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On r-dynamic local irregularity vertex coloring of special graphs

I N Maylisa[1,2], Dafik[1,2], A F Hadi[2,5], A I Kristiana[1,3], R Alfarisi[1,4],

[1]CGANT-Research Group, University of Jember, Indonesia
[2]Department of Mathematics Education Post Graduate, University of Jember, Indonesia
[3]Department of Mathematics Education, University of Jember, Indonesia
[4]Department of Primary Education, University of Jember, Indonesia
[5]Department of Mathematics, University of Jember, Indonesia

E-mail: `ikamaylisa46@gmail.com`

Abstract. In this paper, we study a new notion of graph coloring, that is a local irregularity vertex r-Dynamic coloring. We combine irregular local and the principles of r-dynamic coloring and we assign color to all vertices by using the weights of local irregularity vertex. We define $l : V(G) \rightarrow \{1, 2, ..., k\}$ as a vertex irregular k-labeling and $w : V(G) \rightarrow N$ where $w(u) = \sum_{v \in N(u)} l(v)$. By a local irregularity vertex coloring, we define a condition for f if for every $uv \in E(G), w(u) \neq w(v)$ and $max(l) = min\{max\{l_i\}; l_i$ vertex irregular labeling$\}$. Each vertex of the weight as a color should satisfy the r-dynamic condition, namely $|w(N(v))| \geq min\{r, d(v)\}$ and each the adjacent vertices must be different. In this paper, we study the local irregularity vertex r-Dynamic coloring of special graph, namely triangular book graph, central of friendship graph, tapol graph, and rectangular book graph.

1. Introduction

One of popular topic in graph theory is graph labeling, especially graph coloring. In this paper we discuss the newest topic in graph coloring, namely local irregularity vertex r-dynamic coloring. Kristiana, et.al [9] defined local irregularity vertex coloring. Suppose $l : V(G) \rightarrow \{1, 2, ..., k\}$ be called vertex irregular k-labeling and $w : V(G) \rightarrow N$ where $w(u) = \Sigma_{v \in N(u)} l(v), l$ be called local irregularity vertex coloring, if $max(l) = min\{maks\{l_i\}; l_i$ vertex irregular labeling$\}$ and for every $uv \in E(G), w(u) \neq w(v)$. Furthermore Kristiana, et.al [10] founded r-dynamic local irregularity vertex coloring of path graph, cycle graph, and complete graph. In this paper, we combine the two concepts, that are local irregularity vertex coloring and vertex r-dynamic coloring.

We use the set of natural numbers {1,2,...,k} to give label in each vertex. In this labeling, numbers can be used repeatedly because they're used irregular concepts. And

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than, we calculate the weight of each vertex. The weight of u is the sum of the labels of all vertices adjacent to u (distance 1 from u), that is $w(u) = \Sigma_{y \in N(u)} \lambda(y)$. In graph $G = (u, v)$, $w(u) \neq w(v)$ if they're adjacent. The last, to find the vertex coloring pattern, we use the principle of vertex coloring r-dynamic, by assuming that the weight of the vertex be the color of thats vertex (do this step until we have the general pattern of r-dynamic color).

Definition 1. *Let $\chi : V(G) \to \{1, 2, ..., k\}$ be called vertex irregular k-labeling and $w : V(G) \to N$ where $w(u) = \Sigma_{v \in N(u)} \chi(v), \chi$ be called r-dynamic local irregular vertex coloring, if:*
(i) $max(\chi) = min\{maks\{\chi_i\}; \chi_i$ vertex irregular k-labeling$\}$.
(ii) for every $uv \in E(G), w(u) \neq w(v)$.
(iii) for every vertex in G, $|w(N(v))| \geq min\{r, d(v)\}$.

Definition 2. *The chromatic number r-dynamic local irregular denoted by $\chi_r^{lis}(G)$, is minimum of cardinality r-dynamic local irregular vertex coloring, defined by $\chi_r^{lis}(G) = min\{|w(V(G))|; w\}$ local irregular vertex r-dynamic coloring .*

Observation 1. *Let u and w be any two adjacent vertices in a connected graph G. If $N(u) - \{w\} = N(w) - \{u\}$ then the labels of u and w must be distinct, namely $l(u) \neq l(w)$.*

2. Main Results

We study the presence of local irregularity vertex r-dynamic coloring of triangular book graph, central of friendship graph, tapol graph, and rectangular book graph.

