Problem solving analysis of rational inequality based on IDEAL model

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Abstract. This research was conducted using descriptive qualitative method to describe the process of teacher candidates in solving the rational inequality problem. The IDEAL model was used as indicator of this research which tested on 35 students. In the step of identifying and defining the problem's objective, students have understood the meaning of set of completion was value x which accomplished inequality to the problem. Mostly, students had the same strategy, yet there was difference in grouping rational expression, such as S2 and S3 who did "cross-multiplication" while another detracted two internodes with mathematical expression in the right internode. It turned out that S5 was the only student who found the whole critical points even though made a mistake in writing set of completion. In the step of "looking back", none of students did the correction whether in calculation or concept even used another strategy to check the answer.

1. Introduction

Mathematics is one of the science playing a notable role in current technological advancements. The development of technology that is growing rapidly is the outcome of the collaboration between mathematics and other sciences. In this case, mathematics is used as the language of all sciences to create a new product. Mathematics is the science that underlies the development of other sciences since every branch of science requires the calculation [1]. In addition, mathematics is a fundamental part of knowledge and one of the main parts of the development of modern technology [2].

The linearity between mathematics ability and technology advance is very important. Consequently, students must have a better understanding of the essence of mathematics itself, in terms of content and application in real life to contribute both of the country and the world. In fact, based on the results of the 2015 PISA (Program for International Student Assessment) study, mathematics literacy skills of students in Indonesia are still ranked 63 out of 70 countries and are still very far from the average score set in the study [3]. Mostly, the students have the same problem which is the problem-solving ability, hence the way students solve the mathematical problems must be changed to enhance students' mathematical achievement.

Teachers must have good pedagogical capabilities (in material content or teaching strategies) to be able to encourage students to learn how to solve problems well [4]. The solving is the process of organizing concepts and skills into a new pattern to achieve the objective of the problem that is not easily solved using routine procedures [5]. The question referred to as a problem depends on the cognitive possessed by the problem solver. It is very possible that an issue/question is not a problem for one person but a problem for another. For example, the issue/question of a linear system of two variables is no longer a problem for junior high school students because it is a routine problem, but the same question is a problem when given to elementary students. Based on that problem solving requires a deep thought and combining some knowledge that has been previously owned. Problem-solving is the process of creating relations between existing problems in front of students and problems that may have been encountered before, besides solving problems also involves choosing an approach method, knowing when to step back and reconsidering the method when it seems unproductive, looking back and reflecting on the solution obtained [6]. From those experts' opinions above, it can be concluded that problem-solving is a student's effort using all of student's knowledge, skills and understanding to find solutions toward the problems given using a particular approach.

There may be differences in skills in the problem-solving process. This is due to differences in knowledge, expertise, and even experience related to problem-solving. Meanwhile, to observe the ability and process in problem-solving, a problem-solving model is needed. The Polya model is a wellknown problem-solving model which classifies problem-solving into 4 stages namely understanding problem, planning the strategy, carrying on the strategy and rechecking [5]. However, this research will use a problem-solving model developed by Bransford and Stein, the IDEAL model. According to its name, this model has 5 stages which identifies the problem, defines and represents the problem, explores the possible strategies, acts on strategies, looks back and evaluates the effect [7]. The main difference between Polya and IDEAL models is at a stage of the understanding problem. In IDEAL model, the understanding problem stage has 2 parts that are identifying the problem and defining and representing the problem. Researchers used the IDEAL model in this study with various considerations. First, the IDEAL model is considered more detail in describing the problem-solving process. Second, the stages of the IDEAL model can be easily remembered because it is an acronym of its name so that prospective teachers can use it to solve the problem and moreover teach it to the students later.

From previous studies, researchers have not encountered an analysis of the problem-solving process using the IDEAL model that should be easy to remember. Besides, most research only focuses on elementary or middle school students regardless of how the problem is solved by prospective teachers, even though it is necessary to know the problem-solving abilities of prospective teachers and fix them if there are weaknesses at several stages of problem-solving. Thus, they are expected to transfer their skills in problem-solving to their students. Therefore in this study, researchers consider it is important to see how the process of students in solving the problem of rational inequality material.

1.1. Mathematics

Many experts have tried to define what mathematics is about. However, there is no such definition that is agreed upon, used and accepted together. Mathematics is definitely not talking about easy things to be complicated, but on the contrary, mathematics is how to turn complicated things into simpler things, how to infer a pattern of events so that conclusions can be drawn, such as this statement that mathematics is the basis of reasoning deductive through experiences in inductive reasoning [8]. Elsewhere, mathematics is the core of knowledge and the main basis of the development of modern technology with the aim of functional numeration (so that it can use mathematical skills in everyday life), and can connect and apply knowledge with skills through information provided [2]. While the other mentioned that mathematics is the science that underlies the development of other sciences because every branch of science requires calculation [1]. Based on those opinions, it can be concluded that mathematics is a science to get solutions based on available information through knowledge and experience about calculation whose main characteristics are logical, systematic, consistent, and requires creativity and innovation.

