On total H-irregularity strength of diamond ladder, three circular ladder, and prism graphs

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Abstract. Let G be a graph with vertex set V and edge set E. A total labeling $\varphi : V(G) \cup E(G) \rightarrow \{1, 2, 3, ..., \alpha\}$ is called a total α-labeling of a graph G. For the subgraph $H \subseteq G$ under the total α-labeling, H-weight is defined as $wt_\varphi(H) = \sum_{v \in V(H)} \varphi(v) + \sum_{e \in E(H)} \varphi(e)$. A total α-labeling is called an H-irregular total α-labeling of the graph G if $wt_\varphi(H') \neq wt_\varphi(H")$ for any two distinct subgraphs H' and $H"$ isomorphic to H. The minimum α for which the graph G has a total H-irregular α-labeling is called the total H-irregularity strength of G, denoted by $tHs(G)$. In this paper we initiate to study the total H-irregularity strength of G and we have obtained the tHs of diamond ladder, three circular ladder and prism graphs.

1. Introduction

We use a simple, connected, and finite graph, especially planar graph in this research. G is a graph which has the vertex set is given as $V(G)$ and the edge set is given as $E(G)$. Graph labeling is mapping graph elements to positive or non-negative integers number. The most common choices of domain are the set of all vertices (vertex labellings), the only edge set (edge labellings), or the set of either vertices or edges (total labelings). Other domains are possible [5]. The graph G contains H includes each H_j isomorphic subgraph which conditions each $E(G)$ edge included in every one of the H_j subgraphs, $j = 1, 2, \ldots s$ [4].

The total irregular vertex of α-labeling on the graph G is the assignment of the $1, 2, ..., \alpha$ for vertices and edges such that the weights calculated at the different vertices. The vertex weight $v \in V$ in G is defined as the sum of label v and labels all incident edges with v, that is $wt(v) = \lambda(v) + \sum_{uv \in E} \lambda(uv)$ [9]. The vertex-irregularity strength of G is the smallest α integer on the H-irregular label of G and denoted by $vhs(G, H)$ [3]. Indriati et al. [7] obtain a for the total vertex irregularity strength of generalized helm graphs and prisms with outer pendant edges.

The irregular total edge α-labeling of a graph $G = (V, E)$ is labeling $\phi : V \cup E = 1, 2, ..., \alpha$ so that the total edge weight of $wt(xy) = \phi(x) + \phi(xy) + \phi(y)$ is different for all different edge pairs. The minimum α where graph G has an irregular total edge α-labeling is called the total irregular edge strength of G and denoted by $vhs(G, H)$ [2]. Bača and Siddiqui [6] investigate the total edge irregularity strength of generalized prism.

Ashraf et al. in [3] introduce total H-irregularity strength as a natural extension of the $tes(G)$ and $tvs(G)$ parameters. G is a graph that recognizes H-covering. For subgraph $H \subseteq G$ under total α-labeling φ associated with H-weight is defined as

$$wt_{\varphi}(H) = \sum_{v \in V(H)} \varphi(v) + \sum_{e \in E(H)} \varphi(e).$$

The total α-labeling is called H-irregular total α-labeling of the graph G if $wt_{\varphi}(H') \neq wt_{\varphi}(H")$ for every two different subgraphs of H' and $H"$ isomorphic to H.

The smallest integer α for which an H-irregular total α-labeling of exists is known as the total H-irregularity strength of G and denoted by $tHs(G, H)$.

Theorem 1. [4] Let G be a graph that recognizes H-covering provided by the t isomorphic subgraph to H. Then

$$tHs(G, H) \geq \left\lceil 1 + \frac{t-1}{|V(H)| + |E(H)|} \right\rceil.$$

Agustin et al [1] have conducted research and obtained results from $tHs(G, H)$ of shackle and amalgamation graphs. Nisviasari [8] have conducted research and obtained results from $tHs(G, H)$ of triangular ladder and grid graphs. Ashraf et al. [3] have conducted research and obtained results from $tHs(G, H)$ of ladder and fan graphs. We use diamond ladder, three circular ladders, and prism graph to get $tHs(G, H)$.

2. Results

In this paper, we provide the results of $tHs(G, H)$ of diamond ladder, three circular ladders, and prism graphs, is as follows.