Theorem 1. *Let Tb_n for $n \geq 2$. The local irregular vertex r-dynamic chromatic number of Tb_n is:*

$$\chi_{lis}^r(Tb_n) = \begin{cases} 3, 1 \leq r \leq 2 \\ \sim, r \geq 3 \end{cases}$$

Proof. Let Tb_n be a connected graph with $V(Tb_n) = \{x, y\} \cup \{x_i; 1 \leq i \leq n\}$ and $E(Tb_n) = \{xy\} \cup \{xx_i; 1 \leq i \leq n\} \cup \{yx_i; 1 \leq i \leq n\}$, $|V(Tb_n) = n+2$, $|E(Tb_n)| = 2n+1$, $\partial(Tb_n) = 2$, and $\triangle(Tb_n) = n+1$. In order to proof the r-dynamic chromatic number of this graph, we devide to two cases since they have different condition.
Case 1: for $1 \leq r \leq 2$

If every vertex $x, y \in V(Tb_n)$ is labeled by 1, then $w(x) = w(y) = n+1$. It contradicts to the $Definition 2.1$. It implies that $l(x) = 1$ and $l(y) = 2$ or $l(x) = 2$ and $l(y) = 1$, so $w(x) \neq w(y)$. We have $max(l) = 2$. We prove that the lower bound of chromatic number local irregular r-dynamic is $\chi_{lis}^r(Tb_n) \geq 3$. We can't use $\chi_{lis}^r(Tb_n) < 3$ since Tb_n has a subgraph C_3. $\chi(C_3) = 3$, thus $\chi_r \geq \chi \geq 3$. It means, $\chi_r \geq 3$. Furthermore, we prove that $\chi_r \leq 3$, define the function $l : V(Tb_n) \to \{1, 2\}$ as follows:

$$l(v) = \begin{cases} 1; v = x \text{ and } x_i, 1 \leq i \leq n \\ 2; v = y \end{cases}$$

Based on the label in vertex of triangular book graph (Tb_n), we have the vertex weights of Tb_n as follows

$$w(v) = \begin{cases} n+2; v = x \\ n+1; v = y \\ 3; v = x_i, 1 \leq i \leq n \end{cases}$$

It is clear that $\chi^r_{lis}(Tb_n) \leq 3$. Thus, we have the local irregularity vertex r-dynamic of triangular book graph $\chi^r_{lis}(Tb_n) = 3$ for $1 \leq r \leq 2$.

Case 2: for $r \geq 3$

The triangular book graph Tb_n has a subgraph C_3, we know that $\chi(C_3) = 3$. It means that $\chi_r(Tb_n) \geq 3$ such that we use 3 colors in triangular book graph (Tb_n). Such that, every vertex x, y and x_i in Tb_n have different colors. If $l(x) = l(y)$, then $w(x) = w(y)$, it is contradicts. It implies that $l(x) = 1$ and $l(y) = 2$ or $l(x) = 2$ and $l(y) = 1$, so $w(x) \neq w(y)$. In Observation 1, if u and w be any two adjacent vertices in Tb_n and $N(u) - \{w\} = N(w) - \{u\}$, then the labels of u and w must be distinct, that is $l(u) \neq l(w)$. By observation 1,

$$|N(x)| = |\{w(y), w(x_i)\}| = 2 \tag{1}$$

By the definition of r-dynamic coloring that

$$|N(x)| \leq min\{r, n+1\} = r \tag{2}$$

Based on the equation (1) and (2) that $|N(x)| = 2 \neq r$. Thus, $\chi^r_{lis}(Tb_n) = \sim$ for $r \geq 3$.

Theorem 2. *Let $C(F_n)$ for $n \geq 4$. The local irregularity vertex r-dynamic chromatic number of $C(F_n)$ is:*

$$\chi^r_{lis}(C(F_n)) = \begin{cases} 3, r = 1 \\ 5, r = 2 \\ 7, r \geq 3 \end{cases}$$

Proof. Let $C(F_n)$ be a connected graph with $V(C(F_n)) = \{x_i; 1 \leq i \leq 2n+1\} \cup \{y_i; 1 \leq i \leq n+1\} \cup \{z\}$ and $E(C(F_n)) = \{x_i x_(i+1); 1 \leq i \leq 2n\} \cup \{x_i y_i; 1 \leq i \leq n+1\} \cup \{y_i z; 1 \leq i \leq n+1\}$, $|V(C(F_n)) = 3n+3$, $|E(C(F_n))| = 4n+2$, $\partial(C(F_n)) = 2$, and $\triangle(C(F_n)) = n+1$. In order to proof the r-dynamic chromatic number of this graph, we devide to three cases since they have different condition.

