for any natural number $n \geq 3$ and n is odd. The edge local antimagic total labeling chromatic number of $G_{n,2}$ is $4 \leq \chi_{late}(G_{n,2}) \leq 5$

Proof. Let $G_{n,2}$ be grid graph with $V(G_{n,2}) = \{a_i, b_i, c_i; 1 \leq i \leq n\}$ and $E(G_{n,2}) = \{a_i b_i, b_i c_i; 1 \leq i \leq n\} \cup \{a_i a_{i+1}, b_i b_{i+1}, c_i c_{i+1}; 1 \leq i \leq n - 1\}$. The cardinality of vertices and edges are $|V(G_{n,2})| = 3n$ and $|E(G_{n,2})| = 5n - 3$.

To proof $4 \leq \chi_{late}(G_{n,2}) \leq 5$, firstly we show $\chi_{late}(G_{n,2}) \geq 4$. To show the $\chi_{late}(G_{n,2}) \geq 4$, based on Lemma 2.1[8] to proof the lowerbound of edge local antimagic total labeling we identify the maximum degree of graph $G_{n,2}$. The maximum degree of graph $G_{n,2}$ is $\Delta(G_{n,2}) = 4$. From the Lemma, we have $\chi_{late}(G_{n,2}) \geq \Delta(G_{n,2}) = 4$. It concludes that $\chi_{late}(G_{n,2}) \geq 4$. Furthermore, we will show $\chi_{late}(G_{n,2}) \leq 5$.

Assume that a bijection $f : V(G_{n,2}) \cup E(G_{n,2}) \rightarrow \{1, 2, 3, \dots, 8n - 3\}$, the function of labeling vertex and edge of are as follows.

The label of vertices:

$$f(b_i) = \begin{cases} \frac{i+1}{2}, & \text{for } i = 1, 3, 5, \dots, n \\ \frac{i+n+1}{2}, & \text{for } i = 2, 4, 6, \dots, n - 1 \end{cases}$$

$$f(a_i) = \begin{cases} \frac{3n+i}{2}, & \text{for } i = 1, 3, 5, \dots, n \\ n + \frac{i}{2}, & \text{for } i = 2, 4, 6, \dots, n - 1 \end{cases}$$

$$f(c_i) = \begin{cases} \frac{5n+i}{2}, & \text{for } i = 1, 3, 5, \dots, n \\ 2n + \frac{i}{2}, & \text{for } i = 2, 4, 6, \dots, n - 1 \end{cases}$$

The label of edges:

$$f(c_i c_{i+1}) = \begin{cases} 4n - i - 1, & \text{for } i = 1, 3, 5, \dots, n - 2 \\ 4n - i + 1, & \text{for } i = 2, 4, 6, \dots, n - 1 \end{cases}$$

$$f(a_i a_{i+1}) = \begin{cases} 6n - i - 2, & \text{for } i = 1, 3, 5, \dots, n - 2 \\ 6n - i, & \text{for } i = 2, 4, 6, \dots, n - 1 \end{cases}$$

$$f(b_i b_{i+1}) = \begin{cases} 8n - i - 3, & \text{for } i = 1, 3, 5, \dots, n - 2 \\ 8n - i - 1, & \text{for } i = 2, 4, 6, \dots, n - 1 \end{cases}$$

$$f(a_i b_i) = 7n - i - 1 \text{ for } i = 1, 2, 3, \dots, n$$

$$f(b_i c_i) = 5n - i \text{ for } i = 1, 2, 3, \dots, n$$

From the function of vertex and edge labeling above, the total edge weights are as follows:

Hence, the function of total edge weights above, we can see that $\chi_{late}(G_{n,2}) \leq 5$. Since $\chi_{late}(G_{n,2}) \geq 4$ and $\chi_{late}(G_{n,2}) \leq 5$, thus $4 \leq \chi_{late}(G_{n,2}) \leq 5$. \square

The Example of super edge local antimagic total labeling of graph $G_{5,2}$ can be seen on Figure 1 .