Theorem 2. Let Dl_m be a diamond ladder graph and subgraph $H_1 \equiv Dl_n$. The total H_1-irregularity strength of Dl_m graph for $2 \leq n < m$ is $\left\lceil \frac{m+11n-3}{12n-3} \right\rceil$.

Proof. Let Dl_m, $m \geq 0$, be a diamond ladder graph with the vertex set $V(Dl_m) = \{x_j, y_j : j = 1, 2, 3, \ldots, m\} \cup \{z_j : j = 1, 2, 3, \ldots, 2m\}$ and the edge set $E(Dl_m) = \{x_j y_j, x_j z_{2j-1}, x_j z_{2j}, y_j z_{2j-1}, y_j z_{2j}, : j = 1, 2, 3, \ldots, m\} \cup \{x_i x_{i+1}, y_i y_{i+1} : j = 1, 2, 3, \ldots, m-1\} \cup \{z_{2j-2} z_{2j-1} : j = 2, 3, \ldots, m\}$. The diamond ladder graph Dl_m, m is positive integer, admits a Dl_n covering with exactly $(m - n + 1)$ diamond ladder Dl_n, where n is a positive integer and $2 \leq n < m$. Based on Theorem 2, we have $tHs((Dl_m), Dl_n) \geq \left\lceil \frac{m+11n-3}{12n-3} \right\rceil$. Put $l = \left\lceil \frac{m+11n-3}{12n-3} \right\rceil$. The following function of Dl_n-irregular total α-labeling $\varphi_n : V(Dl_m) \cup E(Dl_m) \to \{1, 2, \ldots, l\}, n = 2, 3, ..., m$ is prove that α as an upper bound for the total Dl_n-irregularity strength of Dl_m.

$$\varphi_n(y_j) = \begin{cases} \left\lceil \frac{j+20n-6}{24n-6} \right\rceil, & \text{for j is even} \\ \left\lceil \frac{j+16n-5}{24n-6} \right\rceil, & \text{for j is odd} \end{cases}$$

$$\varphi_n(x_j) = \left\lceil \frac{j+11n-3}{12n-3} \right\rceil, \text{for } j \in [1,m], \qquad \varphi_n(z_j) = \left\lceil \frac{j+9n-3}{12n-3} \right\rceil, \text{for } j \in [1,m]$$
$$\varphi_n(x_jx_{j+1}) = \left\lceil \frac{j+2n-2}{12n-3} \right\rceil, \text{for } j \in [1,m-1], \qquad \varphi_n(x_jy_j) = \left\lceil \frac{j+3n-3}{12n-3} \right\rceil, \text{for } j \in [1,m]$$
$$\varphi_n(z_jz_{j+1}) = \left\lceil \frac{j}{12n-3} \right\rceil, \text{for } j \in [1,m-1], \quad \varphi_n(x_jy_{2j-1}) = \left\lceil \frac{j+4n-3}{12n-3} \right\rceil, \text{for } j \in [1,m]$$
$$\varphi_n(x_jy_{2j}) = \left\lceil \frac{j+7n-3}{12n-3} \right\rceil, \text{for } j \in [1,m], \quad \varphi_n(y_{2j-1}z_j) = \left\lceil \frac{j+5n-3}{12n-3} \right\rceil, \text{for } j \in [1,m]$$
$$\varphi_n(y_{2j}z_j) = \left\lceil \frac{j+6n-3}{12n-3} \right\rceil, \text{for } j \in [1,m], \quad \varphi_n(y_jy_{j+1}) = \left\lceil \frac{j+2n-2}{24n-6} \right\rceil, \text{for } j \text{ is even.}$$

We get the upper bound from the function of Dl_n-irregular total Dl_m-labeling. We take the largest label from $\varphi_n(x_j) = \left\lceil \frac{j+11n-3}{12n-3} \right\rceil$ for $j = m$ $\varphi_n(x_j) = \left\lceil \frac{m+11n-3}{12n-3} \right\rceil$. We get to present the upper bound of the graph in the Theorem 2, $tHs((Dl_m), Dl_n) \leq \left\lceil \frac{m+11n-3}{12n-3} \right\rceil$.