Case 1: for $r = 1$

If vertex in $C(F_n)$ is labeled by 1, then $w(x) = w(y) = n+1$. It contradicts to the $Definition$2.1. It implies that $l(x)$ and $l(y)$ must be coloring by different color, so $w(x) \neq w(y)$. We have $max(l) = 2$. We prove that the lower bound of chromatic number local irregularity vertex r-dynamic coloring is $\chi^r_{lis}(C(F_n)) \geq 3$. Furthermore, we prove that $\chi^r_{lis}(C(F_n)) \leq 3$ as the upper bound in $r = 1$, define the function $l : V(C(F_n)) \rightarrow$ {1,2,3,4,5,... $|V(C(F_n))|$} for $n =$ even and $n \geq 4$ as follows:

$$l(x_i) = \begin{cases} 1 \; ; \; i \; = \; 0,1,3 \bmod 4 \\ 2 \; ; \; i \; = \; 2 \bmod 4 \end{cases}$$

$$l(y_i) = 1; l(z) = 1$$

Hence l is a local irregularity vertex r-dynamic coloring of $C(F_n)$ and the function of vertex weights are: For $n =$ even and $n \geq 4$:

$$w(x_i) = \begin{cases} 3 \ ; \ i & = 1, n \\ 2 \ ; \ i & = \text{ even} \\ 4 \ ; \ i & = \text{ odd} \end{cases}$$

$$w(y_i) = 2; w(z) = n$$

Clearly, w is local irregularity vertex r-dynamic coloring of central of friendship graph. Furthermore, $\chi^1_{lis}(C(F_n)) = 3$
Case 2: for $r = 2$

Based on $Definition$ 1 the lower bound chromatic number r-dynamic is $\chi^2_{lis}(C(F_n)) \geq 5$. We can't use color less than 5 because it contradict with $Definition$ 2.1. Furthermore, it shown the upper bound is $\chi^2_{lis}(C(F_n)) < 5$, define the function l : $V(C(F_n)) \rightarrow$ {1,2,3,4,5,... $|V(C(F_n))|$} for $n =$ even and $n \geq 4$ as follows :

$$l(x_i) = \begin{cases} 1 \ ; \ i & = \ 0, 1 \bmod 6 \\ 2 \ ; \ i & = \ 2, 3, 4, 5 \bmod 6 \end{cases} ; l(y_i) = \begin{cases} 1 \ ; \ i & = \ 2 \bmod 4 \\ 2 \ ; \ i & = \ 1, 3 \bmod 4 \\ 3 \ ; \ i & = \ 0 \bmod 4 \end{cases}$$

$$l(z) = 1$$

Hence l is a local irregularity vertex r-dynamic coloring of central of friendship graph and the function of vertex weights are: For $n =$ even and $n \geq 4$:

$$w(x_i) = \begin{cases} 4 \ ; \ i & = 1, 4 \bmod 6 \\ 3 \ ; \ i & = \ 0, 2 \bmod 6 \\ 5 \ ; \ i & = \ 3, 5 \bmod 6 \end{cases} ; w(y_i) = \begin{cases} 2 \ ; \ i & = \ 1 \bmod 3 \\ 3 \ ; \ i & = \ 0, 2 \bmod 3 \end{cases}$$

$$w(y_i) = \begin{cases} 2 \ ; \ i & = \ 1 \bmod 3 \\ 3 \ ; \ i & = \ 0, 2 \bmod 3 \end{cases} ; w(z) = 2n$$

Clearly, w is local irregularity vertex r-dynamic coloring of central of friendship graph. Furthermore, $\chi^2_{lis}(C(F_n)) = 5$
Case 3: for $r \geq 3$

