Super edge local antimagic total labeling of some graph operation

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Abstract. Let G be a simple connected, and undirected graph. Graph G has a set of vertex denoted by $V(G)$ and a set of edge denoted by $E(G)$. $d(v)$ is the degree of vertex $v \in V(G)$ and $\Delta(G)$ is the maximum degree of G. A total labeling of graph $G(V,E)$ is said to be local edge antimagic total labeling if a bijection $f : V(G) \cup E(G) \longrightarrow \{1, 2, 3, ..., |V(G)| + |E(G)|\}$ such that for any two adjacent edges e_1 and e_2, $w_t(e_1) \neq w_t(e_2)$, where for $e = uv \in G$, $w_t(e) = f(u) + f(uv) + f(v)$. The local edge antimagic total labeling induces a proper edge coloring of G if each edge e is assigned the color $w_t(e)$. The edge local antimagic chromatic number of G denoted by $\gamma_{elat}(G)$, is the minimum number of distinct color induced by edge weights over all local antimagic total labeling of G. In this paper, we determined the edge local antimagic chromatic number of Diamond Ladder graph, $P_n \odot P_m$Th ree Circular ladder graphht, and $shack(F_2, v, n)$.

1. Introduction

Mathematics is one of basic sciences which play an important role in the real life. Mathematics is always needed in the advance of modern technology. Mathematics consists of several branches, one of them is a discrete mathematics. Everyone needs math in every way and everyday life, so it be is very useful and sought after by many people. There are several components that missing object during the learning process, one of the interesting topics in discrete mathematics is graph theory. There are many topics in graph theory, such as coloring, graph labeling, etc. We identify edge local antimagic total coloring of graph. Graph $G(V,E)$ consist of vertex set $V(G)$ and edge set $E(G)$. We use finite and connected graph. The definition of graph taken from [6]. The Element of graph such that vertex, edge and face labeled by natural number.

The type of labeling in this paper is antimagic. Hartsfield and Ringel [7] introduced the concept of antimagic labeling. A labeling of graph can be called antimagic if all weight of labels have different values. The result of antimagic total labelings [4, 5] presented by Dafik *et. al.* They determined super edge-antimagic total labelings and super edge-antimagicness of graphs.

A total labeling of graph $G(V,E)$ is said to be edge local antimagic total labeling if a bijection $f : V(G) \cup E(G) \longrightarrow \{1, 2, 3, ..., |V(G)|+|E(G)|\}$ such that for any two adjacent edges e_1 and e_2, $w_t(e_1) \neq w_t(e_2)$, where for $e = uv \in G$, $w_t(e) = f(u) + f(uv) + f(v)$. The edge local antimagic total labeling induces a proper edge coloring of G if each edge e is assigned the color $w_t(e)$. The edge local antimagic chromatic number of G denoted by $\gamma_{elat}(G)$, is the minimum number of distinct color induced by edge weights over all local antimagic total labeling of G.

Local vertex antimagic coloring of graph introduced by Arumugam *et al.* [3]. A different type of local antimagic has been developed by Agustin *et. al.* [1], namely local edge antimagic coloring of graph. They identifed the lower bound and the upper bound of the local antimagic coloring. Agustin *et. al.* [1] studied a different type of local antimagic coloring, namely local edge antimagic coloring. They studied the existence of local edge antimagic coloring of some special graphs and the lower bound of local edge antimagic. The chromatic number of some graph by $\gamma_{elat}(G) \geq \Delta(G)$. Agustin *et. al.* [2] studied super local edge antimagic total coloring of any graph using EAVL technique and they found the lemma by following.