Theorem 3.2. Let Pr_n be prism graph, for any natural number $n \geq 3$ and n is odd. The edge local antimagic total labeling chromatic number of Pr_n is $3 \leq \chi_{late}(Pr_n) \leq 5$

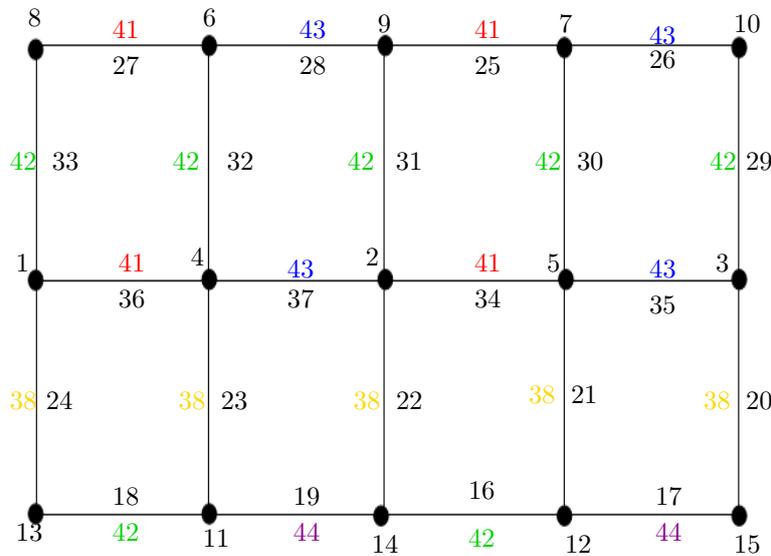


Figure 1. The example of super edge local antimagic total labeling of graph $G_{5,2}$

Proof. Let Pr_n be prism graph with $V(Pr_n) = \{a_i, b_i; 1 \leq i \leq n\}$ and $E(Pr_n) = \{a_i b_i; 1 \leq i \leq n\} \cup \{a_i a_{i+1}, b_i b_{i+1}; 1 \leq i \leq n - 1\} \cup \{a_1 a_n, b_1 b_n\}$. The cardinality of vertices and edges are $|V(Pr_n)| = 2n$ and $|E(Pr_n)| = 3n$.

To proof $3 \leq \chi_{late}(Pr_n) \leq 5$, firstly we show $\chi_{late}(Pr_n) \geq 3$. To show the $\chi_{late}(Pr_n) \geq 3$, based on Lemma 2.1[8] to proof the lowerbound of edge local antimagic total labeling we identify the maximum degree of graph Pr_n . The maximum degree of graph Pr_n is $\Delta(Pr_n) = 3$. From the Lemma, we have $\chi_{late}(Pr_n) \geq \Delta(Pr_n) = 3$. It concludes that $\chi_{late}(Pr_n) \geq 3$. Furthermore, we will show $\chi_{late}(Pr_n) \leq 5$.

Assume that a bijection $f : V(Pr_n) \cup E(Pr_n) \rightarrow \{1, 2, 3, \dots, 5n\}$, the function of labeling vertex and edge of are as follows.

The label of vertices:

$$f(a_i) = \begin{cases} \frac{n+i}{2}, & \text{for } i = 1, 3, 5, \dots, n \\ \frac{i}{2}, & \text{for } i = 2, 4, 6, \dots, n - 1 \end{cases}$$

$$f(b_i) = \begin{cases} n + \frac{i+1}{2}, & \text{for } i = 1, 3, 5, \dots, n \\ \frac{i+3n+1}{2}, & \text{for } i = 2, 4, 6, \dots, n - 1 \end{cases}$$

The label of edges:

$$f_1(e) = \begin{cases} 3n, & \text{for } e = a_1 a_2 \\ 5n - i + 2, & \text{for } e = a_i a_{i+1}, i = 3, 5, 7, \dots, n - 2 \\ 5n - i, & \text{for } e = a_i a_{i+1}, i = 2, 4, 6, \dots, n - 3 \\ 4n + 1, & \text{for } e = a_{n-1} a_n \\ 4n + 2, & \text{for } e = a_1 a_n \end{cases}$$

$$f_2(e) = \begin{cases} 5n, & \text{for } e = b_1b_n \\ 3n - i - 1, & \text{for } e = b_ib_{i+1}, i = 1, 3, 5, \dots, n - 2 \\ 3n - i + 1, & \text{for } e = b_ib_{i+1}, i = 2, 4, 6, \dots, n - 1 \end{cases}$$