Based on the labeling above, we can show the all weights are different by the following equation:

$$\begin{aligned} wt\varphi_n(Dl_n^{j+1}) - wt\varphi_n(Dl_n^j) &= \varphi_n(x_{j+n}) + \varphi_n(y_{2j+2n-1}) + \varphi_n(y_{2j+2n}) + \varphi_n(z_{j+n}) + \\ &\quad \varphi_n(x_{j+n-1}x_{j+n}) + \varphi_n(y_{2j}y_{2j+1}) + \varphi_n(z_jzj+1) + \varphi_n(x_jz_j) + \\ &\quad \varphi_n(x_jy_{2j-1}) + \varphi_n(x_jy_{2j}) + \varphi_n(z_jy_{2j-1}) + \varphi_n(z_jy_{2j}) - \varphi_n(x_j) - \\ &\quad \varphi_n(y_{2j-1}) - \varphi_n(y_2j) - \varphi_n(z_j) - \varphi_n(x_jx_{j+1}) - \varphi_n(y_{2j}y_{2j+1}) - \\ &\quad \varphi_n(z_jz_{j+1}) - \varphi_n(x_jz_j) - \varphi_n(x_jy_{2j-1}) - \varphi_n(x_jy_{2j}) - \\ &\quad \varphi_n(z_jy_{2j-1}) - \varphi_n(z_jy_{2j}) \\ &= 1 \end{aligned}$$

We respect to $wt_{\varphi_n}(Dl_n^j) < wt_{\varphi_n}(Dl_n^{j+1})$, $j = 1, 2, \ldots, m-n$ then $wt\varphi_n(Dl_n^{j+1}) - wt\varphi_n(Dl_n^j) = 1$. The all H_1-weights are distinct. This matter concludes that $tHs((Dl_m), Dl_n) = \left\lceil \frac{m+11n-3}{12n-3} \right\rceil$. The example of total Dl_n-irregularity of diamond ladder graph labeling, we can see on Figure 1, and we get $tHs((Dl_8), Dl_2) = 2$.

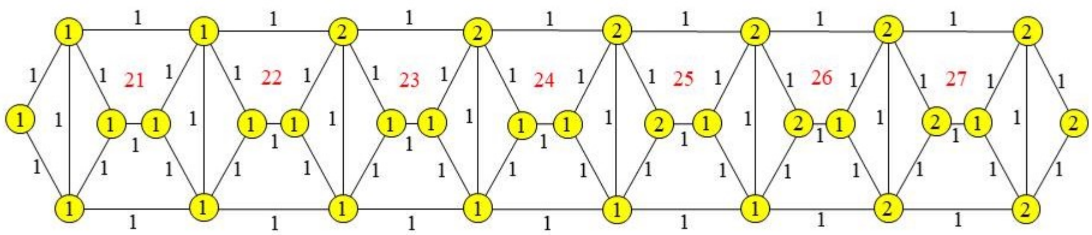

Figure 1. The Example of Total Dl_2-Irregularity of Dl_8 labeling

Theorem 3. Let TCl_m be a three circular ladder graph and subgraph $H_2 \equiv C_3$. The total H_2-irregularity strength of TCl_m graph is $\left\lceil \frac{3m+5}{6} \right\rceil$.

Proof. Let TCl_m, $m \geq 0$, be a three circular ladder graph with the vertex set $V(TCl_m) = \{x_j, z_j : j = 1, 2, 3, \ldots, m+1\} \cup \{y_j : j = 1, 2, 3, \ldots, m\}$ and the edge set $E(TCl_m) = \{x_jy_j, x_{j+1}y_j, y_jz_j, z_jz_{j+1} : j = 1, 2, 3, \ldots, m\} \cup \{x_jz_j : j = 1, 2, 3, \ldots, m+1\} \cup \{y_jz_{j+1} : j = 1, 2, 3, \ldots, m-1\}$. The three circular ladder graph TCl_m, m is positive integer, admits a C_3-covering with exactly m cycles C_3. Based on Theorem 3, we have $tHs((TCl_m), C_3) \geq \left\lceil \frac{3m+5}{6} \right\rceil$. Put $l = \left\lceil \frac{3m+5}{6} \right\rceil$. The following function of C_3-irregular total α-labeling $\varphi_3 : V(TCl_m) \cup E(TCl_m) \to \{1, 2, \ldots, l\}$ is prove that α as an upper bound for the total C_3-irregularity strength of TCl_m.