Lemma 1.1. *[2] If $\Delta(G)$ is maximum degrees of G, then we have $\gamma_{leat}(G) \geq \Delta(G)$.*

We use two operations, namely Corona and Shackles. For any integer $k \geq 2$, the corona product of graph $G \odot_k H$ recursively of $G \odot H$ as $G \odot_k H = (G \odot_{k\ -1} H) \odot H$. The graph $G \odot_k H$ is named as corona product or multicorona product of graph G and H for more definition details can be seen in Furmanczyk *et. al.* [8]. Let k ≥ 2 be an integer number. We define a shackle as a graph constructed by non-trivial connected graphs $G_1, G_2, ..., G_k$ such that G_s and G_t have no a common vertex for every $s, t \in [1, k]$ with $|s - t| \geq 2$, and for every $i \in [1, k-1]$, G_i and $G_i + 1$ share exactly one common vertex, called linkage vertex, and the k_1 linking vertices are all distinct. We denote a shackle graph by $shack(G_1, G_2, ..., G_k)$.

2. Main Result

In this paper we present and give a result of super edge local antimagic total labeling of some graph operation. Furthermore we have determine chromatic number of super edge local antimagic total labeling of DL_n, $P_n \odot P_m$, TCL_n, $shack(F_2, v, n)$.

Theorem 2.1. *Let DL_n be Diamond Ladder graph for $n \geq 2$, the local edge antimagic total chromatic number of DL_n is $5 \leq \chi_{elat}(DL_n) \leq 7$*

Proof. The graph DL_n is a connected graph that with the vertex set $\mathrm{V(DL}_n) = \{x_p, y_p; 1 \leq p \leq n\} \cup \{a_{1,p}; 1 \leq p \leq n\} \cup \{a_{2,p}; 1 \leq p \leq n\}$ and edge set $E(DL_n) = \{x_p x_{p+1}, y_p y_{p+1}; 1 \leq p \leq n-1\} \cup \{x_p y_p; 1 \leq p \leq n\} \cup \{x_p a_{1,p}; 1 \leq p \leq n\} \cup \{x_p a_{2,p}; 1 \leq p \leq n\} \cup \{a_{1,p} y_p; 1 \leq p \leq n\} \cup \{a_{2,p} y_p; 1 \leq p \leq n\} \cup \{a_{2,p} a_{1,p+1}; 1 \leq p \leq n-1\}$. Hence we have cardinality of graph (DL_n) are $|V(DL_n)| = 4n$ and $|E(DL_n)| = 8n - 3$.

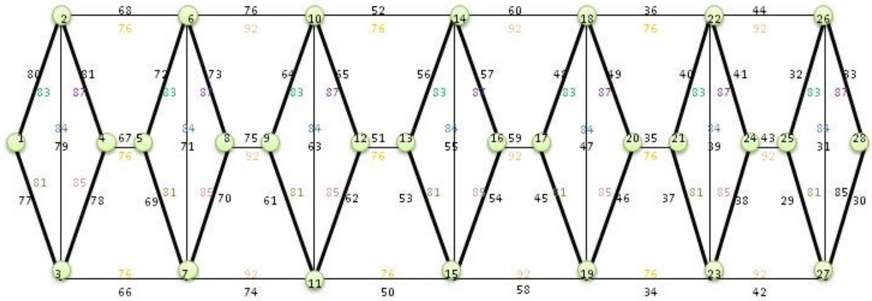

Figure 1. Diamond Ladder Graph

The proof of the edge local antimagic total chromatic number of (DL_n) is $5 \leq \chi_{elat}(DL_n) \leq 7$, we will show $\chi_{elat}(DL_n) \geq 5$ and $\chi_{elat}(DL_n) \leq 7$. To show $\chi_{elat}(DL_n) \geq 5$, we identify the maximum degree of graph DL_n by the lemma of agustin *et. al.* [2] the lower bound of leat chromatic number of graph is $\chi_{elat} \geq \Delta$. We identify the Diamond Ladder graph have maximum degree $\Delta(DL_n) = 5$ Based on the lower bound in [2] we have $\Delta(DL_n) = 5$. Then we have $\Delta(DL_n) = 5$, so there must be at least have 5 colors. This show that the lower bound of the edge local antimagic is total anti magic coloring (DL_n) is $\gamma_{elat}(DL_n) \geq 5$.