1.2. Rational inequality

Rational inequality is inequality that contains rational function and can be expressed in the form $f(x)/g(x)$ where $g(x)\neq 0$. This material was not chosen randomly but with several considerations, including the issues that are hotly discussed namely higher-order thinking skills. It has been known that Indonesia is in the lower rank based on the result of PISA, and this is caused by the low problemsolving ability of students in Indonesia. Therefore, improvement is strongly necessary. The initial improvement is on prospective teachers. It is difficult for Indonesian students to have good problemsolving skills if the teachers are not capable either. Rational inequality is indeed a semi-procedural material, but skills are still needed to find the right approach so that the strategy used is valid. If semiprocedural problems cannot be solved properly, then it will be hard to deal with the complicated ones.

1.3. High order thinking skills
High-level thinking is a form of Bloom's top-level taxonomy such as analyzing, evaluating and making in her book stated [7]. Some high order thinking skills include problem-solving skills, critical thinking, analytical thinking, creative thinking, communication skills, and others. High-level thinking is broad thinking that leads students to combine, apply, and manipulate existing information to reach solutions to the new situation in question [9]. The low student's problem-solving level will have an impact on the results obtained. The main focus of mathematical problem-solving learning for students is to improve higher-order thinking skills, thus students must use problem-solving to enhance their skills [10].

1.4. Problem solving
One of higher-order thinking skills component is problem-solving. Before discussing what problemsolving is, the meaning of the problem itself must be defined. A question can be called as a problem if it uses embedded mathematics, is important, can develop students' conceptual ability, connecting among influential mathematical ideas, and requires high-level thinking, problem-solving and mathematical skills [11, 12]. These points do not have to be entirely contained in a question as it depends on the learning objectives. In other words, a question becomes a problem or not according to the knowledge of the problem solver as well. For example, the question of a linear system of two variables is no longer a problem for junior high school students because it is a routine problem, but the same question is a problem when given to elementary students. Based on these statement, solving problems requires deep thinking and incorporating some of the previously acquired knowledge. Problem-solving is a very important skill when we want to learn mathematics [13]. By having good mathematical problem-solving skills, the basic concepts will be more applicable to face a problem. Father of problem-solving, stated that the definition of problem-solving is the process of organizing concepts and skills into a new pattern to achieve the objective of the problem that is not easily solved using routine procedures [5]. Problem-solving is a process that involves two important things namely representing the problem and then executing it [14]. Another report argued that the situation is said to be a problem when an individual must combine existing information with new ways to solve problems [15]. Moreover, problem-solving is an attempt to find a solution when there is no short solution available [16]. Here, it can be concluded that problem-solving is defined as a process of student’s effort with all the knowledge, experience and skills to be reconstructed and create new algorithms in order to find the right approach and strategy to solve a problem, and also to evaluate all steps that have been taken. People who are often confronted with a problem or situation that is really a problem, will gain a lot of experience, both in terms of new strategies and new knowledge that can be applied when dealing with other problem. Accordingly, problem solving is an important key when dealing with problems in everyday life that are related or not related to math. Each problem provides new experiences and knowledge that can serve as a guide for future problem.

1.5. IDEAL model
To observe the problem-solving process or the differences that occur between subjects, we need a problem-solving model. A well-known problem-solving model is the Polya model which has 4 stages namely understanding the problem, planning the strategy, implementing the strategy and rechecking [5]. However, this research will use a problem-solving model developed by Bransford and Stein that is the IDEAL model. IDEAL stands for identifying the problem, defining and representing the problem, exploring possible strategy, acting on strategy, looking back and evaluating the effect [7]. The main difference between Polya and IDEAL models is on the understanding stage. Bransford and Stein broke this stage into two steps. First is identifying the problem. Second is defining and representing the problem. Researchers used this model because it is considered to be more detail in describing the problem-solving process. Another reason is the stages of the IDEAL model can be easily remembered because it is an acronym of its name so that students can use it to solve the problem.

The first stage is identifying the problem, in this stage the problem solver understands the underlying essence of the problem. This stage can be practiced by trying to constantly reiterate in your own language of the issues presented. Second stage is defining and representing the problem, in this stage the problem solver is able to create a data list of variables that are either required or not (as an intruder). In addition, at this stage the problem solver can simplify the view by presenting the problem into various representations (tables, graphs, etc). The next stage is to explore possible strategies, in which the problem solver uses all the experience and knowledge to sign up, preparing as many strategies as possible to implement in order to find a solution, and then determining and choosing the strategy that is most appropriate. The fourth stage is acting on the strategies, which is the follow-up to the previous step, in this step must be meticulously because the right strategy with the least error can lead to incorrect solutions [7].

