The total H-irregularity strength of triangular ladder graphs

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Abstract. Let G be a graph which has $V(G)$ as a vertex set and $E(G)$ as an edge set. The G graph contain subgraph H that isomorphic with $H_j, j = 1, 2, \ldots s$, if every $e \in E(G)$ include only in one of the edge set of H subgraph ($e \in E(H)$). The total l-labeling is a graph labeling that give label positive integer number until l into vertices and edges. The total H-irregular labeling is a total l-labeling with condition that the sum of vertex labels and edge labels in two distinct subgraphs H_1 and H_2 isomorphic to H is different. We define H-weight as $wt_\varphi(H) = \sum_{v\in V(H)} \varphi(v) + \sum_{e\in E(H)} \varphi(e)$, for the subgraph $H \subseteq G$ under the total l-labeling (φ). The total H-irregularity strength of G graph ($ths(G, H)$) is the smallest l value in label of G graph has total H-irregular labeling. We used plane graphs such as triangular ladder graph and windmill graph.

1. Introduction

We only used plane graphs in this paper, especially triangular ladder graph and windmill graph. Let G be a graph which has $V(G)$ as a vertex set and $E(G)$ as an edge set. The G graph contain subgraph H that isomorphic with $H_j, j = 1, 2, \ldots s$, if every $e \in E(G)$ include only in one of the edge set of H subgraph ($e \in E(H)$) [1]. An assignment of integers to the elements of graph such as vertices, edges, or vertices and edges with certain conditions is called a graph labeling [2]. The type of graph labeling is total l-labeling. The total l-labeling is a graph labeling that give label positive integer number until l into vertices and edges [3].

The total l-labeling has another type called irregular labeling. There are three type of irregular labeling such as total vertex irregular labeling, total edge irregular labeling, and total H irregular labeling. A total vertex irregular labeling is a total l-labeling with condition that the sum of vertex label and edge labels in incident vertex is different [4]. There are many researcher that found the total vertex irregular strengths for various classes of graphs. Susilawati et al. [5] proved total vertex irregularity strength of trees with maximum degree five. Ahmad et al. [6] obtain total vertex irregularity strength of ladder related graphs.

Another type of irregular labeling is a total edge irregular labeling. A total edge irregular labeling is a total l-labeling with condition that the sum of vertex labels and edge label in adjacent edge is different [7]. Putra and Susanti [8] investigate the total edge irregularity strength of centralized uniform theta graphs. Siddiqui et al. [9] get total edge irregularity strength of accordion graphs.

The total H-irregular labeling is a total l-labeling with condition that the sum of vertex labels and edge labels in two distinct subgraphs H_1 and H_2 isomorphic to H is different. We define H-weight as

$$wt_{\varphi}(H) = \sum_{v \in V(H)} \varphi(v) + \sum_{e \in E(H)} \varphi(e),$$

for the subgraph $H \subseteq G$ under the total l-labeling (φ). The total H-irregularity strength of G graph ($ths(G, H)$) is the smallest l value in label of G graph has total H-irregular labeling. We used Theorem 1 as a lower bound of the strength of total H-irregularity.

Theorem 1 *[1] Let G be a graph admitting an H-covering given by t subgraphs isomorphic to H. Then*

$$tHs(G, H) \geq \left\lceil 1 + \frac{t-1}{|V(H)| + |E(H)|} \right\rceil.$$

Agustin et al. [10] determine the total H-irregularity strength of operation graph such as shackle and amalgamation graph. Nisviasari et al. [11] have found the total H-irregularity strength of family graph, especially $tHs(G(3, n, C_4))$, $tHs(TL_n, C_m)$, and $tHs(TL_n, F_3)$.

We used plane graphs such as triangular ladder graph and windmill graph. [12] described triangular ladder graph. Triangular ladder graph is constructed from a ladder (L_m) by adding the edges $u_i v_{i+1}$ for j integer number until $m - 1$.

2. Results

Some result of our research total H-irregularity strength of plane graphs is as follows.

