

Deduction level of undergraduate students' imagination in solving geometrical problem

M Mahfut¹, Sunardi², E Yudianto², R C Purnomo¹ and F F Firmansyah¹

¹Master Student in the Department of Mathematics Education, University of Jember

²Department of Mathematics Education, University of Jember

Corresponding author: mahfut25@gmail.com

Abstract. This research aims to know the imagination process on Mathematics' undergraduate students who had deduction level in solving Geometrics' problem. This research is qualitative which the subjects in the research were 2 students that one female and one male. To know the imagination process, subjects were asked to solve two Geometrics' problems. Analysis was done to determine the indicators of imagination that appeared for each subjects. To improve the data accuracy, the researcher used triangulated data method for the data collection method, such as observation, students' worksheet, and interview. The result showed that there were differences that appeared for each subjects.

1. Introduction

Imagination is someone's ability to draft the picture or idea about an event which they have never seen or experienced before in their thought[1]. Liang et al., divided imagination into two. They are reproductive and creative imagination, composed those two types into some indicators, such as transformation, crystallization, effectiveness, elaboration, exploration, intuition, innovation, productivity, and sensitivity[2].

This research about imagination in education is not the first time, as the example are the researches which were conducted [2,3,4,5,6]. Those researches showed the importance of imagination in solving a problem, especially in Mathematics. It was caused by the immateriality of content in Mathematics or it only existed in people's thought. Alphen said that someone would not be able to understand the history or even problem in Mathematics orally if that person did not have imagination[7]. Based on some saying from the experts before, it can be inferred that imagination is important thing in understanding Mathematics. One of Mathematics' branches is Geometrics. It studies about geometry.

Van Hiele is an influential person in Geometrics. He leveled someone's geometrics' ability into 5 stages. One of the stages is deduction. Haviger and Vojtkůvková explained that someone who was in deduction level could give geometrical evidence deductively[8]. Moreover, that person could understand and applied the use of definition, theorem, and axiom. This article will explain the process of undergraduate students' imagination in deduction level in solving geometrical problem. Therefore, this research aims to explain the imagination process of undergraduate students in solving geometrical problem.



2. Basic Theory

Imagination

Imagination as a new way to provide “natural house” for something that is in thought[9]. While imagination is people ability to draft the picture or idea in their thought about an event that they have ever seen or experienced before[1]. It was clarified by Wibowo et al., who explained that imagination is an ability to shape ideas in solving problem[6]. Imagination is a basic in building creative thinking and promoting innovation[2]. Imagination developed people creative ideas become product or object based on those ideas [10]. Imagination can make people travel world, solve problem, and develop new skill which is within their selves [11]. Imagination affects someone’s idea, language, and experience every day[12, 13, 14, 15, 16, 17]. Based on the previous explanation, it can be concluded that imagination is someone ability to shape the picture or idea in solving a problem.

Liang et al., divided imagination into two types; they are creative and reproductive imagination. Creative imagination is an ability to manage received objects and compile them by using any ways. Meanwhile, reproductive imagination is an ability to maintain simple impression from object. Reproductive imagination is had by people in various levels[2]. From both types of imagination, some indicators were composed. It is in Table 1.

Table 1. Indicator of Imagination

Type of Imagination	Indicator	Explanation
Creative Imagination	Intuition	Individual can associate with the target object soon.
	Sensitivity	Individual is able to issue or move feelings during creation process.
	Productivity	Individual can produce lots idea about an object.
	Exploration	Individual can explore or imagine something that never has been known.
	Innovation	Individual can create anti-mainstream or idea that never has been made.
Reproductive Imagination	Focus	Individual is able to shape structure or shape from an idea through focus.
	Effectiveness	Individual is able to build effective ideas about object or target theme.
	Transformation	Individual can do a task by changing what they had known in any disciplines.
	Crystallization	Individual is able to express abstract ideas by using concrete examples.
	Dialectics	Individual is able to develop idea or object by using logic analysis.

Van Hiele’s Theory

Van Hiele is influential person in Mathematics. He divided someone’s ability in Geometrics into 5 stages, such as; level 0 (visual), level 1 (analysis), level 2 (informal deduction), level 3 (deduction), and level 4 (Rigor). Haviger and Vojkůvková explained that students who were in deduction level

were able to differentiate whether the needed condition or enough condition[18]. They identified characteristics which were implied by others. They understood the use of definition, theorem, axiom, and evidence. The researchers found some indicators on deduction level students, they are ability to correct (clarify) ambiguous problem to proper language, reasoning and answering to verify the problem, ability to understand Mathematics' terms such as axiom, definition, theorem, and evidence, and implicit understanding about postulate of Euclidean Geometry[19, 20, 21, 22, 23, 24].