2. Methodology

This research was conducted at one of the state universities in East Java, Indonesia. The subjects in this study consisted of 35 students at the first semester mathematics education who were taking basic mathematics courses I. The research instruments used tests of problem-solving for rational inequality, interview guidelines, and IDEAL problem-solving indicators. The rational inequality material was chosen because when the research will be conducted, students had just received the material, so it was still warm in their minds. The Interview method was conducted to complete the student problemsolving data which is still not explored from the test method. From various problem-solving that is done by students, the researchers grouped them based on the similarity of the problem-solving process used. Then from each group, the researchers took one subject to be interviewed further about the problem-solving process. The interview was conducted semi-structured. It means that the researcher used interview guidelines in which the questions given can go in to detail due to the problem-solving of each subject, but must still be in accordance with the topic being discussed.

Table 1. Ideal's indicators

IDEAL model	Indicators
Identifying the problem	Understanding every word in the given question Describing the question given using their language
Defining and representing the problem	Writing down or mentioning information that is known in the question completely. Writing down or mentioning the problem asked in the question correctly Using images, tables, symbols or other forms of representation
Exploring possible strategies	Preparing several problem-solving strategies Choosing a strategy from several alternatives
Acting on the strategies	Implementing the chosen strategy correctly
Looking Back and Evaluating	Correcting the concepts or formulas Doing correction in the calculation section Using other strategies to ensure answers

Based on the indicators previously, an analysis of the subject's problem-solving process is conducted. The analysis is done by triangulating all data (problem-solving tests and interview), then described step by step on how the subject do problem-solving. The following question is the problem-solving test instrument used in this research.

A graphic designer wants to make an image consisting of 2 curves. The curves are $y_1 = \frac{3}{x^2-4}$ and $y_2 = \frac{5}{x^2+7x+10}$. The curve y_1 is designed not to be above curve y_2. Determine the limits of value x needed.

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3. Result

The results of this study connect and answer questions that arise from previous research on why students' problem solving skills are still low. Some of the studies that mention this include PISA studies, Rozencwajg & Corroyer's research, and Kurniati & Annizar's research that mentioned the problem solving skills of the subject is being low [3, 17, 13]. This study brings up the fact that none of the subjects find the right solution at the end their work. But not only that, the research shows why this can happen. Based on the problem solving analysis process of the subject, none of the subjects wrote down what was known and asked in the question. This is like what was mentioned that most people are too lazy to write down what is known and asked, some think that it is a waste of time so they can immediately move on to the next stage [17]. Some of them made mistakes at the stage of planning and implementing strategy and none of them went through looking back. This result shows that very few subjects do a stage of looking back which is supporting the results of previous studies [18, 19]. So that the following are the results and discussion of the problem-solving process of each subject of this research.

Based on the 35 students' work, the researchers classified them into 5 different groups in solving problems, so the researchers took 1 representative for each group by considering the smooth communication. Let S1, S2, S3, S4, and S5 be the subjects of group 1 to 5 respectively. The result of the problem-solving process of each subject can be compared. The differences and similarities can be seen in the following table.

Table 2. Problem solving subject based on ideal

IDEAL model	S1	S2	S3	S4	S5
Identifying the problem	At first, the subjects understood every word in the problem, so that they could draw the essence of the problem that is the problem related to the material rational inequality.				
Defining and representing the problem	Next, the subjects did not write what was known and asked, but based on the results of their works, the subjects knew what information was given, and from interviews, the subjects stated that the problem was aimed at finding the set of solutions of value x that meet the existing inequality.				
Exploring possible strategies	Furthermore, the subject immediately thought of a strategy and stopped when getting one. The steps are making a model of	Although not writing down the design of the strategy first, it is clearly illustrated from the subject's work that the subject directly modelled the problem in	The subject only has 1 strategy to solve the problem, namely after modelling the inequality, the subject grouped rational expressions into one segment then	The subject only has 1 strategy, namely by modelling the existing problems, grouping rational expressions into one segment, simplifying them,	The subject claimed to only have 1 strategy, namely modelling and grouping rational expressions into one segment, simplifying it, looking for