Theorem 2 *Let TL_n be a triangular ladder graph for $n \geq 3$ admitting a $H \equiv Wd_{3,2}$-covering. Then*

$$tHs(TL_n, Wd_{3,2}) = \left\lceil \frac{2n+6}{11} \right\rceil.$$

Proof. Let TL_n, $n \geq 3$, be a triangular ladder graph with the vertex set $V(TL_n) = \{x_i, y_i : i = 1, 2, 3, \ldots, n\}$ and the edge set $E(TL_n) = \{x_i x_{i+1}, y_i y_{i+1}, x_i y_{i+1} : i = 1, 2, 3, \ldots, n-1\} \cup \{x_i y_i : i = 1, 2, 3, \ldots, n\}$. The triangular ladder graph TL_n, $n \geq 3$, contains a $Wd_{3,2}$-covering with exactly $2n - 4$ windmills $Wd_{3,2}$. To get the lower bound, we must input the information above into Theorem 1. We get $tHs(TL_n, Wd_{3,2}) \geq \left\lceil \frac{2n+6}{11} \right\rceil$ as a lower bound. Put $k = \left\lceil \frac{2n+6}{11} \right\rceil$. We shown l as an upper bound of the total $Wd_{3,2}$-irregularity strength of TL_n, we define a $Wd_{3,2}$-irregular total l-labeling $\varphi_{Wd_{3,2}} : V(TL_n) \cup E(TL_n) \to \{1, 2, \ldots, k\}$.

A $Wd_{3,2}$-irregular total l-labeling $\varphi_{Wd_{3,2}} : V(TL_n) \cup E(TL_n) \to \{1, 2, \ldots, l\}$ is as follows: for $s = 1, 2, 3, \ldots$

$$\varphi_{Wd_{3,2}}(x_i) = \begin{cases} 1, & 1 \leq i \leq 3, \\ 2, & 4 \leq i \leq 9, \\ 2s+1, & 11s - 1 \leq i \leq 11s + 2, \\ 2s+2, & 11s + 3 \leq i \leq 11s + 9, \end{cases}$$

$$\varphi_{Wd_{3,2}}(y_i) = \begin{cases} 1, & 1 \leq i \leq 3, \\ 2, & 4 \leq i \leq 9, \\ 2s+1, & 11s - 1 \leq i \leq 11s + 4, \\ 2s+2, & 11s + 5 \leq i \leq 11s + 8, \\ 2s+3, & i = 11s + 9, \end{cases}$$

$$\varphi_{Wd_{3,2}}(x_iy_i) = \begin{cases} 1, & 1 \le i \le 6, \\ 2, & 7 \le i \le 9, \\ 2s, & i = 11s-1, \\ 2s+1, & 11s \le i \le 11s+6, \\ 2s+2, & 11s+7 \le i \le 11s+9, \end{cases}$$

$$\varphi_{Wd_{3,2}}(x_ix_{i+1}) = \begin{cases} 1, & 1 \le i \le 4, \\ 2, & 5 \le i \le 9, \\ 2s, & 11s-1 \le i \le 11s, \\ 2s+1, & i = 11s+1, \\ 2s+2, & 11s+2 \le i \le 11s+9, \end{cases}$$

$$\varphi_{Wd_{3,2}}(y_iy_{i+1}) = \begin{cases} 1, & i = 1, \\ 2, & 2 \le i \le 7, \\ 3, & 8 \le i \le 9, \\ 2s+1, & 11s-1 \le i \le 11s+6, \\ 2s+2, & i = 11s+7, \\ 2s+3, & 11s+8 \le i \le 11s+9, \end{cases}$$

$$\varphi_{Wd_{3,2}}(x_iy_{i+1}) = \begin{cases} 1, & 1 \le i \le 5, \\ 2, & 6 \le i \le 9, \\ 2s, & 11s-1 \le i \le 11s, \\ 2s+1, & 11s+1 \le i \le 11s+4, \\ 2s+2, & 11s+5 \le i \le 11s+9, \end{cases}$$

We can see that every vertex and edge labels under $\varphi_{Wd_{3,2}}$-labeling are almost l. We get the $Wd_{3,2}$-weight of ${Wd_{3,2}}^j$, $j = 1, 2, \dots, 2n-4$, under the total labeling $\varphi_{Wd_{3,2}}$, we get

$$wt_{\varphi_{Wd_{3,2}}}(Wd^j_{3,2}) = \sum_{v \in V(Wd^j_{3,2})} \varphi_{Wd_{3,2}}(v) + \sum_{e \in E(Wd^j_{3,2})} \varphi_{Wd_{3,2}}(e) \tag{1}$$

From the $wt_{\varphi_{Wd_{3,2}}}({Wd_{3,2}}^j)$ that the sum of vertex and edge labels, we get increasing sequences. And it is enough to prove that $wt_{\varphi_{Wd_{3,2}}}({Wd_{3,2}}^j) < wt_{\varphi_{Wd_{3,2}}}({Wd_{3,2}}^{j+1})$, $j = 1, 2, \dots, 2n-5$.