$$f_3(a_ib_i) = 4n - i + 1 \text{ for } i = 1, 2, 3, \dots, n$$

From the function of vertex and edge labeling above, the total edge weights are as follows:

$$w_t(e) = \begin{cases} \frac{11n+3}{2}, & \text{for } e = a_ib_i; i = 1, 2, 3, \dots, n \\ \frac{7n+3}{2}, & \text{for } e = a_1a_2 \\ \frac{11n+1}{2}, & \text{for } e = a_ia_{i+1}; i = 2, 4, 6, \dots, n - 1 \text{ and } e = b_ib_{i+1}; i = 1, 3, 5, \dots, n - 2 \\ \frac{11n+5}{2}, & \text{for } e = a_ia_{i+1}; i = 1, 3, 5, \dots, n - 2, e = b_ib_{i+1}; i = 2, 4, 6, \dots, n - 1 \text{ and } \\ & e = a_1a_n \\ \frac{15n+3}{2}, & \text{for } e = b_1b_n \end{cases}$$

Hence, the function of total edge weights above, we can see that $\chi_{late}(Pr_n) \leq 5$. Since $\chi_{late}(Pr_n) \geq 3$ and $\chi_{late}(Pr_n) \leq 5$, thus $3 \leq \chi_{late}(Pr_n) \leq 5$. \square

The Example of super edge local antimagic total labeling of graph Pr_5 can be seen on Figure 2.

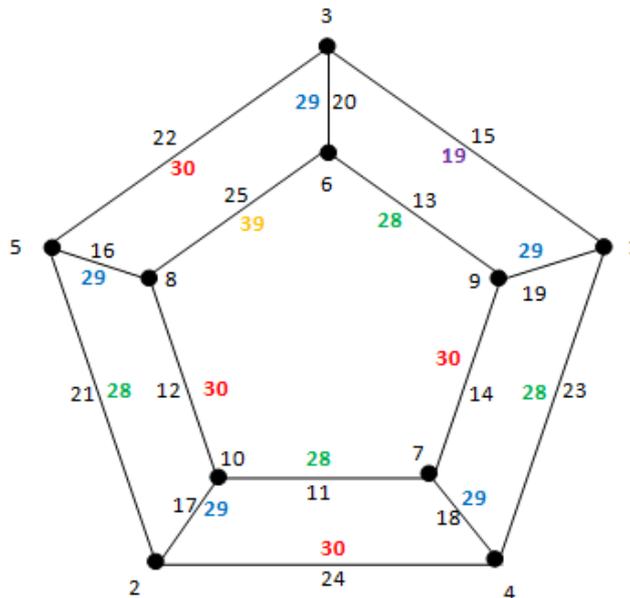


Figure 2. The example of super edge local antimagic total labeling of graph Pr_5

Theorem 3.3. *Let M_n be mobious ladder graph, for any natural number $n \geq 3$ and n is odd. The edge local antimagic total labeling chromatic number of M_n is $3 \leq \chi_{late}(M_n) \leq 5$*

Proof. Let Pr_n be prism graph with $V(M_n) = \{a_i, b_i; 1 \leq i \leq n\}$ and $E(M_n) = \{a_i b_i; 1 \leq i \leq n\} \cup \{a_i a_{i+1}, b_i b_{i+1}; 1 \leq i \leq n - 1\} \cup \{a_1 b_n, a_n b_1\}$. The cardinality of vertices and edges are $|V(M_n)| = 2n$ and $|E(M_n)| = 3n$.

To proof $3 \leq \chi_{late}(M_n) \leq 5$, firstly we show $\chi_{late}(M_n) \geq 3$. To show the $\chi_{late}(M_n) \geq 3$, based on Lemma 2.1[8] to proof the lowerbound of edge local antimagic total labeling we identify the maximum degree of graph M_n . The maximum degree of graph M_n is $\Delta(M_n) = 3$. From the Lemma, we have $\chi_{late}(M_n) \geq \Delta(M_n) = 3$. It concludes that $\chi_{late}(M_n) \geq 3$. Furthermore, we will show $\chi_{late}(M_n) \leq 5$.

Assume that a bijection $f : V(M_n) \cup E(M_n) \rightarrow \{1, 2, 3, \dots, 5n\}$, the function of labeling vertex and edge of are as follows.