	inequality, gathering rational expressions into one segment, simplifying it, then determining the critical point by finding zero makers in the denominator and numerator. From that critical point, the sign of each interval (positive or negative) was determined until the final set of solutions made.	mathematical inequality and collected rational expressions on one segment, then factorized them to find a critical point. The sign of each interval was then determined which resulted in a set of solutions. The subject looks more systematic by writing the stage title at each step.	considered the inequality as an equation, and simplified it then continued by finding its critical point and determining the set of solutions.	looking for critical points, then determining the sign of each interval, and finally managing the set of solutions.	critical points, determining signs at intervals bounded by these critical points, and then arranging the set of solutions.
Acting on the strategies	The strategy was initially implemented quite well, but then there was a difficulty in which only obtaining 2 of the 3 zeromaker factors of the denominator and not finding a zero-maker of the numerator. Consequently, the subject concluded that there were only 2 critical points and then determined the sign of each interval. Based on interviews, the set of solution was $-2 \leq x \leq 2$.	The plan was carried out without regard to the concept of rational inequality. It is seen when gathering rational expressions into one segment, the subject performed crossmultiplication so that only 3 of the 4 critical points were found. S2 was also sure if there were only 3 critical points. Therefore, S2 determined the sign (positive or negative) by substituting several points to the new rational inequality so that some signs are still wrong.	S3 made a concept fault, namely multiplying the two segments with the multiple of the denominators, so this is similar to cross-multiplication. The subject continued to look for a critical point and performed a mistake in factorizing so that the critical point obtained is not appropriate. Finally, in determining the set of solutions, the subject immediately inputted the critical value into a closed interval and considered it a set of solutions.	After modelling in rational inequality, the subject grouped rational expressions into one segment by subtracting the two segments by $\frac{5}{x^2+7x+10}$. The subject made a carelessness in simplifying (can be seen in Figure 5) so that the subject only found 3 of the 4 critical points. After that, the subject determined the sign at each interval by substituting particular x to a new inequality, then made the set	The strategy was implemented well by S5, but in the last strategy which is to write the set of solutions, S5 did not reexamine how the inequality value for $x = 5$, and $x = \pm 2$, so that it's still not right to write the final answer.

			of solutions even though it is not appropriate to use the operator " ≤ " at the denominator's critical point.
Looking Back and Evaluating	The subject claimed not to make corrections because time was limited	The subject claimed not to make corrections again because he was confident in his work	

4. Discussion

The first group consists of 8 students. S1 identified the problem given by looking at the essence of the problem. Based on the interview, S1 stated that the question given is a matter of rational inequality. The subject was also able to mention that the final goal of the problem is to find a set of solutions. At the stage of planning the strategy, the subject just claimed have 1 strategy. First is collecting rational expressions into one segment. After simplification, the next step is to determine the critical point by finding zeroes in the denominator and numerator. From the critical point, the sign will be determined (positive or negative) of each interval, and finally makes the set of solutions. In the first place, this plan was implemented by reducing the two sections with $\frac{5}{x^2+7x+10}$ so that rational expressions converge on one of the segments and then simplify their form by equating the denominator. However, S1 had problems in determining the zero makers of the denominator and the numerator. S1 only found 2 of 3 factors of zero makers of the denominator and did not find the zero makers of the numerator. Therefore, S1 concluded that there were only 2 critical points and then determined the sign of each interval which resulted in $-2 \leq x \leq 2$ as the set of solutions. After determining the sign of each interval, S1 claimed not to make corrections again because the time had passed used up.