The function of vertex and edges label $\varphi_{Wd_{3,2}}$ is periodic function with s variable. For every s is natural number, $s = 1, 2, 3, \dots$, function $\varphi_{Wd_{3,2}}$ used to give label for every vertex and edges on TL_n graph for $n \equiv 10 \ (mod\ 11)$ where n is natural number which $n \ge 21$. Every weight on $Wd_{3,2}$ covering:

$$\begin{aligned} w_1 &= \varphi_{Wd_{3,2}}(x_i) + \varphi_{Wd_{3,2}}(x_{i+1}) + \varphi_{Wd_{3,2}}(x_{i+2}) + \varphi_{Wd_{3,2}}(x_ix_{i+1}) + \varphi_{Wd_{3,2}}(x_{i+1}x_{i+2}) \\ &\quad + \varphi_{Wd_{3,2}}(y_{i+1}) + \varphi_{Wd_{3,2}}(y_{i+2}) + \varphi_{Wd_{3,2}}(x_iy_{i+1}) + \varphi_{Wd_{3,2}}(x_{i+1}y_{i+2}) \\ &\quad + \varphi_{Wd_{3,2}}(x_{i+1}y_{i+1}) + \varphi_{Wd_{3,2}}(x_{i+2}y_{i+2}) \\ w_2 &= \varphi_{Wd_{3,2}}(x_i) + \varphi_{Wd_{3,2}}(x_{i+1}) + \varphi_{Wd_{3,2}}(y_i) + \varphi_{Wd_{3,2}}(y_{i+1}) + \varphi_{Wd_{3,2}}(y_{i+2}) \\ &\quad + \varphi_{Wd_{3,2}}(y_iy_{i+1}) + \varphi_{Wd_{3,2}}(y_{i+1}y_{i+2}) + \varphi_{Wd_{3,2}}(x_iy_{i+1}) + \varphi_{Wd_{3,2}}(x_{i+1}y_{i+2}) \\ &\quad + \varphi_{Wd_{3,2}}(x_iy_i) + \varphi_{Wd_{3,2}}(x_{i+1}y_{i+1}) \end{aligned}$$

For every s, we got the weight of covering for $n \ge 21$, $n \equiv 10 \ (mod\ 11)$ is

$$\begin{aligned} w_1 &= 11, 13, 15, \dots, 2n+5 \\ w_2 &= 12, 14, 16, \dots, 2n+6 \end{aligned}$$

Hence,

$$wt_{\varphi_{Wd_{3,2}}} = w_1 + w_2 = 11, 12, 13, 14, 15, 16, \dots, 2n+5, 2n+6$$

Based on (1), we get $wt_{\varphi_{Wd_{3,2}}}(Wd^{j+1}_{3,2}) = 1 + wt_{\varphi_{Wd_{3,2}}}(Wd^{j}_{3,2})$. Otherwise, the weight of $Wd_{3,2}$ covering of TL_n has different weight which is aritmathic sequence with difference 1. The weight of $Wd_{3,2}$ is $wt_{\varphi_{Wd_{3,2}}}(Wd^{j}_{3,2}) = \sum_{v\in V(Wd^{j}_{3,2})} \varphi_{Wd_{3,2}}(v) + \sum_{e\in E(Wd^{j}_{3,2})} \varphi_{Wd_{3,2}}(e) = \{11, 12, 13, 14, \dots, 2n+6\}$.