$$\varphi_3(x_i) = \left\lceil \frac{j+1}{2} \right\rceil, \text{for } j \in [1, m+1], \quad \varphi_3(y_i) = \left\lceil \frac{j+1}{2} \right\rceil, \text{for } j \in [1, m]$$

$$\varphi_3(z_j) = \left\lceil \frac{j}{2} \right\rceil, \text{for } j \in [1, m+1], \quad \varphi_3(x_jy_j) = \left\lceil \frac{j}{2} \right\rceil, \text{for } j \in [1, m]$$

$$\varphi_3(x_{j+1}y_j) = \left\lceil \frac{j}{2} \right\rceil, \text{for } j \in [1, m], \quad \varphi_3(y_jz_j) = \left\lceil \frac{j}{2} \right\rceil, \text{for } j \in [1, m]$$

$$\varphi_3(y_jz_{j+1}) = \left\lceil \frac{j+1}{2} \right\rceil, \text{for } j \in [1, m-1], \quad \varphi_3(x_jz_j) = \left\lceil \frac{j}{2} \right\rceil, \text{for } j \in [1, m+1]$$

$$\varphi_3(z_jz_{j+1}) = \left\lceil \frac{j}{2} \right\rceil, \text{for } j \in [1, m].$$

We get the upper bound from the function of C_3-irregular total TCl_m-labelling. We take the largest label from $\varphi_3(x_i) = \left\lceil \frac{j+1}{2} \right\rceil$ for $j = m+1$ $\varphi_3(x_i) = \left\lceil \frac{m+2}{2} \right\rceil$. We get to present the upper bound of the graph in the Theorem 3, $tHs((TCl_m), C_3) \leq \left\lceil \frac{3m+5}{6} \right\rceil$.

Based on the labeling above, we can show the all weights are different by the following equation:

$$\begin{aligned} wt_3(C_3^{j+1}) - wt\varphi_3(C_3^j) &= \varphi_3(x_{j+1}) + \varphi_3(y_{j+1}) + \varphi_3(z_{j+1}) + \varphi_3(x_{j+1}y_{j+1}) + \\ &\quad \varphi_3(x_{j+1}z_{j+1}) + \varphi_3(y_{j+1}z_{j+1}) - \varphi_3(x_j) - \varphi_3(y_j) - \\ &\quad \varphi_3(z_j) - \varphi_3(x_jy_j) - \varphi_3(x_jz_j) - \varphi_3(y_jz_j) \\ &= 3 \end{aligned}$$

$$\begin{aligned} wt_3(C_3^{j+1}) - wt\varphi_3(C_3^j) &= \varphi_3(z_{j+1}) + \varphi_3(y_{j+1}) + \varphi_3(z_{j+2}) + \varphi_3(y_{j+1}z_{j+1}) + \\ &\quad \varphi_3(y_{j+1}z_{j+2}) + \varphi_3(z_{j+1}z_{j+2}) - \varphi_3(z_j) - \varphi_3(y_jz_j) - \\ &\quad \varphi_3(y_j) - \varphi_3(z_{j+1}) - \varphi_3(y_jz_{j+1}) - \varphi_3(z_jz_{j+1}) \\ &= 3 \end{aligned}$$

$$\begin{aligned} wt_3(C_3^{j+1}) - wt\varphi_3(C_3^j) &= \varphi_3(y_{j+1}) + \varphi_3(x_{j+2}) + \varphi_3(z_{j+2}) + \varphi_3(y_{j+1}x_{j+2}) + \\ &\quad \varphi_3(y_{j+1}z_{j+2}) + \varphi_3(x_{j+2}z_{j+2}) - \varphi_3(y_j) - \varphi_3(x_{j+1}) - \\ &\quad \varphi_3(z_{j+1}) - \varphi_3(y_jx_{j+1}) - \varphi_3(y_jz_{j+1}) - \varphi_3(x_{j+1}z_{j+1}) \\ &= 3 \end{aligned}$$

We respect to $wt_{\varphi_n}(C_3^j) < wt_{\varphi_n}(C_3^{j+1})$, $j = 1, 2, \ldots, m$ then $wt\varphi_n(C_3^{j+1}) - wt\varphi_n(C_3^j) = 3$. The all H_2-weights are distinct. This matter concludes that $tHs((TCl_m), C_3) = \left\lceil \frac{3m+5}{6} \right\rceil$. The example of total C_3-irregularity of three circular ladder graph labeling, we can see on Figure 2, and we get $tHs((TCl_m), C_3) = 5$.