Based on $Definition$ 1 the lower bound chromatic number r-dynamic is $\chi^r_{lis}(C(F_n)) \geq 7$. We can't use color less than 7 because it contradict with $Definition$ 2.1. Furthermore, it shown the upper bound is $\chi^r_{lis}(C(F_n)) \leq 7$, define a bijection l : $V(C(F_n)) \rightarrow$ {1,2,3,... $|V(C(F_n))|$} for $r \geq 3$; $n =$ even and $n \geq 4$ with the following function :

$$l(x_i) = \begin{cases} 1 \ ; \ i & = \ 1 \bmod 6 \\ 2 \ ; \ i & = \ 2 \bmod 6 \\ 3 \ ; \ i & = \ 3, 4 \bmod 6 \\ 4 \ ; \ i & = \ 0, 5 \bmod 6 \end{cases}$$

$$l(y_i) = 1; l(z) = max\{l_i\} + 1$$

Hence l is a local irregularity vertex r-dynamic coloring of $C(F_n)$ and the function of vertex weights are : For n = even and $n \geq 4$:

$$w(x_i) = \begin{cases} 3 \ ; \ i \ = \ 1 \\ 5 \ ; \ i \ = \ n \\ 4 \ ; \ i \ = \ 2, n-1 \\ 6 \ ; \ i \ = \ 1 \text{ mod } 3 \\ 7 \ ; \ i \ = \ 2 \text{ mod } 3 \\ 8 \ ; \ i \ = \ 0 \text{ mod } 3 \end{cases} ; w(y_i) = \begin{cases} 6 \ ; \ i \ = \ 1 \text{ mod } 4 \\ 8 \ ; \ i \ = \ 2 \text{ mod } 4 \\ 9 \ ; \ i \ = \ 3 \text{ mod } 4 \\ 8 \ ; \ i \ = \ 0 \text{ mod } 4 \end{cases}$$

$$w(z) = 4n$$

Clearly, w is local irregularity vertex r-dynamic coloring of central of friendship graph. Furthermore, $\chi^r_{lis}(C(F_n)) = 7; r \geq 3$

Theorem 3. *Let $T_{n,m}$ be a tapol graph with $n \geq 8$; n = even, and $m \geq 5; m$ = odd, $\chi_r^{lis}(T_{n,m})$ is:*

$$\chi_r^{lis}(T_{n,m}) \leq \begin{cases} 4 \ ; \ r = 1 \\ 5 \ ; \ r \geq 2 \end{cases}$$

Proof. Let $T_{n,m}$ be a special graph with $V(T_{n,m}) = \{x_i; 1 \leq i \leq n\} \cup \{y_i; 1 \leq i \leq n\}$ and $E(T_{n,m}) = \{x_i x_(i+1); 1 \leq i \leq n-1\} \cup \{x_i x_n\} \cup \{x_1 y_1\} \cup \{y_i y_(i+1); 1 \leq i \leq n-1\}$, $|V(T_{n,m}) = n+m$, $|E(T_{n,m})| = n+m$, $\partial(T_{n,m}) = 1$, and $\triangle(T_{n,m}) = 3$. In order to proof the r-dynamic chromatic number of this graph, we split to two cases since they have different condition.

Case 1: for $r = 1$

Based on $Definition$ 1 the lower bound chromatic number r-dynamic is $\chi^r_{lis}(T_{n,m}) \geq 7$. We can't use color less than 7 because it contradict with $Definition$ 2.1. Furthermore, it shown the upper bound is $\chi^r_{lis}(C(F_n)) \leq 7$, define a bijection $l : V(T_{n,m}) \to$ {1,2,3,... $|V(T_{n,m})|$} for n = even; $n \geq 8$ and m = odd; $m \geq 5$ with the following function :

$$l(x_i) = \begin{cases} 1 \ ; \ i \ = \ 1, 2, 3 \text{ mod } 4 \\ 2 \ ; \ i \ = \ 2 \text{ mod } 4 \end{cases} ; l(y_i) = \begin{cases} 1 \ ; \ i \ = \ 1, 2, \text{ mod } 3 \\ 2 \ ; \ i \ = \ 0 \text{ mod } 3 \end{cases}$$

Hence l is a local irregularity vertex r-dynamic coloring of $T_{n,m}$ and the function of vertex weights are : For n = even; $n \geq 8$ and m = odd; $m \geq 5$:

$$w(x_i) = \begin{cases} 4 \ ; \ i \ = \ 1 \\ 2 \ ; \ i \ = \ \text{even} \\ 3 \ ; \ i \ = \ \text{odd} \ , i \geq 3 \end{cases} ; w(y_i) = \begin{cases} 2 \ ; \ i \ = \ \text{odd}; 1 \leq i \leq n \\ 3 \ ; \ i \ = \ \text{even} \ ; 1 \leq i \leq n \\ 1 \ ; \ i \ = \ w(y_{n-1}) \end{cases}$$