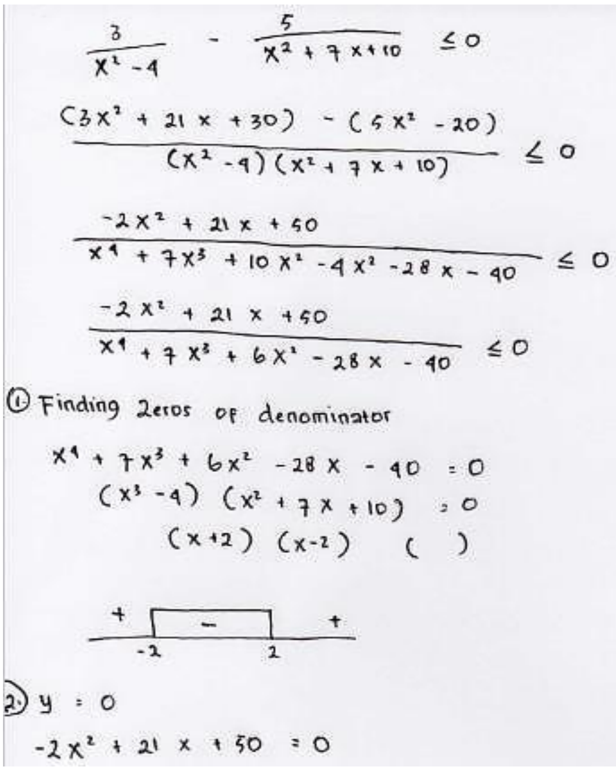

Figure 1. The result of S1

The second group consists of 10 students. At the stage of identifying and determining the problem, S2 understood the essence of the problem by stating that the problem is a rational inequality and the objective is to determine the set of solutions which is a value x that satisfies the inequality. S2's strategy is collecting rational expressions on one segment, then factoring it to find a critical point, then determining the sign of each interval, and making a set of solutions. However, the plan was carried out without regard to the concept of rational inequality. It was shown when S2 tried to gather rational expressions into one segment. S2 did cross-multiplication so that only 3 of the 4 existed critical points were found. Based on the interview, S2 also believed that if there are only 3 critical points. S2 then determined the sign (positive or negative) by substituting several points to the new rational inequality obtained from the cross-multiplication. Consequently, there are some wrong signs, nevertheless, S2 wrote the set of solutions. S2 claimed not to make any corrections again because S2 was confident of the work.

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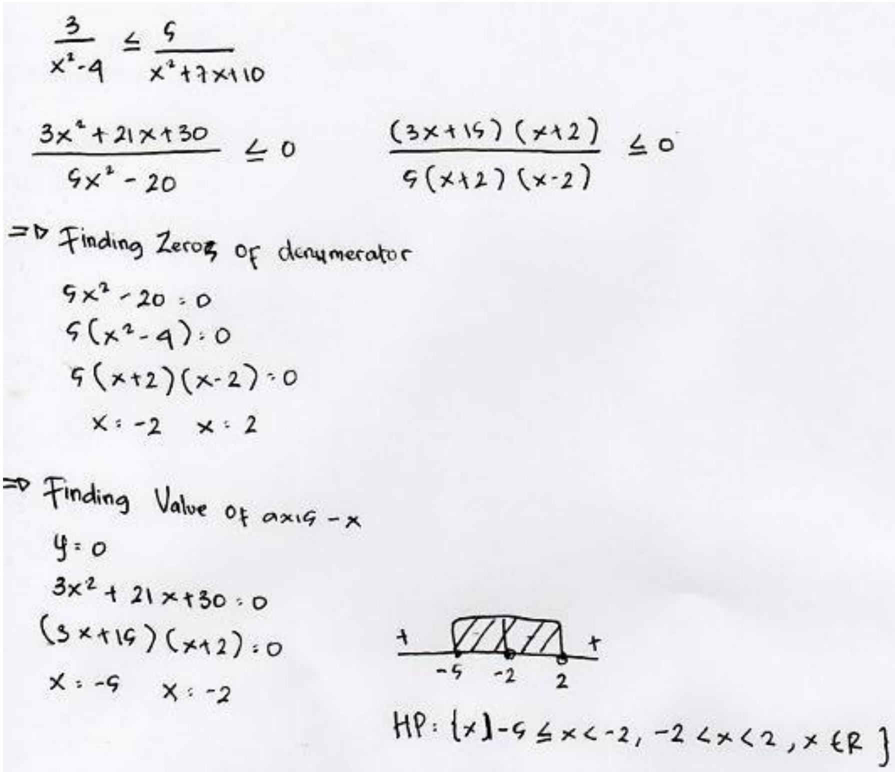

Figure 2. The result of S2

The third group consists of 6 students. At the stage of identifying and determining the problem, S2 understood the essence of the problem by stating that the problem is a rational inequality and the objective is to determine the set of solutions which is a value x that satisfies the inequality. S3 claimed to have 1 strategy to solve the problem, namely by grouping rational expressions into one segment, then considering inequality as equality, and simplifying it then looking for the critical points and determining the set of solutions. However, in the process of implementing the strategy, S3 made a misconception that was simplifying the equation by multiplying the two segments with the multiple of the denominators. This is as same as cross-multiplying. However, based on the interview, S3 was not aware of the error, and continued to look for the critical points, and made a mistake in factoring which results in obtaining the inexact critical point. The last part of the strategy is to determine the set of solutions. At this step, S3 directly input the critical points into a closed interval and consider it as a set of solutions. S3 claimed not to make corrections again because the subject was quite sure of the answer.