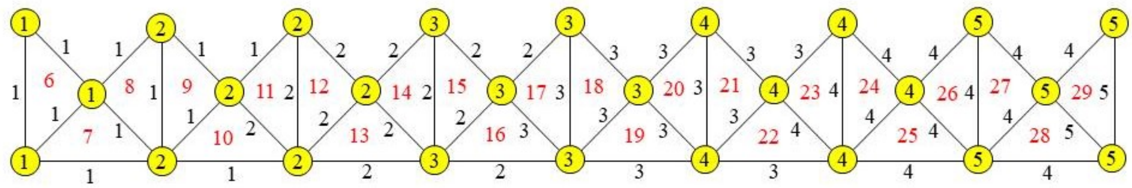

Figure 2. The Example of Total C_3-Irregularity of TCl_m labeling

Theorem 4. Let Pr_m be a prism graph and subgraph $H_3 \equiv C_4$. The total H_3-irregularity strength of Pr_m graph for $m \geq 3$, $m \equiv 0 \bmod 4$ and $m \equiv 1 \bmod 4$ is $\left\lceil \frac{n+7}{8} \right\rceil$.

Proof. Let Pr_m, $m \geq 3$, be a prism graph with the vertex set $V(Pr_m) = \{x_j, y_j : j = 1, 2, 3, \ldots, m\}$ and the edge set $E(Pr_m) = \{x_jx_{j+1}, y_jy_{j+1} : j = 1, 2, 3, \ldots, m-1\} \cup \{x_jy_j : j = 1, 2, 3, \ldots, m\} \cup \{x_mx_1\} \cup \{y_my_1\}$. The prism graph Pr_m, $m \geq 3$, contains a C_4-covering with exactly m cycles C_4. Based on Theorem 4, we have $tHs(Pr_m), C_4) \geq \left\lceil \frac{m+7}{8} \right\rceil$. Put $\alpha = \left\lceil \frac{m+7}{8} \right\rceil$. The following function of C_4-irregular total α-labeling $\varphi_4 : V(Pr_m) \cup E(Pr_m) \to \{1, 2, \ldots, \alpha\}$ is prove that α as an upper bound for the total C_4-irregularity strength of Pr_m.

A C_4-irregular total α-labeling $\varphi_4 : V(Pr_m) \cup E(Pr_m) \to \{1, 2, \ldots, \alpha\}$ is as follows:
for $j = 1, 2, \ldots, \left\lceil \frac{m}{2} \right\rceil$,

$$\begin{aligned} \varphi_4(x_j) &= \left\lceil \frac{j+2}{4} \right\rceil, & \varphi_4(y_j y_{j+1}) &= \left\lceil \frac{j}{4} \right\rceil \\ \varphi_4(y_j) &= \left\lceil \frac{j+2}{4} \right\rceil, & \varphi_4(x_j y_j) &= \left\lceil \frac{j}{4} \right\rceil \\ \varphi_4(x_j x_{j+1}) &= \left\lceil \frac{j+1}{4} \right\rceil, \end{aligned}$$

for $j = \left\lceil \frac{m}{2} \right\rceil + 1, \ldots, m-1$, and $m \equiv 0 \bmod 4$

$$\begin{aligned} \varphi_4(x_j) &= \left\lceil \frac{m}{4} \right\rceil - \left\lceil \frac{j}{4} \right\rceil + 2, & \varphi_4(y_j y_{j+1}) &= \left\lceil \frac{m}{4} \right\rceil - \left\lceil \frac{j}{4} \right\rceil + 1 \\ \varphi_4(y_j) &= \left\lceil \frac{m}{4} \right\rceil - \left\lceil \frac{j+1}{4} \right\rceil + 2, & \varphi_4(x_j y_j) &= \left\lceil \frac{m}{4} \right\rceil - \left\lceil \frac{j+2}{4} \right\rceil + 2 \\ \varphi_4(x_j x_{j+1}) &= \left\lceil \frac{m}{4} \right\rceil - \left\lceil \frac{j+3}{4} \right\rceil + 2, \end{aligned}$$