Clearly, w is local irregularity vertex r-dynamic coloring of tapol graph. Furthermore, $\chi^1_{lis}(T_{n,m}) = 4$

Case 2: for $r \geq 2$

Based on $Definition$ 1 the lower bound chromatic number r-dynamic is $\chi^r_{lis}(T_{n,m}) \geq 7$. We can't use color less than 7 because it contradict with $Definition$ 2.1. Furthermore, it shown the upper bound is $\chi^r_{lis}(C(F_n)) < 7$, define the function $l: V(T_{n,m}) \rightarrow$ {1,2,3,... $|V(T_{n,m})|\}$ for $n =$ even; $n \geq 8$ and $m =$ odd; $m \geq 5$ as follows :

$$l(x_i) = \begin{cases} 1 \ ; \ i \ = \ 1,2,7 \bmod 8 \\ 2 \ ; \ i \ = \ 0,3 \bmod 8 \\ 3 \ ; \ i \ = \ 4,5,6, \bmod 8 \end{cases} ; l(y_i) = \begin{cases} 3 \ ; \ i \ = \ 1, \text{even} \ ; 1 \leq i \leq m \\ 2 \ ; \ i \ = \ \text{odd} \ ; 3 \leq i \leq m \end{cases}$$

Hence l is a local irregularity vertex r-dynamic coloring of $T_{n,m}$ and the function of vertex weights are : For $n =$ even; $n \geq 8$ and $m =$ odd; $m \geq 5$:

$$w(x_i) = \begin{cases} 3 \ ; \ i \ = \ 1 \bmod 8 \\ 4 \ ; \ i \ = \ 2,5 \bmod 8 \\ 5 \ ; \ i \ = \ 3,6 \bmod 8 \\ 6 \ ; \ i \ = \ 0,4 \bmod 8 \\ 2 \ ; \ i \ = \ 7 \bmod 8 \end{cases} ; w(y_i) = \begin{cases} 4 \ ; \ i \ = \ 1 \bmod 3 \\ 5 \ ; \ i \ = \ 2 \bmod 3 \\ 6 \ ; \ i \ = \ 0 \bmod 3 \\ l(y_i) \ ; \ i \ = \ n \end{cases}$$

Clearly, w is local irregularity vertex r-dynamic coloring of tapol graph. Furthermore, $\chi^1_{lis}(T_(n,m)) = 5; r \geq 2$

Theorem 4. *Let Rb_n be a connected graph with $n \geq 4$, $\chi^r_{lis}(G)$ of Rb_n is :*

$$\chi^r_{lis}(Rb_n) \leq \begin{cases} 4, 1 \leq r \leq 2 \\ 5, r = 3 \\ 6, r = 4 \\ 7, r \geq 5 \end{cases}$$

Proof. Rb_n is a connected graph with $V(Rb_n) = \{x_i; 1 \leq i \leq n\} \cup \{y_i; 1 \leq i \leq n\} \cup \{z_i; 1 \leq i \leq 2\}$ and $E(Rb_n) = \{x_i z_1; 1 \leq i \leq n\} \cup \{y_i z_2; 1 \leq i \leq n\} \cup \{x_i y_i; 1 \leq i \leq n\}$. $|V(Rb_n)| = 2n + 2$ and $|E(Rb_n)| = 3n + 1$. $\partial(Rb_n) = 2$, and $\triangle(Rb_n) = n + 1$. In order to proof the r-dynamic chromatic number of this graph, we split to four cases since they have different condition.