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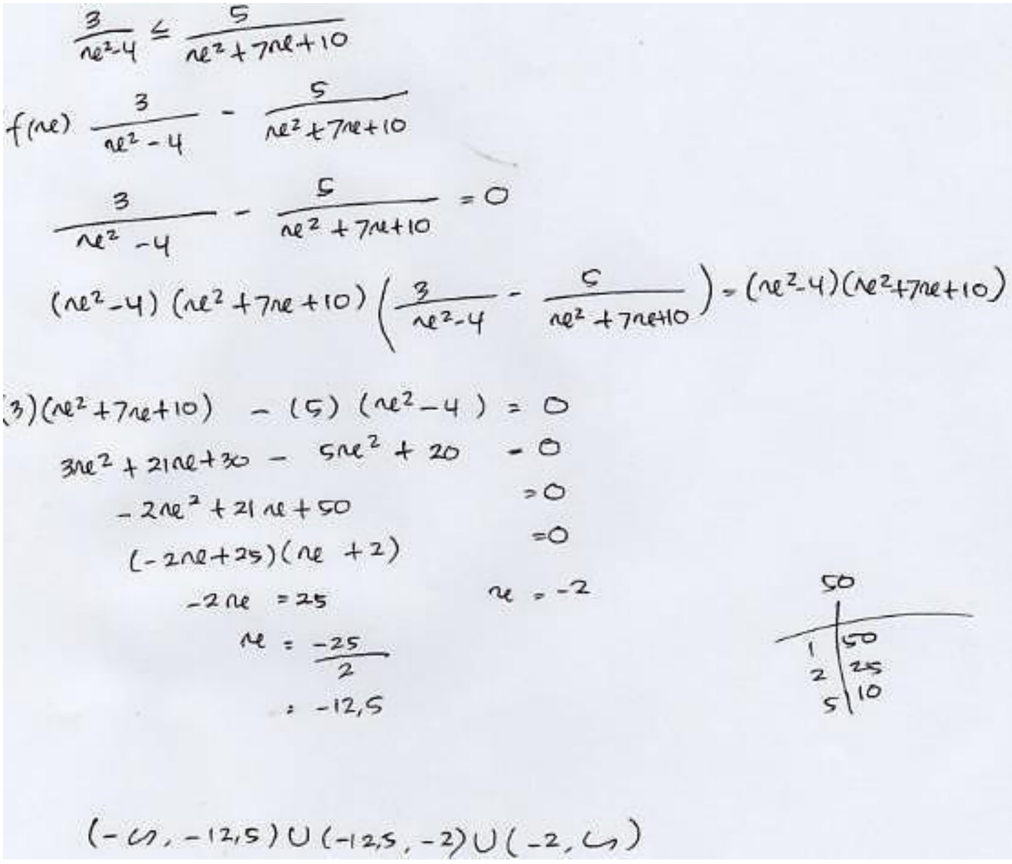

$$\frac{3}{x^2-4} \le \frac{5}{x^2+7x+10}$$

$$f(x)\ \frac{3}{x^2-4} - \frac{5}{x^2+7x+10}$$

$$\frac{3}{x^2-4} - \frac{5}{x^2+7x+10} = 0$$

$$(x^2-4)(x^2+7x+10)\left(\frac{3}{x^2-4} - \frac{5}{x^2+7x+10}\right) = (x^2-4)(x^2+7x+10)$$

$$(3)(x^2+7x+10) - (5)(x^2-4) = 0$$

$$3x^2+21x+30 - 5x^2+20 = 0$$

$$-2x^2+21x+50 = 0$$

$$(-2x+25)(x+2) = 0$$

$$-2x = 25 \qquad x = -2$$

$$x = \frac{-25}{2}$$

$$= -12{,}5$$

50	
1	50
2	25
5	10

$$(-\infty, -12{,}5) \cup (-12{,}5, -2) \cup (-2, \infty)$$

Figure 3. The result of S3

The fourth group consists of 4 students. At the stage of identifying and determining the problem, S4 understood the essence of the problem by stating that the problem is a rational inequality and the objective is to determine the set of solutions which is a value x that satisfies the inequality. S4's strategy is quite good, namely by grouping rational expression into one segment, then simplifying it, looking for the critical points, determining the sign of each interval, and finally determining the set of solutions. However, this right strategy is not supported by its implementation. After grouping rational expression into one segment by subtracting the two segments with $\frac{5}{x^2+7x+10}$, S4 was careless in simplifying that can be seen in Figure 4. Therefore, S4 only found 3 of the 4 existed critical points. After that, S4 determined the sign at each interval by substituting certain x to a new inequality, then makes the set of solutions even though it is not appropriate to use the operator "≤" at the critical point of the denominator. S4 claimed not to re-do the correction because the subject felt quite confident with the results obtained.