for $j = \left\lceil \frac{m}{2} \right\rceil + 1, \ldots, m-1$, and $m \equiv 1 \bmod 4$

$$\begin{aligned} \varphi_4(x_j) &= \left\lceil \frac{m}{4} \right\rceil - \left\lceil \frac{j+1}{4} \right\rceil + 2, & \varphi_4(y_j y_{j+1}) &= \left\lceil \frac{m}{4} \right\rceil - \left\lceil \frac{j+1}{4} \right\rceil + 1 \\ \varphi_4(y_j) &= \left\lceil \frac{m}{4} \right\rceil - \left\lceil \frac{j+2}{4} \right\rceil + 2, & \varphi_4(x_j y_j) &= \left\lceil \frac{m}{4} \right\rceil - \left\lceil \frac{j+3}{4} \right\rceil + 2 \\ \varphi_4(x_j x_{j+1}) &= \left\lceil \frac{m}{4} \right\rceil - \left\lceil \frac{j}{4} \right\rceil + 1, \end{aligned}$$

for $i = m$

$$\begin{aligned} \varphi_4(x_m) &= 2, & \varphi_4(x_m y_m) &= 1 \\ \varphi_4(y_m) &= 1, & \varphi_4(x_m x_1) &= 1 \\ \varphi_4(y_m y_1) &= 1. \end{aligned}$$

We get the upper bound from the function of C_4-irregular total Pr_m-labeling. We take from the largest label of graph . We get to present the upper bound of the graph in the Theorem 4, $tHs(Pr_m), C_4) \leq \left\lceil \frac{m+7}{8} \right\rceil$.

Based on the labeling above, we can show the all weights are different by the following equation:

for every $j = 1, 2, \ldots, \left\lceil \frac{m}{2} \right\rceil$, we have

$$\begin{aligned} wt\varphi_n(C_4^{j+1}) - wt\varphi_n(C_4^{j}) &= \varphi_4(x_{j+1}) + \varphi_4(x_{j+2}) + \varphi_4(x_{j+1}x_{j+2}) + \varphi_4(y_{j+1}) + \\ &\quad \varphi_4(y_{j+2}) + \varphi_4(y_{j+1}y_{j+2}) + \varphi_4(x_{j+1}y_{j+1}) + \varphi_4(x_{j+2}y_{j+2}) - \\ &\quad -\varphi_4(x_j) - \varphi_4(x_{j+1}) - \varphi_4(x_jx_{j+1}) - \varphi_4(y_j) - \varphi_4(y_{j+1}) - \\ &\quad \varphi_4(y_jy_{j+1}) - \varphi_4(x_jy_j) - \varphi_4(x_{j+1}y_{j+1}) \\ &= 2 \end{aligned}$$

for every $j = \left\lceil \frac{m}{2} \right\rceil + 1, \ldots, m-1$, we have

$$\begin{aligned} wt\varphi_n(C_4^{j+1}) - wt\varphi_n(C_4^{j}) &= \varphi_4(x_{j+1}) + \varphi_4(x_{j+2}) + \varphi_4(x_{j+1}x_{j+2}) + \varphi_4(y_{j+1}) + \\ &\quad \varphi_4(y_{j+2}) + \varphi_4(y_{j+1}y_{j+2}) + \varphi_4(x_{j+1}y_{j+1}) + \varphi_4(x_{j+2}y_{j+2}) - \\ &\quad \varphi_4(x_j) - \varphi_4(x_{j+1}) - \varphi_4(x_jx_{j+1}) - \varphi_4(y_j) - \varphi_4(y_{j+1}) - \\ &\quad \varphi_4(y_jy_{j+1}) - \varphi_4(x_jy_j) - \varphi_4(x_{j+1}y_{j+1}) \\ &= -2 \end{aligned}$$

for every $j = m$, we have

$$\begin{aligned} wt_{\varphi_4}(C_4^{m}) &= \varphi_4(x_m) + \varphi_4(y_m) + \varphi_4(x_my_m) + \varphi_4(x_mx_1) + \\ &\quad \varphi_4(y_my_1) + \varphi_4(x_1) + \varphi_4(y_1) + \varphi_4(x_1y_1) \\ &= 2+1+1+1+1+\left\lceil \frac{j+2}{4} \right\rceil + \left\lceil \frac{j+2}{4} \right\rceil + \left\lceil \frac{j}{4} \right\rceil \\ &= 6+\left\lceil \frac{1+2}{4} \right\rceil + \left\lceil \frac{1+2}{4} \right\rceil + \left\lceil \frac{1}{4} \right\rceil \\ &= 9 \end{aligned}$$