3. Methods

Subject

To achieve the expected aim in this research, undergraduate students of Mathematics education department of Jember University which have deduction level based on van Hiele's test was chosen. It was started from giving van Hiele's test to 140 undergraduate students. The result showed there were 14 students in level 0 (pre-visualization), 21 students in level 1 (visualization), 39 students in level 2 (analysis), 61 students in level 3 (informal deduction), and 5 students in level 4 (deduction), while there was no student in level 5 (rigor). 2 students who were in deduction level were chosen as the research subject. The selection was based on some aspects such as fluent communication, flexible time, and the subject's willingness to be involved in research.

Data Collection

Data collection in this research was done through test and interview. There are two test methods that used by van Hiele to select research subject and Geometrics problem test. Van Hiele's test that used was taken from Usiskin that had been translated to Bahasa Indonesia by Sunardi at 2010. Then, the second test was Geometrics problem test. In this test, the subject had been given two problems of Geometrics, then they were asked to solve those problems, and the result of their works was followed by interviewing the subject. Interview was done after the subject had done the given problem about Geometrics. The questions referred to interview guidelines that had been made and validated. The process of research from the test up to interviewing the subject was documented in the form of audio-visual. It was done to simplify the researchers in analyzing the result of this research.

Data Analysis

The first step in analyzing the data was writing the dialogue in interview between the researchers and the subject. It was done to simplify the process of data analyzing. Analysis was done based on the result of subject's work, dialogues in interview, and subject's attitude when solving the problem. those three elements were combined, so that it was obtained which indicators that appeared on those subjects.

4. Results and Discussion

Researches about imagination had been done, one of them is a research by Liang et al., which entitled The Exploration of Indicators of Imagination. In this research, Liang divided the imagination's indicators became 10 and this research is one of the references for this research[2]. The next one is the research by Wibowo et al., which entitled Characteristics of Students Sensory Mathematical Imagination in Solving Mathematics Problem. In that research, Wibowo explained about the students' characteristics which had sensory imagination in solving Mathematics problem. Subject of that research was VII grade students. This research used different indicator from the one that used by Wibowo, and also the subjects of this research were given test to find out subject's van Hiele's level. The subjects were who had deduction level[6].

Imagination process on undergraduate students who was in deduction level based on the indicators that appeared when the subjects did the problems about geometry. Subjects in this research were one female undergraduate student which called as S1 and one male undergraduate student which called S2. Those two subjects were undergraduate students who were in deduction level based on van Hiele's test. Next, those subjects were given 2 Geometrics' problems which can be seen in Table 2.

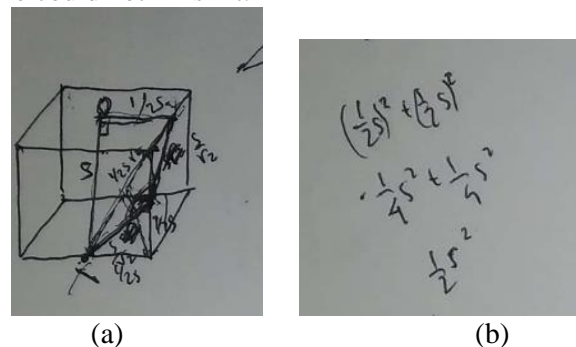
Table 2. Geometrics' Problem

Number	Problem
1	There is ABCD.EFGH cube. Midpoint of the AB, BF, and FG side were symbolized with X, Y, Z. Determine the dimensions of $\angle XYZ$!
2	There is ABCD.EFGH cube. Measure the projection's length from DE line to BDHF area!

After the subjects finished the given problems, the interview was done to them. It was done to dig up the information about their imagination process.

S1's Imagination Proses

The steps of finishing the problem by S1 was started from reading the problem, then S1 illustrated the problem in question number 1 became a picture. After that, S1 thought solution of that problem and applied it. The solution from S1 could not solve the problem, so that S1 re-read the problem and repeated the steps from the beginning. S1 illustrated the problem became a picture and searched another solution. She did it repeatedly. Concept of sinus and cosine were given since the subject did not find the solution, yet. After that, S1 searched the third solution and applied it. Unfortunately, S1 found a dead end so that she could not finish it.