Then $tHs(TL_n, Wd_{3,2}) \geq \left\lceil \frac{2n+6}{11} \right\rceil$ and $tHs(TL_n, Wd_{3,2}) \leq \left\lceil \frac{2n+6}{11} \right\rceil$, $tHs(TL_n, Wd_{3,2}) = \left\lceil \frac{2n+6}{11} \right\rceil$. □

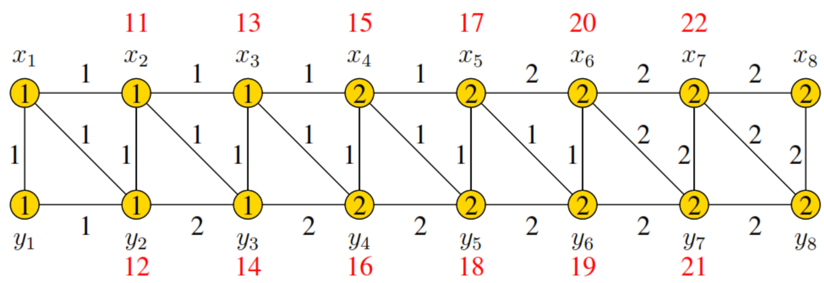

Figure 1. The example of total $Wd_{3,2}$-irregular labeling of triangular ladder graph

Theorem 3 *Let TL_n be a triangular ladder graph for $n \geq 4$ admitting a $H \equiv TL_m$-covering, $3 \leq m < n$. Then*

$$tHs(TL_n, TL_m) = \left\lceil \frac{n+5m-3}{6m-3} \right\rceil.$$

Proof. Let TL_n, $n \geq 3$, be a triangular ladder with the vertex set $V(TL_n) = \{x_i, y_i : i = 1, 2, 3, \dots, n\}$ and the edge set $E(TL_n) = \{x_i x_{i+1}, y_i y_{i+1}, x_i y_{i+1} : i = 1, 2, 3, \dots, n-1\} \cup \{x_i y_i : i = 1, 2, 3, \dots, n\}$. The triangular ladder TL_n, $n \geq 3$, contains a TL_m-covering with exactly $n-m+1$ triangular ladder TL_m, $3 \leq m < n$.

To get the lower bound, we must input the information above into 1. We get $tHs(TL_n, TL_m) \geq \left\lceil \frac{n+5m-3}{6m-3} \right\rceil$ as a lower bound. Put $k = \left\lceil \frac{n+5m-3}{6m-3} \right\rceil$. To show that l is an upper bound for the total TL_m-irregularity strength of TL_n, we define a TL_m-irregular total l-labeling φ_{TL_n} from vertex and edge labels to integer number until l.

A TL_3-irregular total l-labeling φ_{TL_3} from vertex and edge labels to integer number until l is as follows:

for $i = 1, 2, \dots, n$,

$$\varphi_{TL_3}(x_i) = \left\lceil \frac{i+12}{15} \right\rceil, \quad \varphi_{TL_3}(x_i y_i) = \left\lceil \frac{i+6}{15} \right\rceil,$$
$$\varphi_{TL_3}(y_i) = \left\lceil \frac{i+9}{15} \right\rceil.$$

for $i = 1, 2, \ldots, n-1$,

$$\varphi_{TL_3}(x_i x_{i+1}) = \left\lceil \frac{i+4}{15} \right\rceil, \quad \varphi_{TL_3}(x_i y_{i+1}) = \left\lceil \frac{i}{15} \right\rceil,$$
$$\varphi_{TL_3}(y_i y_{i+1}) = \left\lceil \frac{i+2}{15} \right\rceil.$$

A TL_4-irregular total l-labeling φ_{TL_4} from vertex and edge labels to integer number until l is as follows:
for $i = 1, 2, \ldots, n$,

$$\varphi_{TL_4}(x_i) = \left\lceil \frac{i+17}{21} \right\rceil, \quad \varphi_{TL_4}(x_i y_i) = \left\lceil \frac{i+9}{21} \right\rceil,$$
$$\varphi_{TL_4}(y_i) = \left\lceil \frac{i+13}{21} \right\rceil.$$

for $i = 1, 2, \ldots, n-1$,

$$\varphi_{TL_4}(x_i x_{i+1}) = \left\lceil \frac{i+6}{21} \right\rceil, \quad \varphi_{TL_4}(x_i y_{i+1}) = \left\lceil \frac{i}{21} \right\rceil,$$
$$\varphi_{TL_4}(y_i y_{i+1}) = \left\lceil \frac{i+3}{21} \right\rceil.$$