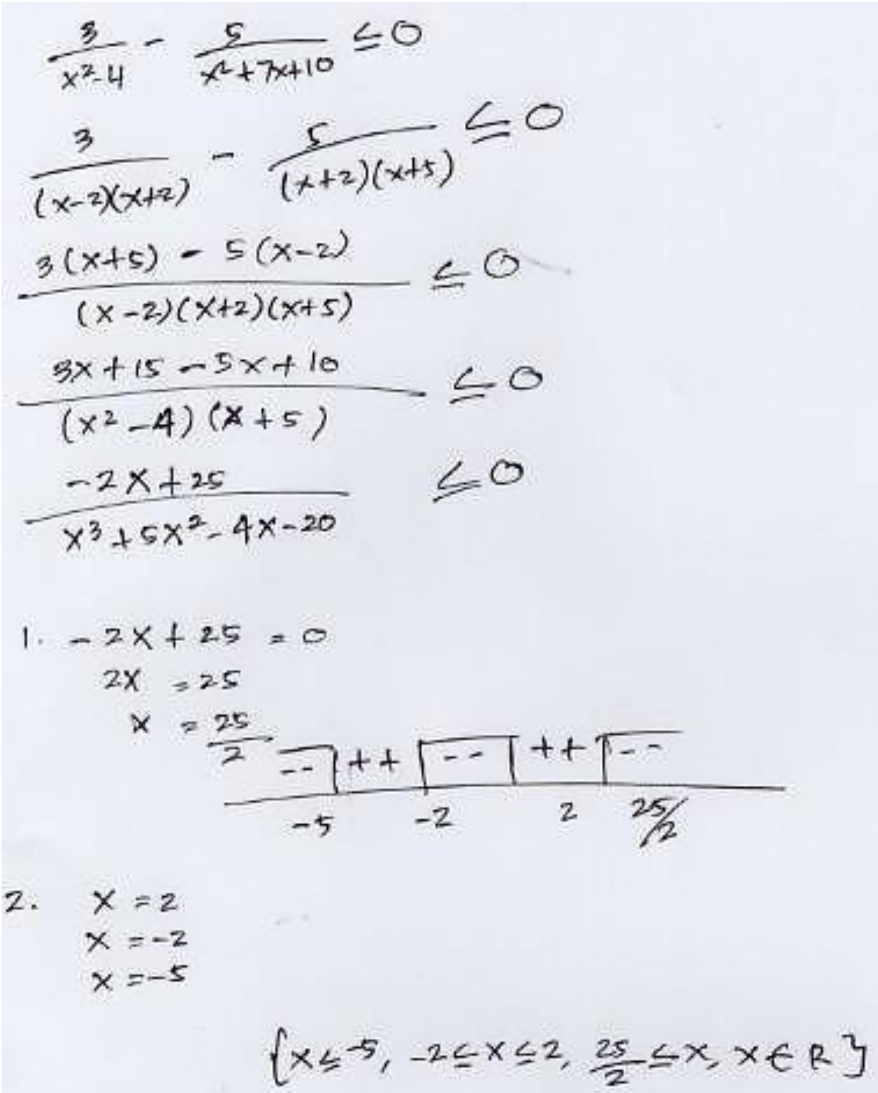

Figure 4. The result of S4

The fifth group consists of 7 students. At the stage of identifying and determining the problem, S5 understood the essence of the problem by stating that the problem is a rational inequality and the goal is to determine the set of solutions which is a value x that satisfies the inequality. S5 claimed to have 1 strategy, namely by grouping rational expressions into one segment, simplifying it, looking for the critical points, determining the signs at intervals bounded by these critical points, then setting the set of solutions. The strategy was implemented very well by S5, but in the last part which is writing the set of solutions, S5 did not re-examine how the values are for $x=5$, $x=\pm 2$, so that the final answer is not quite right. S5 also claimed not to make any corrections on these points and the steps that have been implemented.