**Figure 1.** S1 Answers to The First Problem

S1 started the second problem by reading it, then she illustrated the problem became a picture. After that, S1 thought the solution of that problem and applied it. The solution could not solve the problem, so that S1 re-read the problem and repeated the steps from the beginning. S1 illustrated the problem became a picture and searched the solution. Projection concept was given as help for S1 since she could not find the solution, yet. After that, S1 searched the third solution and applied it. Through this solution, S1 could solve the given problem. After finishing the problem using the found solution, S1 obtained other solution and it could solve the problem, too. Based on those two solution, S1 was able to decide which solution was more effective for the problem.

Figure 2. S1 Answers to The Second Problem

The indicators that appeared when S1 solved problem number 1 were intuition and focus. In this research, intuition meant that the subject was able to find the solution soon after reading the problem. S1's intuition could be seen when doing the problem number 1. Soon after S1 read the problem, she immediately applied the idea. Focus in this research meant that subject was able to construct the idea. S1's focus could be seen when she started the problem when she searched what she needed first to solve the problem.

The appeared indicator when S1 solve the problem number 2 was crystallization, intuition, dialectics, productivity, transformation, effectiveness, and focus. Nuthall and Old stated that intuition is important component in decision-making and underlie human asset[25]. In this research, intuition was solution that appeared soon after reading the problem. S1's intuition in solving problem number 2 could be seen when she finished reading the problem, she applied the idea. Crystallization of S1 could be seen when she played her hands to help herself imagining the problem and when she used cake box as geometry in problem. S1's dialectics could be seen when a question was given to S1 as could be seen in Table 3.

Table 3. S1's Dialogue Interview

People	Question
Researcher	Which one is the slanted angle of that triangle?
S1	(Saw the box and move her fingers) It is wrong, $DP^2 = ED^2 - EP^2$ so, $(S\sqrt{2})^2 - \left(\frac{S}{\sqrt{2}}\right)^2 \rightarrow 2S^2 - \frac{1}{2}S^2 \rightarrow \frac{3}{2}S^2$

S1's productivity was appeared at the end of solving. After she successfully finished the problem, she obtained other ways that more effective from the first way (effectiveness). Not only one way, but he also found two other ways besides the way she used to solve the problem. Next, transformation indicator on S1 could be seen from the combination of concept she used, such as Pythagoras, multiplication, one-line projection to area, division, and many more. Focus in this research meant that subjects were able to construct a structure from the idea they obtained. S1's focus could be seen when she started to solve the problem. She searched what she needed to solve it first.

Based on the analysis before, S1's imagination process in solving the problem was illustrated and could be seen in Figure 3.

S2's Imagination Proses

2 started doing the first problem by reading the problem. Then, he S2 guessed the answer. S2 searched a way to prove the answer. S2 guessed that the answer for number 1 were 90° , 120° , and 135° . For 90° angle, S2 could not find its proof, and also S2 thought that the searched angle was not perpendicular angle, but obtuse angle. Next, S2 illustrated the problem in the form of dimension 2. Based on the picture, S2 assumed that the formed angle was 135° then the concept of geometry was given. After that, S2 realized his fault then re-thought the way to solve the problem. Next, S2 found that if the angle of XYZ was rotated would be as same as XYF angle, so that S2 concluded that XYZ angle was 120° .