Furthermore, we have total labeling on vertices and edges of (DL_n) graph, $f : B(DL_n) \cup E(DL_n) \longrightarrow \{1, 2, 3, ..., |B(DL_n)| + |E(DL_n)|\}$ as the following.

$$f(v) = \begin{cases} 4p & \text{if } v = c_{2,p} \\ 4p-1 & \text{if } v = y_p \\ 4p-2 & \text{if } v = x_p \\ 4p-3 & \text{if } v = c_{2,p} \end{cases}$$

$$f(e) = \begin{cases} 12n-8p+1 & \text{if } e = c_{1,p}, y_p \\ 12n-8p+2 & \text{if } e = c_{2,p}, y_p \\ 12n-8p+3 & \text{if } e = x_p, y_p \\ 12n-8p+4 & \text{if } e = c_{1,p}, xp \\ 12n-8p+5 & \text{if } e = c_{2,p}, xp \\ 4n+8+\frac{16n-16p}{2}\text{- } 16 & \text{if } e = x_p x_p+1, for\ p = 1,3,5,...n \\ 4n+16+\frac{16n-16p+16}{2}\text{- } 16 & \text{if } e = x_p x_p+1, for\ p = 2,4,6,...n \\ 4n+6+\frac{16n-16p}{2}\text{- } 16 & \text{if } e = y_p y_p+1, for\ p = 1,3,5,...n \\ 4n+14+\frac{16n-16p+16}{2}\text{- } 16 & \text{if } e = y_p y_p+1, for\ p = 2,4,6,...n \\ 4n+7+\frac{16n-16p}{2}\text{- } 16 & \text{if } e = c_{2,p}, c_{1,p}+1, for\ p = 1,3,5,...n \\ 4n+15+\frac{16n-16p+16}{2}\text{- } 16 & \text{if } e = c_{2,p}, c_{1,p}+1, for\ p = 2,4,6,...n \end{cases}$$

From total labeling on vertices and edges of (DL_n) above, we have the total edge weighs are as follows,

$$w_t(e) = \begin{cases} 12n-1 & \text{if } e = x_p, c_{1,p} \\ 12n-3 & \text{if } e = c_{1,p}, y_p \\ 12n & \text{if } e = y_p, x_p \\ 12n+1 & \text{if } e = y_p, x_{2,p} \\ 12n+3 & \text{if } e = c_{2,p}, x_p \\ 12n-8 & \text{if } e = x_px_p+1\cup \text{ y}_py_p+1\cup \text{ c}_{2,p}, c_{1,p}, for\ p = 1,3,5,...n \\ 12n+8 & \text{if } e = x_px_p+1\cup \text{ y}_py_p+1\cup \text{ c}_{2,p}, c_{1,p}, for\ p = 2,4,6,...n \end{cases}$$

Therefore, it gives $\chi_{elat}(DL_n) \leq 7$. It concludes that $5 \leq \gamma_{elat}(DL_n) \leq 7$. □

Figure 1 shows an example edge local antimagic total coloring of DL_n

Theorem 2.2. *For n, m natural number $n \geq 3$ and $m \geq 3$, then the cromatic number is the total edge local antimagic coloring of the graph $P_n \odot P_m$ is:*

$$\gamma_{elat}(P_n \odot P_m) = \begin{cases} m+4, & n \text{ odd, } m \text{ odd} \\ m+5, & n \text{ odd, } m \text{ even} \end{cases}$$

Proof. $P_n \odot P_m$ where n odd, $n \geq 3$ and $m \geq 3$,is a connected graph with the cardinality of vertices on this graph is $|B(P_n \odot P_m)| = mn + n$ and furthermore for edge cardinality is $|E(P_n \odot P_m)| = 2mn - 1$. We devide the proof into two cases, the first is for m odd and the socond is for m even.

Case 1. For n odd and m odd,we will show the edge local antimagic total chromatic number $\gamma_{elat}(P_n \odot P_m) = m + 4$. The chromatic number of path graph for n odd is $\gamma_{elat}(P_n) = 2$ and fan graph for n odd is $\gamma_{elat}(P_n) = n + 2$, thus, we got $P_n \odot P_m$ with n odd and m odd is $\gamma_{elat}(P_n \odot P_m) \geq \gamma_{elat}(P_n) + \gamma_{elat}(P_m) = 2 + (m + 2) = m + 4$. Based on the explaination, we got $\gamma_{elat}(P_n \odot P_m) \geq m + 4$.