A TL_5-irregular total l-labeling φ_{TL_5} from vertex and edge labels to integer number until l is as follows:
for $i = 1, 2, \ldots, n$,

$$\varphi_{TL_5}(x_i) = \left\lceil \frac{i+22}{27} \right\rceil, \quad \varphi_{TL_5}(x_i y_i) = \left\lceil \frac{i+12}{27} \right\rceil,$$
$$\varphi_{TL_5}(y_i) = \left\lceil \frac{i+17}{27} \right\rceil.$$

for $i = 1, 2, \ldots, n-1$,

$$\varphi_{TL_5}(x_i x_{i+1}) = \left\lceil \frac{i+8}{27} \right\rceil, \quad \varphi_{TL_5}(x_i y_{i+1}) = \left\lceil \frac{i}{27} \right\rceil,$$
$$\varphi_{TL_5}(y_i y_{i+1}) = \left\lceil \frac{i+4}{27} \right\rceil.$$

If we continue the function of vertex and edge labels, then we get general function of vertex and edge labels for φ_{TL_m}. A TL_m-irregular total l-labeling φ_{TL_m} from vertex and edge labels to integer number until l is as follows:
for $i = 1, 2, \ldots, n$,

$$\varphi_{TL_m}(x_i) = \left\lceil \frac{i+5m-3}{6m-3} \right\rceil, \quad \varphi_{TL_m}(x_i y_i) = \left\lceil \frac{i+3m-3}{6m-3} \right\rceil,$$
$$\varphi_{TL_m}(y_i) = \left\lceil \frac{i+4m-3}{6m-3} \right\rceil.$$

for $i = 1, 2, \ldots, n-1$,

$$\varphi_{TL_m}(x_i x_{i+1}) = \left\lceil \frac{i+2m-2}{6m-3} \right\rceil, \quad \varphi_{TL_m}(x_i y_{i+1}) = \left\lceil \frac{i}{6m-3} \right\rceil,$$
$$\varphi_{TL_m}(y_i y_{i+1}) = \left\lceil \frac{i+m-1}{6m-3} \right\rceil.$$

We can see that every vertex and edge labels under φ_{TL_m}-labeling and φ_{TL_m}-labeling are almost l. We get the φ_{TL_m}-weight of TL_m^j, $j = 1, 2, \ldots, n-m+1$, under the total labeling TL_m, $3 \leq m < n$, we get

$$wt_{\varphi_{TL_m}}(TL_m^j) = \sum_{v \in V(TL_m^j)} \varphi_(v) + \sum_{e \in E(TL_m^j)} \varphi_(e) \tag{2}$$

From the $wt_{\varphi_{TL_m}}(TL_m^j)$ that the sum of vertex and edge labels, we get increasing sequences. We can prove increase sequences with $wt_{\varphi_{TL_m}}(TL_m^{j+1}) - wt_{\varphi_{TL_m}}(TL_m^j) = 1$, $3 \leq m < n$ and $j = 1, 2, \ldots, n-m$.

For every $j = 1, 2, \ldots, n-m$, we have

$$\begin{aligned}
wt_{\varphi_m}(TL_m^{j+1}) - wt_{\varphi_m}(TL_m^j) &= \varphi_{TL_m}(x_{j+m}) + \varphi_{TL_m}(y_{j+m}) + \varphi_{TL_m}(x_{j+m}y_{j+m}) \\
&\quad + \varphi_{TL_m}(x_{j+m-1}x_{j+m}) + \varphi_{TL_m}(y_{j+m-1}y_{j+m}) \\
&\quad + \varphi_{TL_m}(x_{j+m-1}y_{j+m}) - \varphi_{TL_m}(x_j) - \varphi_{TL_m}(y_j) \\
&\quad - \varphi_{TL_m}(x_j y_j) - \varphi_{TL_m}(x_j x_{j+1}) - \varphi_{TL_m}(y_j y_{j+1}) \\
&\quad - \varphi_{TL_m}(x_j y_{j+1}) \\
&= \left\lceil \frac{j+m+5m-3}{6m-3} \right\rceil + \left\lceil \frac{j+m+4m-3}{6m-3} \right\rceil \\
&\quad + \left\lceil \frac{j+m+3m-3}{6m-3} \right\rceil + \left\lceil \frac{j+m-1+2m-2}{6m-3} \right\rceil \\
&\quad + \left\lceil \frac{j+m-1+m-1}{6m-3} \right\rceil + \left\lceil \frac{j+m-1}{6m-3} \right\rceil \\
&\quad - \left\lceil \frac{j+5m-3}{6m-3} \right\rceil - \left\lceil \frac{j+4m-3}{6m-3} \right\rceil - \left\lceil \frac{j+3m-3}{6m-3} \right\rceil \\
&\quad - \left\lceil \frac{j+2m-2}{6m-3} \right\rceil - \left\lceil \frac{j+m-1}{6m-3} \right\rceil - \left\lceil \frac{j}{6m-3} \right\rceil \\
&= \left\lceil \frac{j+6m-3}{6m-3} \right\rceil - \left\lceil \frac{j}{6m-3} \right\rceil \\
&= \left\lceil \frac{j}{6m-3} \right\rceil + \left\lceil \frac{6m-3}{6m-3} \right\rceil - \left\lceil \frac{j}{6m-3} \right\rceil \\
wt_{\varphi_m}(TL_m^{j+1}) - wt_{\varphi_m}(TL_m^j) &= 1
\end{aligned}$$