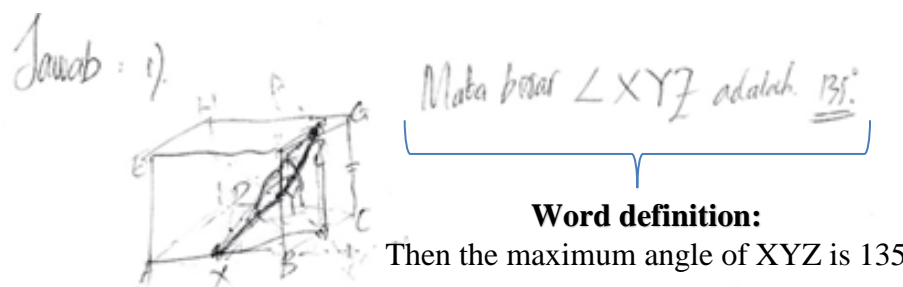


Figure 3. S2 Answers to The First Problem

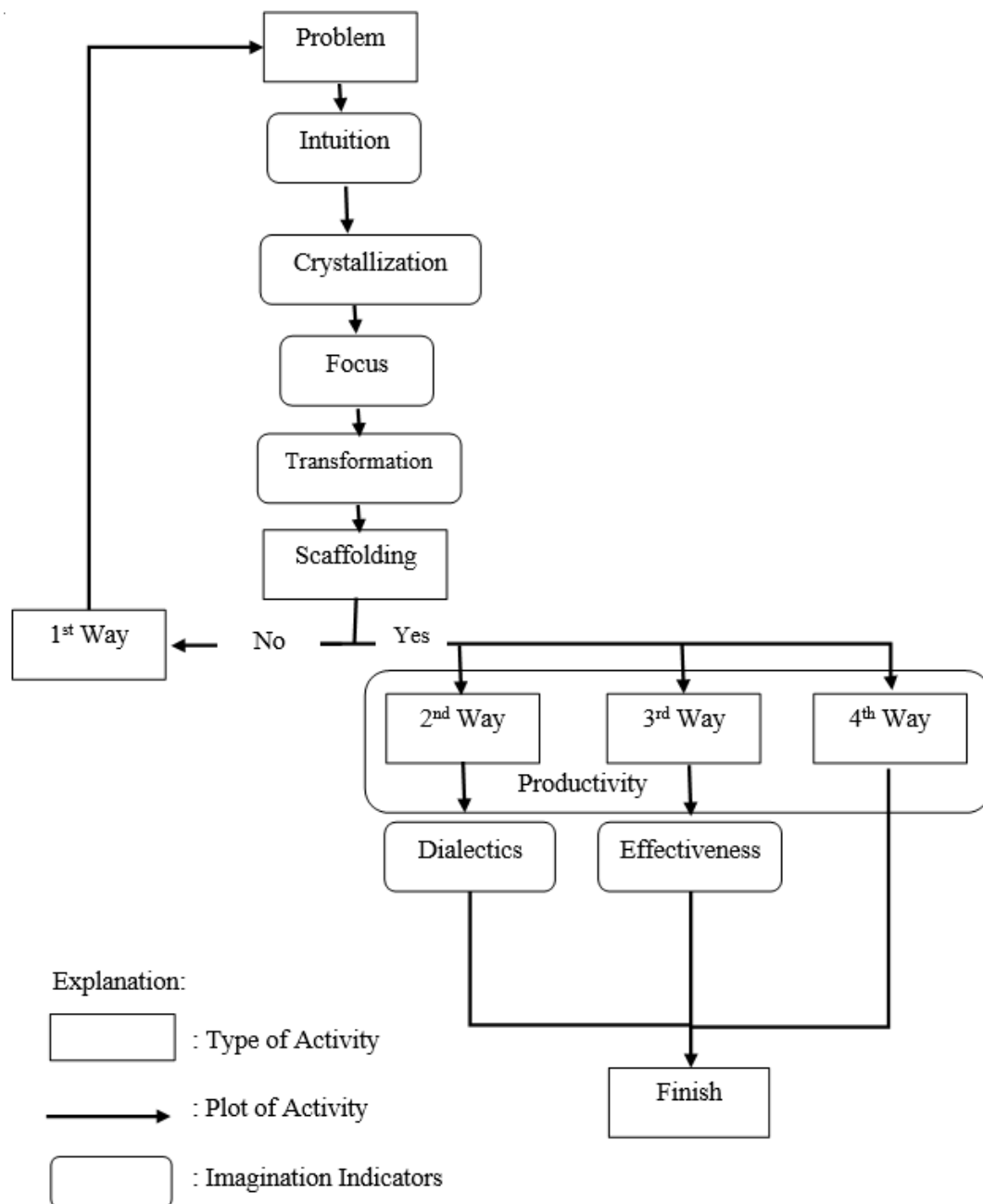
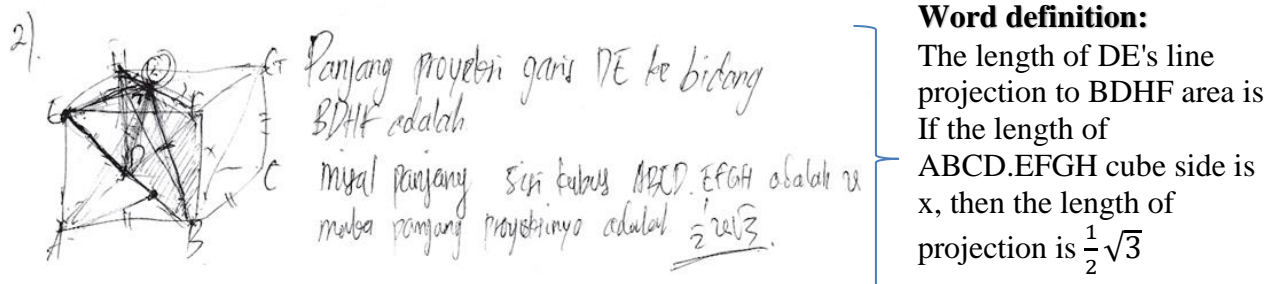