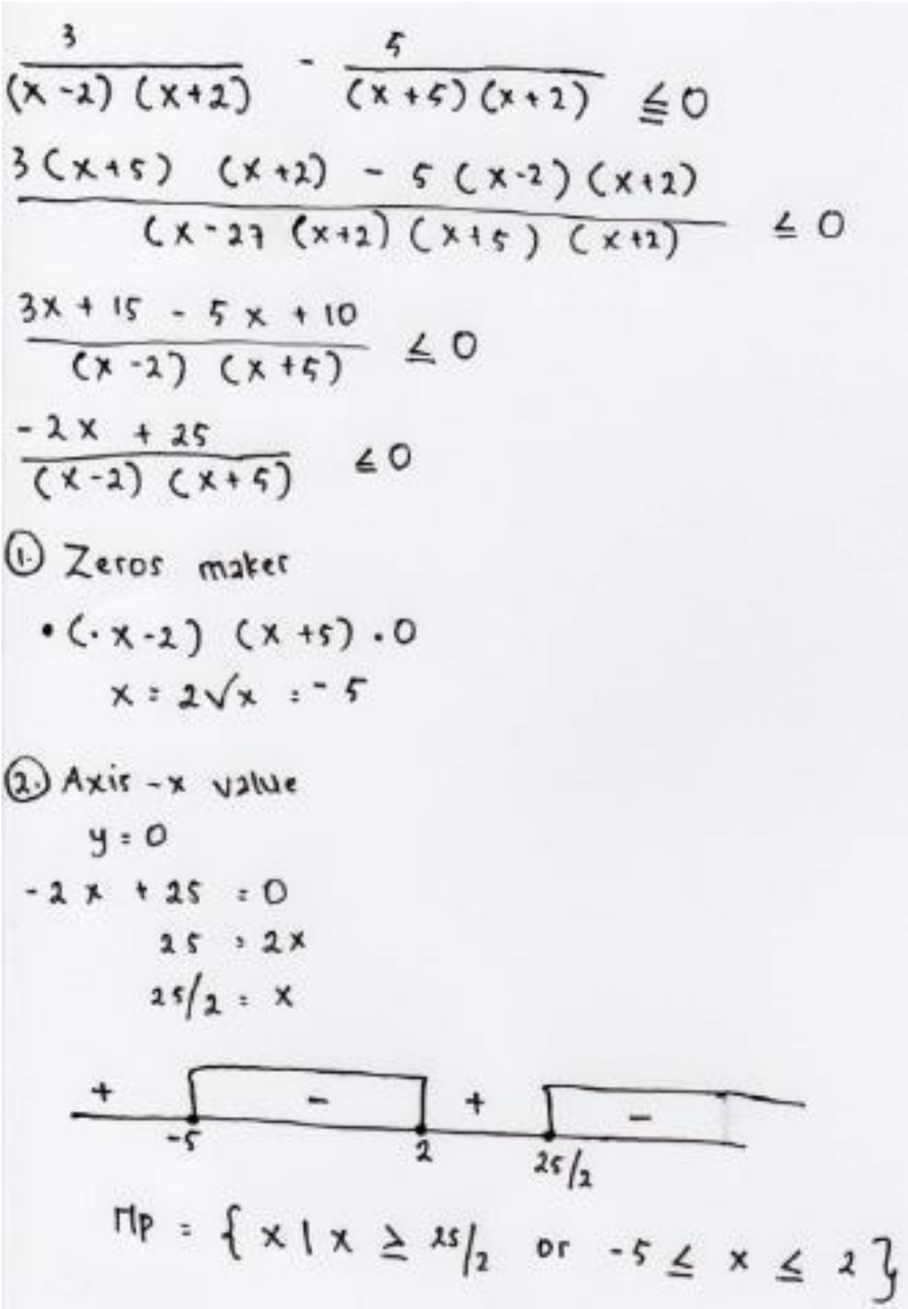

Figure 5. The result of S5

Based on the explanation previously, the whole subject could identify the problem well. In the defining and representing the problem stage, no writing what is known or asked. However, the whole subject interviewed understands that the goal of the problem is to find a solution to solving inequality. This is like what was mentioned that most people are too lazy to write down what is known and asked, some think that it is a waste of time so they can immediately move on to the next stage [17]. Some of them made mistakes at the stage of planning and implementing strategy and none of them went through looking back. This result shows that very few subjects do a stage of looking back which is supporting the results of previous studies [18, 19]. In fact, by looking back which is checking the set of resolutions obtained, especially on the subject of S5, will have a greater chance of getting the right answer. Furthermore, based on 35 students' work, none of them has the right final answer.

5. Conclusion

Based on the results and discussion of the study, it can be concluded that at the stage of identifying the problem, all subjects can explain the essence or meaning of the given problem and know that the problem given is rational inequality. In the stage of setting goals, the whole subject also knows that what is being sought is the value x that satisfies the given rational inequality. The strategy generally used by the five subjects which is similar, namely, grouping rational expressions into one segment, looking for a critical point, determining signs at each interval (except S3 which directly substituted a critical point as a set of solutions within a closed interval), and finally specifying the set of solutions. The difference among strategies used is when grouping rational expressions into one segment, such as S2 and S3 whose plan was to classify rational expressions by cross-multiplying while the other subjects to reduce both segments by $\frac{5}{x^2+7x+10}$.

At the stage of implementing the strategy, S1 simplified the rational form but then had difficulty in finding critical points. S1 only figured out 2 of the 4 critical points and immediately proceeded to find the signs of each interval. Next, S1 determined the set of solutions which is not right. On the other hand, S2 and S4 only got 3 critical points. S2 then substituted these critical points into new rational inequalities (after cross-multiplying) which resulted in the set of solutions is still not right. Slightly different from S2, S4 also found 3 critical points and substituted it to a new inequality (after being grouped and simplified) as well but S4 was still confused by the sign ". $\leq$ "
S3 did the mistake in factoring so that the critical point obtained was still not corresponding. After that, S3 directly determined the set of solutions with closed intervals without specifying the sign of each interval. Unlike the other subjects, S5 implemented the strategy with precise calculations and concepts. S5 also managed to find all the critical points but carelessly to see the zero makers from the denominator which resulted in using the sign " $\leq$ ". It happened in the hyphen only. In the stage of looking back, none of the subjects did the correction either in the calculation part or in the concept used.

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