Case 1: for $1 \leq r \leq 2$

Based on $Definition$ 1 the lower bound chromatic number r-dynamic is $\chi^r_{lis}(T_{n,m}) \geq 7$. We can't use color less than 7 because it contradict with $Definition$ 2.1. Furthermore, it shown the upper bound is $\chi^r_{lis}(C(F_n)) < 7$, define a bijection $l: V(Rb_n) \rightarrow$ {1,2,3,... $|V(Rb_n)|\}$ for $n \geq 4$ with the following function :

$$l(x_i) = \begin{cases} 1 \ ; \ i \ = \ 1 \\ 2 \ ; \ i \ = \ 2 \end{cases} ; l(y_i) = 1; l(z_i) = \begin{cases} 1 \ ; \ i \ = \ \text{odd} \\ 2 \ ; \ i \ = \ \text{even} \end{cases}$$

Hence l is a local irregularity vertex r-dynamic coloring of Rb_n and the function of vertex weights are : For $n \geq 4$:

$$w(x_i) = \begin{cases} n+2 \ ; \ i \ = \ 1 \\ n+1 \ ; \ i \ = \ 2 \end{cases} ; w(y_i) = \begin{cases} 2 \ ; \ i \ = \ \text{odd} \\ 3 \ ; \ i \ = \ \text{even} \end{cases}$$

$$w(z_i) = 3$$

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Clearly, w is local irregularity vertex r-dynamic coloring of Rb_n. Furthermore, $\chi^r_{lis}(Rb_n) = 4; 1 \leq r \leq 2$

Case 2: for $r = 3$

Based on $Definition$ 1 the lower bound chromatic number r-dynamic is $\chi^r_{lis}(T_{n,m}) \geq 7$. We can't use color less than 7 because it contradict with $Definition$ 2.1. Furthermore, it shown the upper bound is $\chi^r_{lis}(C(F_n)) < 7$, define a bijection $l : V(Rb_n) \rightarrow \{1,2,3,... |V(Rb_n)|\}$ for $n \geq 4$ with the following function :

$$l(x_i) = \begin{cases} 1 \; ; \; i \; = \; 1 \\ 2 \; ; \; i \; = \; 2 \end{cases} ; l(y_i) = \begin{cases} 1 \; ; \; i \; = \; 1,2 \bmod 4 \\ 2 \; ; \; i \; = \; 0,3 \bmod 4 \end{cases}$$

$$l(z_i) = \begin{cases} 1 \; ; \; i \; = \; \text{odd} \\ 2 \; ; \; i \; = \; \text{even} \end{cases}$$

Hence l is a local irregularity vertex r-dynamic coloring of Rb_n and the function of vertex weights are : For $n \geq 4$:

$$w(x_i) = \begin{cases} 2n \; ; \; i \; = \; 1 \\ 2n - 1 \; ; \; i \; = \; 2 \end{cases}$$

$$w(y_i) = \begin{cases} 2 \; ; \; i \; = \; \text{odd} \\ 3 \; ; \; i \; = \; \text{even} \end{cases} ; w(z_i) = \begin{cases} 3 \; ; \; i \; = \; 1,2 \bmod 4 \\ 4 \; ; \; i \; = \; 0,3 \bmod 4 \end{cases}$$

Clearly, w is local irregularity vertex r-dynamic coloring of Rb_n. Furthermore, $\chi^3_{lis}(Rb_n) = 5$.

Case 3: for $r = 4$

Based on $Definition$ 1 the lower bound chromatic number r-dynamic is $\chi^r_{lis}(T_{n,m}) \geq 7$. We can't use color less than 7 because it contradict with $Definition$ 2.1. Furthermore, it shown the upper bound is $\chi^r_{lis}(C(F_n)) < 7$, define a bijection $l : V(Rb_n) \rightarrow \{1,2,3,... |V(Rb_n)|\}$ for $n \geq 4$ with the following function :

$$l(x_i) = 1; 1 \leq i \leq 2$$

$$l(y_i) = \begin{cases} 2 \; ; \; i \; = \; 1 \bmod 3 \\ 1 \; ; \; i \; = \; 2 \bmod 3 \\ 4 \; ; \; i \; = \; 0 \bmod 3 \end{cases} ; l(z_i) = \begin{cases} 1 \; ; \; i \; = \; 1 \bmod 3 \\ 2 \; ; \; i \; = \; 2 \bmod 3 \\ 3 \; ; \; i \; = \; 0 \bmod 3 \end{cases}$$

Hence l is a local irregularity vertex r-dynamic coloring of Rb_n and the function of vertex weights are : For $n \geq 4$:

$$w(x_i) = \begin{cases} 2n + 2 \; ; \; i \; = \; 1 \\ 2n \; ; \; i \; = \; 2 \end{cases}$$

$$w(y_i) = \begin{cases} 2 \; ; \; i \; = \; 1 \bmod 3 \\ 3 \; ; \; i \; = \; 2 \bmod 3 \\ 4 \; ; \; i \; = \; 0 \bmod 3 \end{cases} ; w(z_i) = \begin{cases} 3 \; ; \; i \; = \; 1 \bmod 3 \\ 2 \; ; \; i \; = \; 2 \bmod 3 \\ 5 \; ; \; i \; = \; 0 \bmod 3 \end{cases}$$

Clearly, w is local irregularity vertex r-dynamic coloring of Rb_n. Furthermore, $\chi^4_{lis}(Rb_n) = 6$.