We get that every weight TL_m covering of TL_n graph, $n \geq 3$ is different which is arithmathics sequence with different 1. The total weight of TL_m covering is $wt_{\varphi_{TL_m}}(TL_m^j) = \sum_{v \in V(TL_m^j)} \varphi_{TL_m}(v) + \sum_{e \in E(TL_m^j)} \varphi_4(e) = \{6m-3, 6m-2, 6m-1, \ldots, n+5m-3\}$. Otherwise, we take the largest label function, $\varphi_{TL_m}(x_i) = \left\lceil \frac{i+5m-3}{6m-3} \right\rceil$; $1 \leq i \leq n$ for $i = n$, then we get,

$$
\begin{aligned}
tHs(TL_n) &\leq \left\lceil \frac{i+5m-3}{6m-3} \right\rceil \\
&= \left\lceil \frac{n+5m-3}{6m-3} \right\rceil
\end{aligned}
$$

Hence, $tHs(TL_n, TL_m) \geq \left\lceil \frac{n+5m-3}{6m-3} \right\rceil$ and $tHs(TL_n, TL_m) \leq \left\lceil \frac{n+5m-3}{6m-3} \right\rceil$, $tHs(TL_n, TL_m) = \left\lceil \frac{n+5m-3}{6m-3} \right\rceil$. □

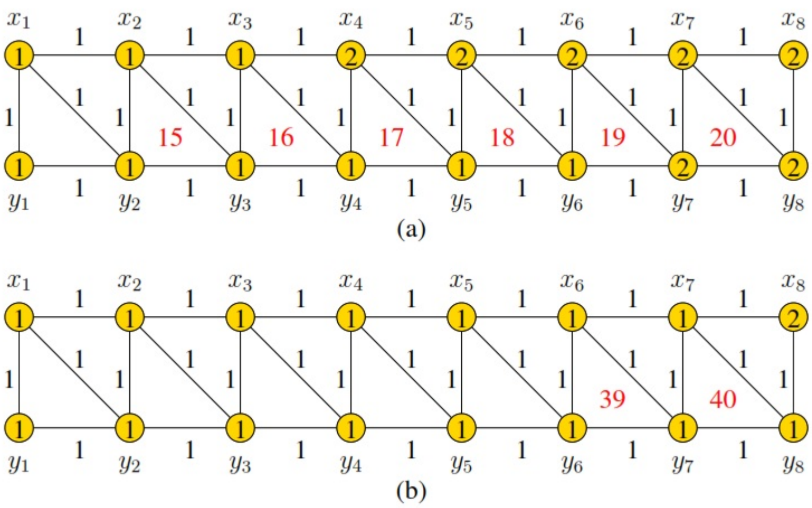

Figure 2. The Illustration of Total (a) TL_3-Irregularity of TL_8 (b) TL_7-Irregularity of TL_8

The result of this research contain two theorem that both of theorems reach the lower bound as 1. We can see the proof of theorem has been proved by the lower bound and the upper bound. We also give the example and illustration of the total H-irregular labeling.

3. Conclusion

We have given the result of total H-irregularity strength of triangular ladder graphs which admitting H equivalent to windmill graph and triangular ladder graph. We give suggestion for further researcher to find the total H-irregularity strength of related tree graphs.

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