The label of vertices:

$$f(a_i) = \begin{cases} \frac{i+1}{2}, & \text{for } i = 1, 3, 5, \dots, n \\ \frac{n+1+i}{2}, & \text{for } i = 2, 4, 6, \dots, n - 1 \end{cases}$$

$$f(b_i) = \begin{cases} \frac{3n+i}{2}, & \text{for } i = 1, 3, 5, \dots, n \\ n + \frac{i}{2}, & \text{for } i = 2, 4, 6, \dots, n - 1 \end{cases}$$

The label of edges:

$$f(b_i b_{i+1}) = \begin{cases} 3n - i - 1, & \text{for } i = 1, 3, 5, \dots, n - 2 \\ 3n - i + 1, & \text{for } i = 2, 4, 6, \dots, n - 1 \end{cases}$$

$$f(a_i a_{i+1}) = \begin{cases} 5n - i - 2, & \text{for } i = 1, 3, 5, \dots, n - 2 \\ 5n - i, & \text{for } i = 2, 4, 6, \dots, n - 1 \end{cases}$$

$$f(a_i b_i) = 4n - i \text{ for } i = 1, 2, 3, \dots, n$$

$$f(a_1 b_n) = 5n - 1$$

$$f(a_n b_1) = 5n$$

From the function of vertex and edge labeling above, the total edge weights are as follows:

$$w_t(e) = \begin{cases} \frac{11n-1}{2}, & \text{for } e = a_i a_{i+1}, b_i b_{i+1}; i = 1, 3, 5, \dots, n - 2 \\ \frac{11n+3}{2}, & \text{for } e = a_i a_{i+1}, b_i b_{i+1}; i = 2, 4, 6, \dots, n - 1 \\ \frac{11n+1}{2}, & \text{for } e = a_i b_i; i = 1, 2, 3, \dots, n \\ 7n, & \text{for } e = a_1 b_n \\ 7n + 1, & \text{for } e = a_n b_1 \end{cases}$$

Hence, the function of total edge weights above, we can see that $\chi_{late}(M_n) \leq 5$. Since $\chi_{late}(M_n) \geq 3$ and $\chi_{late}(M_n) \leq 5$, thus $3 \leq \chi_{late}(M_n) \leq 5$. □

The Example of super edge local antimagic total labeling of graph M_5 can be seen on Figure 3.

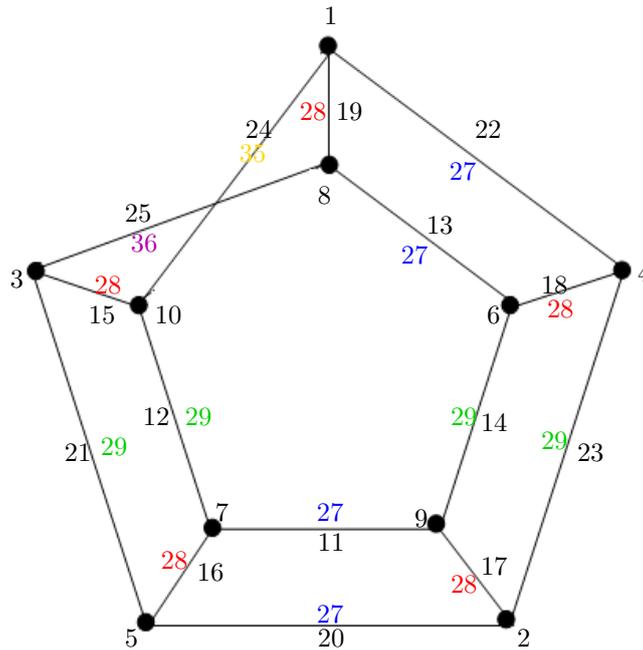


Figure 3. The example of super edge local antimagic total labeling of graph M_5

4. Conclusion

In this paper we have study and determine three theorems about the concept of super edge local antimagic total labeling. The theorems has determined the chromatic number of grid graph $G_{n,2}$, prism graph Pr_n and mobius ladder graph M_n as a follows: $4 \leq \chi_{late}(G_{n,2}) \leq 5$, $3 \leq \chi_{late}(Pr_n) \leq 5$ and $3 \leq \chi_{late}(M_n) \leq 5$.

Open Problem 4.1. find the best chromatic number of edge local antimagic total labeling of another graphs.

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