Then, we will proof $\gamma_{elat}(P_n \odot P_m) \leq m+4$. In order to proof $\gamma_{elat}(P_n \odot P_m) \leq m+4$, we will show them by giving vertex label and edges label using a bijective function $f : V(P_n \odot P_m) \cup E(P_n \odot P_m) \longrightarrow \{1, 2, 3, ..., |V(P_n \odot P_m)| + |E(P_n \odot P_m)|\}$.

The function of the edge label and the vertex label of $P_n \odot P_m$ as follows:

$$f(e) = \begin{cases} \frac{p+1}{2} + 2nm - nj - n, & \text{if } e = x_{p,j}x_{p,j+1},\ p, \text{ for } p = 1,\ 3,\ 5,...\text{n } 1 \leq p \leq n \text{ and} \\ & j = 1,\ 3,\ 5,...\text{n}, 1 \leq j \leq m \\ \frac{p+n+1}{2} + 2nm - nj - n, & \text{if } e = x_{p,j}x_{p,j+1},\ p \text{ for } p\text{= } 2,\ 4,\ 6,...\text{n } 2 \leq p \leq n \text{ and} \\ & j\text{= } 1,\ 3,\ 5,...\text{n}, 1 \leq j \leq m \\ \frac{p+1}{2} + 2nm - nj + n, & \text{if } e = x_{p,j}x_{p,j+1},\ p \text{ for } p = 1,3,5,...\text{n } 1 \leq p \leq n \text{ and} \\ & j\text{= } 2,\ 4,\ 6,...\text{n}, 2 \leq j \leq m \\ \frac{p+n+1}{2} + 2nm - nj + n, & \text{if } e = x_{p,j}x_{p,j+1},\ p \text{ for } p\text{= } 2,\ 4,\ 6,...\text{n } 2 \leq p \leq n \text{ and} \\ & j\text{= } 2,\ 4,\ 6,...\text{n}, 2 \leq j \leq m \\ \frac{5nm+nj+p}{2} - n, & \text{even } e = x_px_{p,j},\ p \text{ even } 2 \leq p \leq n \text{ and} \\ & j = 1,\ 3,\ 5,...\text{n}, 1 \leq j \leq m \\ \frac{nj}{2} + 2nm - p + 1, & \text{if } e = x_px_{p,j},\ 1 \leq p \leq n \text{ and } j\text{= } 2,\ 4,\ 6,\ ...\text{n}, \\ & 2 \leq j \leq m \\ 3nm + n - p - 1, & \text{if } e = x_px_{p+},\ p \text{ for } p = 1,\ 3,\ 5,...\text{n },\ 1 \leq p \leq n-1 \\ 3nm + n - p + 1, & \text{if } e = x_px_{p+1},\ p \text{ for } P = 2,\ 4,\ 5,\ ...\text{n}, 2 \leq p \leq n-1 \end{cases}$$