Figure 4. S1's Imagination Process in Solving the Problem

In problem number 2, S2 had difficulty because he forgot the projection material. Therefore, projection concept was given as a help. After S2 remembered the concept, S2 re-read the problem and then started to solve the problem. S2 started to construct the geometry that illustrated in the problem then S2 guessed the probable answer. S2 started to imagine the point of projection from DE line to BDHF area. After he found the projection point, S2 imagined the shaped area which the area was right triangle area. In the second problem, many ways were explained by S2. Those ways using pythagoras and the rule of sinus and cos which was varied by various angle.

**Word definition:**

The length of DE's line projection to BDHF area is
If the length of
ABCD.EFGH cube side is
x, then the length of
projection is $\frac{1}{2}\sqrt{3}$

Figure 5. S2 Answers to The Second Problem

The appeared indicators on S2 when she solved the first problem were intuition, sensitivity, productivity, innovation, focus, effectiveness, transformation, crystallization, and dialectics. Intuition on S2 could be illustrated when he guessed the probable answer. At that time, S2 guessed XYZ angle was between 90° , 120° , and 135° . That intuition was important component of decision-making and underlie human's asset[25]. Sensitivity indicator was appeared because S2 was curious when he solve first and second problem. It made S2 tried harder to solve the problem. It is explained in the interview with S2 that can be seen in the Table 4.

Table 4. Interview Dialogue with S2

People	Question
Researcher	How was your feeling when you were solving the problem?
S2	I was curious. I will search the solution until success if I have Mathematics' problem and I could not solve it.

The next indicator was crystallization. Crystallization is individual ability to expresses abstract ideas to concrete example[2]. In this research, crystallization can be interpreted as subject's attitude in imagining cube into real shape (box, room, and etc.). On S2, crystallization was marked by playing his ballpoint and moved it to the room (assumed the room as cube) and imagining the line in the first problem. Transformation was individual ability to do the task by using familiar information from any disciplines[2]. On S2, transformation indicator appeared when S2 solving the problem, when S2 combined some mastered materials to solve the problem. However, there was a mistake in that process so that a help in the form of geometry's concept was given to S2. After that S2 re-investigated his work and did a mistake in finishing process so that he repeated the process. It was one of imagination indicator which was dialectics. Productivity indicator appeared when S2 solved the first problem. It was marked by the way S2 solved the problem not using just single way. The ways used were the rule of cosine and by rotating XYZ angle became XYF angle, so that he found that XYZ angle was 120° . The solution from S2 by rotating XYZ angle became XYF angle could be said as new way because it was never done by people, even only S2 who thought about this way. For S2, this way was the most effective way because it did not need more material and also it was faster in finishing (effectiveness). The way done by S2 proved that innovation indicator was appeared in this research. S2's focus could be seen when S2 was able to find needed thing to solve the problem.

On the second problem, the indicator that appeared on S2 was intuition, sensitivity, productivity, innovation, focus, effectiveness, transformation, and crystallization. On the second problem, the process that was done by S2 was as same as the first problem. The activity was started by reading the problem, then S2 guessed the probable answer (intuition). However, on the second problem, S2 forgot about the projection so that the material about projection was given at first. Next, S2 read the problem and guessed the probable answer again. After that, he constructed the construction of geometry on the problem. S2 imagined the room he occupied as cube and S2 started fidgeting his fingers to help him imagining the geometry of that problem (crystallization). S2 started to solve the problem. There were many ways that were stated by S2 for the second problem (productivity). Those ways were the concept

of cosine, Pythagoras function, and rule of cosine (transformation). From all those ways, S2 compared with any sides of cube so that he obtained various solutions. S2 thought that solved the problem using Pythagoras function with DE, EO, and DO sides was the most effective way. It was caused by that way was the simplest and it was still appropriate with projection concept (effectiveness). Innovation indicator was indicated by solving the problem using the rule of sinus and cosine.

Briefly, imagination process on S2 in solving the problem can be seen in Figure 6.