Case 4: for $r \geq 5$

Based on $Definition$ 1 the lower bound chromatic number r-dynamic is $\chi^r_{lis}(T_{n,m}) \geq 7$. We can't use color less than 7 because it contradict with $Definition$ 2.1. Furthermore, it shown the upper bound is $\chi^r_{lis}(C(F_n)) < 7$, define a bijection $l : V(Rb_n) \to \{1,2,3,...|V(Rb_n)|\}$ for $n \geq 4$ with the following function :

$$l(x_i) = 1; 1 \leq i \leq 2$$

$$l(y_i) = \begin{cases} 2 \ ; \ i \ = \ 1 \bmod 4 \\ 3 \ ; \ i \ = \ 2 \bmod 4 \\ 4 \ ; \ i \ = \ 3 \bmod 4 \\ 5 \ ; \ i \ = \ 0 \bmod 4 \end{cases} ; l(z_i) = \begin{cases} 1 \ ; \ i \ = \ 1 \bmod 4 \\ 2 \ ; \ i \ = \ 2 \bmod 4 \\ 3 \ ; \ i \ = \ 3 \bmod 4 \\ 4 \ ; \ i \ = \ 0 \bmod 4 \end{cases}$$

Hence l is a local irregularity vertex r-dynamic coloring of Rb_n and we have the vertex weight as follows : For $n \geq 4$:

$$w(x_i) = \begin{cases} 4n-1 \ ; \ i \ = \ 1 \\ 3n-1 \ ; \ i \ = \ 2 \end{cases}$$

$$w(y_i) = \begin{cases} 2 \ ; \ i \ = \ 1 \bmod 4 \\ 3 \ ; \ i \ = \ 2 \bmod 4 \\ 4 \ ; \ i \ = \ 3 \bmod 4 \\ 5 \ ; \ i \ = \ 0 \bmod 4 \end{cases} ; w(z_i) = \begin{cases} 3 \ ; \ i \ = \ 1 \bmod 4 \\ 4 \ ; \ i \ = \ 2 \bmod 4 \\ 5 \ ; \ i \ = \ 3 \bmod 4 \\ 6 \ ; \ i \ = \ 0 \bmod 4 \end{cases}$$

Clearly, w is local irregularity vertex r-dynamic coloring of Rb_n. Finally, $\chi^5_{lis}(Rb_n) = 7; r \geq 5$.

For an example, local irregularity vertex r-dynamic coloring of Triangular Book (Tb_n) is provided in Figure 1.

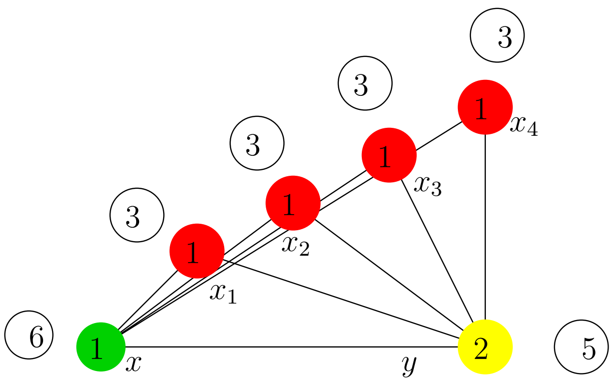

Figure 1. Local Irregularity Vertex r-Dynamic Coloring for $r = 1$ and $r = 2$

3. Concluding Remarks

With this paper, we have found the exact value from local irregularity vertex r-dynamic coloring of some special graphs, namely triangular book graph, central of friendship graph, tapol graph and rectangular book graph.

Open Problem 1. *Let G be a special graph, determine the local irregularity vertex r-dynamic coloring of G, especially for other special graphs and their operation!*

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