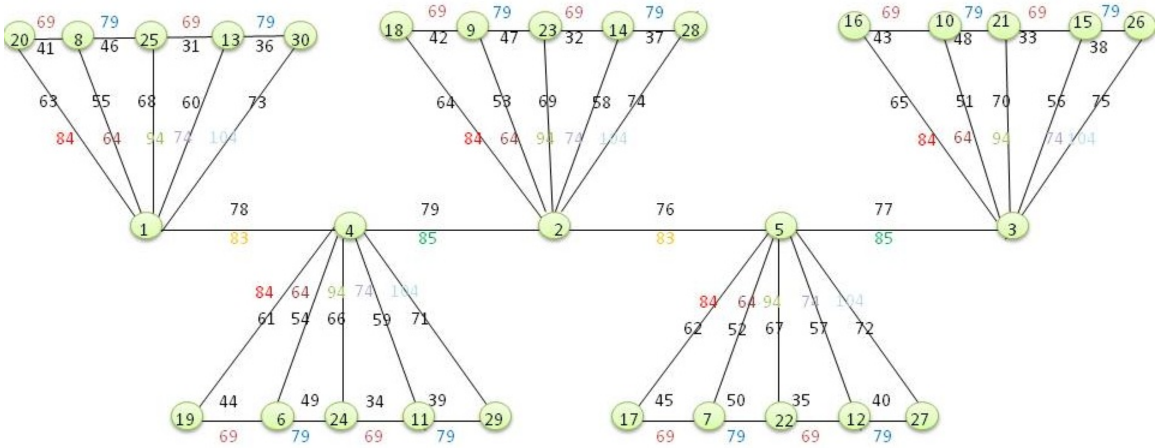

Figure 2. $P_n \odot P_m$ Graph

$$f(v) = \begin{cases} \frac{i+1}{2}, & \text{if } v = x_p,\ p = 1, 3, 5,...\text{n},\ 1 \le p \le n \\ \frac{p+n+1}{2}, & \text{if } v = x_p,\ p = 2, 4, 6,..\text{n},\ 1 \le p \le n \\ n - p + 1 + \frac{nm+nj}{2}, & \text{if } v = x_{p,j},\ 1 \le p \le n \text{ and } j\text{= 1, 3, 5,...n} \\ & 1 \le j \le m \\ \frac{p+nj}{2}, & \text{if } v = x_{p,j},\ p = 2, 4, 6,...\text{n},\ 1 \le p \le n \text{ and } j\text{= 2, 4, 6,...n} \\ & 1 \le j \le m \\ \frac{p+nj+n}{2}, & \text{if } v = x_{p,j},\ p = 1, 3, 5,...\text{n},\ 1 \le p \le n \text{ dan } j\text{=2, 4, 6,...n} \\ & 1 \le j \le m \end{cases}$$

Determine the total side weight of the graph $P_n \odot P_m$ by adding two point labels and one side label that is $w_t(e) = f(u) + f(uv) + f(v)$, so we get the total side weights as follows:

$$w_t(e) = \begin{cases} \frac{6nm+2nj+n+3}{2}, & \text{if } e = x_p x_{p,j},\ 1 \le p \le n \text{ and } j\text{= 1, 3, 5,...n },\ 1 \le j \le m \\ \frac{4nm+2nj+n+3}{2}, & \text{if } e = x_p x_{p,j},\ 1 \le p \le n \text{ and } j = \text{2, 4, 6,...n},\ 2 \le j \le m \\ \frac{5nm+2n+3}{2}, & \text{if } e = x_{p,j} x_{p,j+1},\ 1 \le p \le n \text{ and } j = \text{1, 3, 5,...n},\ 1 \le j \le m \\ \frac{5nm+6n+3}{2}, & \text{if } e = x_{p,j} x_{p,j+1},\ 1 \le p \le n \text{ and } j\text{= 2, 4, 6,...n},\ 2 \le j \le m \\ \frac{6nm+3n+1}{2}, & \text{if } e = x_p x_{p+1},\ p = \text{1, 3, 5, ...n},\ 1 \le p \le n-1 \\ \frac{6nm+3n+5}{2}, & \text{if } e = x_p x_{p+1},\ p = \text{2, 4, 6,...n},\ 1 \le p \le n-1 \end{cases}$$

Based on the total edge weight function, then $\gamma_{elat}(P_n \odot P_m) \le m + 4$. In the total edge weight function, it can be seen that the edge weights on the neighboring edges have dierent side weights, this shows that the colors on the neighboring edges are also different. Because $\gamma_{elat}(P_n \odot P_m) \ge m+4$ dan $\gamma_{elat}(P_n \odot P_m) \le m+4$ it can be concluded that for n odd dan m odd is $\gamma_{elat}(P_n \odot P_m) = m + 4$.