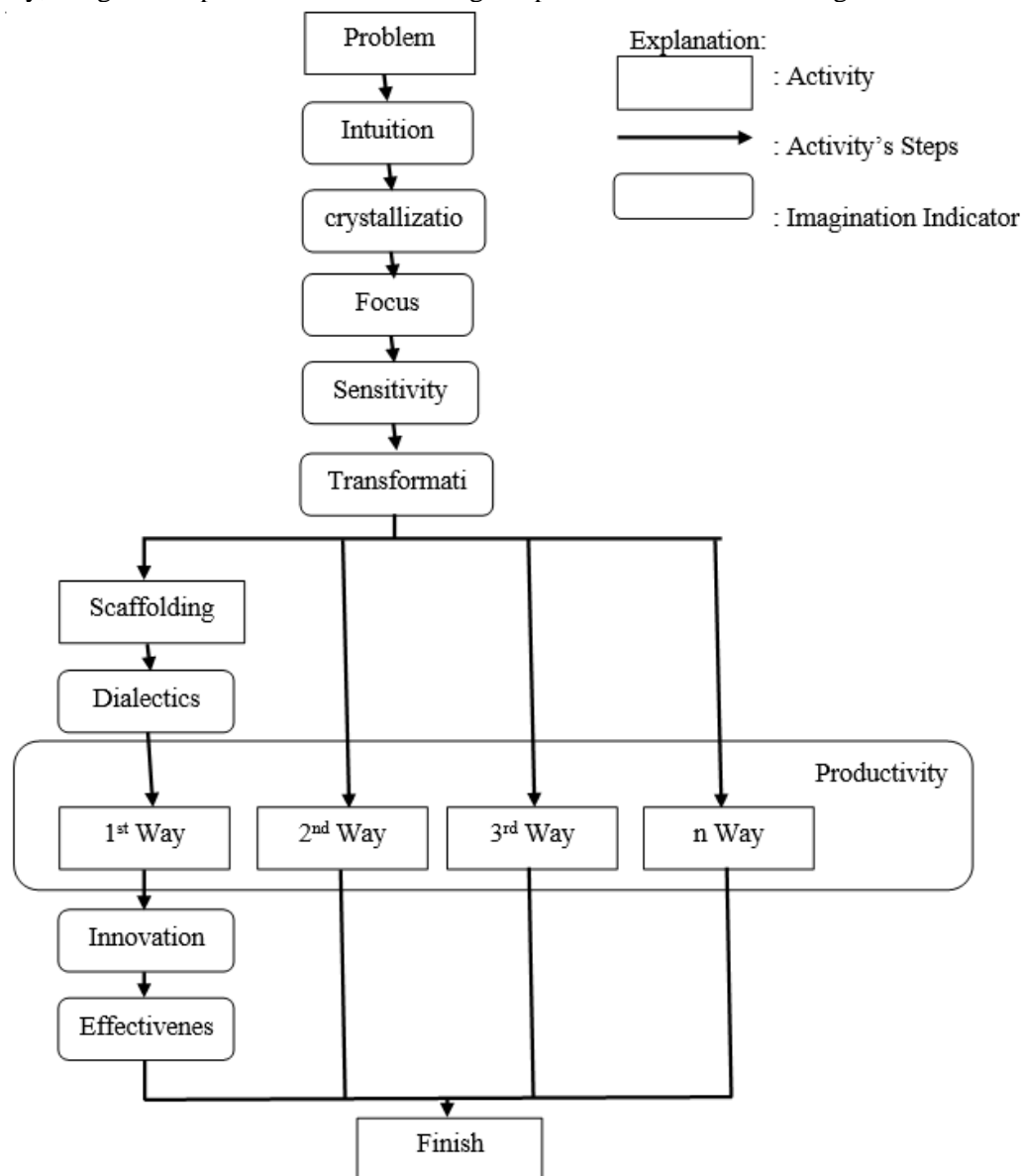


Figure 6. S2's Imagination Process in Solving Problem

5. Conclusion

Imagining process of the S1 was started by intuition, and it was continued by crystallization, focus, transformation, dialectic, productivity, and effectiveness. While, for the S2, imagining process was started by intuition indicator, then it was continued to crystallization, focus, sensitivity, transformation, dialectic, productivity, innovation, and effectiveness. Imagining process of both subjects can be said as the same. It comes from the indicators that appeared from both subject are

same, however sensitivity did not appear on the first subject. Besides the same indicators, the sequence of indicators that appeared on both subject can be said as the same

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