We respect to $wt_{\varphi_4}(C_4^{j}) < wt_{\varphi_4}(C_4^{j+1})$, $j = 1, 2, \ldots, m$. If every $j = 1, 2, \ldots, \left\lceil \frac{m}{2} \right\rceil$ then $wt_{\varphi_4}(C_4^{j+1}) - wt_{\varphi_4}(C_4^{j}) = 2$. If every $j = \left\lceil \frac{m}{2} \right\rceil + 1, \ldots, m-1$ then $wt_{\varphi_4}(C_4^{j+1}) - wt_{\varphi_4}(C_4^{j}) = -2$. If every $j = m$ then $wt_{\varphi_4}(C_4^{m}) = 9$. The all H_3-weights are distinct. This matter concludes that $tHs((Pr_m), C_4) = \left\lceil \frac{m+7}{8} \right\rceil$. We know that example of total C_4-irregularity of prism graph on Figure 3, and we get $tHs((Pr_{16}), C_4) = 2$ which j is even. But We can see the example of total C_4-irregularity of prism graph labeling on Figure 4, and we get $tHs((Pr_{17}), C_4) = 2$ which j is odd.

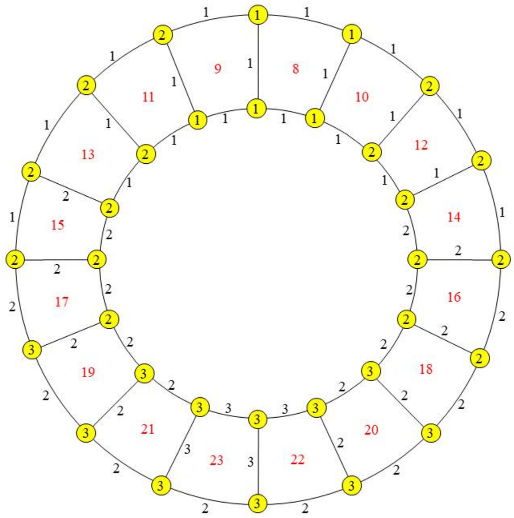

Figure 3. The Example of Total C_4-Irregularity of Pr_{16} labeling if j is even

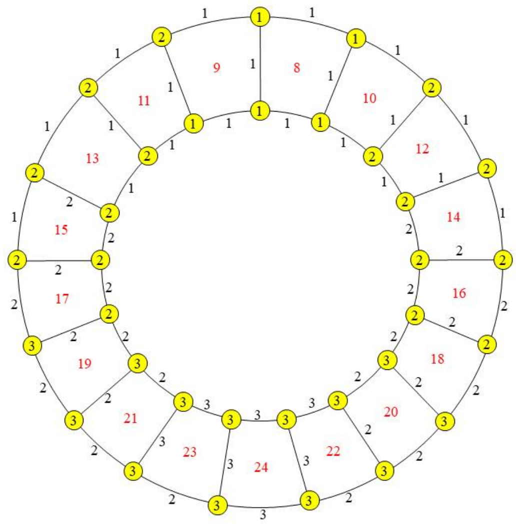

Figure 4. The Example of Total C_4-Irregularity of Pr_{17} labeling if j is odd

3. Concluding Remarks

In this research we have obtained of the total H-irregularity strength of prism graphs, diamond ladder graphs, and three circular ladder graphs. We recognize H-covering on prism graphs and three circular ladder graphs for which H is cyclical. But, we recognize H-covering on diamond ladder graph that H is a diamond ladder graph.

Open Problem 1 *Find the total H-Irregularity Strength (tHs) of the Pr_m, $m \geq 3$.*

Acknowledgment

We gratefully acknowledge CGANT University of Jember 2019.

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