case 2. For n odd and m even the chromatic number of the coloring will be shown *edge local antimagic total* $\gamma_{elat}(P_n \odot P_m) = m+5$. To show $\gamma_{elat}(P_n \odot P_m) = m+5$ then it must be proven $\gamma_{elat}(P_n \odot P_m) \geq m+5$ dan $\gamma_{elat}(P_n \odot P_m) \leq m+5$. First, it will show the lower bound of coloring *edge local antimagic total* from graph $P_n \odot P_m$ is $\gamma_{elat}(P_n \odot P_m) \geq m+5$. Based on Lemma**??**, $\gamma_{elat}(P_n \odot P_m) \geq \gamma_{elat}(P_n) + \gamma_{elat}(P_m)$. Chromatic numbers on the path graph for n odd is $\gamma_{elat}(P_n) = 2$ and fan graphs for n even is $\gamma_{elat}(P_n) = n+3$, so obtained $\gamma_{elat}(P_n \odot P_m) \geq \gamma_{elat}(P_n) + \gamma_{elat}(P_m) = 2 + (m+3) = m+5$.

Next, it will be proven $\gamma_{elat}(P_n \odot P_m) \leq 5$. To prove $\gamma_{elat}(P_n \odot P_m) \leq 5$ will be indicated by labeling the points and edges using the function objectively $f : B(P_n \odot P_m) \cup E(P_n \odot P_m) \longrightarrow \{1, 2, 3, ..., |B(P_n \odot P_m)| + |E(P_n \odot P_m)|\}$.

Function of vertex label and edge label from $P_n \odot P_m$ as follows:

$$f(v) = \begin{cases} \frac{p+1}{2}, & \text{if } v = x_p,\ p = 1, 3, 5, \text{...n},\ 1 \leq p \leq n \\ \frac{p+n+1}{2}, & \text{if } v = x_p,\ p = 2, 4, 6, \text{...n},\ 1 \leq p \leq n \\ \frac{p+nj}{2}, & \text{if } v = x_{p,j},\ p = 2, 4, 6, \text{...n},\ 1 \leq p \leq n \text{ and } j = 2, 4, 6, \text{...n} \\ & 1 \leq j \leq m \\ \frac{p+nj+n}{2}, & \text{if } v = x_{p,j},\ p = 1, 3, 5, \text{...n},\ 1 \leq p \leq n \text{ and } j = 2, 4, 6, \text{...n} \\ & 1 \leq j \leq m \\ n - p + 1 + \frac{nm+nj+n}{2}, & \text{if } v = x_{p,j},\ 1 \leq p \leq n \text{ and } j = 1, 3, 5, \text{...n} \\ & 1 \leq j \leq m \end{cases}$$

$$f(e) = \begin{cases} \frac{p+1}{2} + 2nm - n, & \text{if } e = x_{p,j}x_{p,j+1},\ p = 1, 3, 5, \text{...n } 1 \leq p \leq n \text{ and } j = 1 \\ \frac{p+n+1}{2} + 2nm - n, & \text{if } e = x_{p,j}x_{p,j+1},\ p = 2, 4, 6, \text{...n } 2 \leq p \leq n \text{ and } j = 1 \\ \frac{p+1}{2} + 2nm - nj - n, & \text{if } e = x_{p,j}x_{p,j+1},\ p = 1, 3, 5, \text{...n} 1 \leq p \leq n \text{ and} \\ & j = 2, 4, 6, \text{...n},\ 2 \leq j \leq m \\ \frac{p+n+1}{2} + 2nm - nj - n, & \text{if } e = x_{p,j}x_{p,j+1},\ p = 2, 4, 6, \text{...n } 2 \leq p \leq n \text{ and} \\ & j = 2, 4, 6, \text{...n},\ 2 \leq j \leq m \\ \frac{p+1}{2} + 2nm + n - nj, & \text{if } e = x_{p,j}x_{p,j+1},\ p = 1, 3, 5, \text{...n } 1 \leq p \leq n \text{ and} \\ & j = 1, 3, 5, \text{...n},\ 3 \leq j \leq m \\ \frac{p+n+1}{2} + 2nm + n - nj, & \text{if } e = x_{p,j}x_{p,j+1},\ p = 2, 4, 6, \text{...n } 2 \leq p \leq n \text{ and} \\ & j = 1, 3, 5, \text{...n},\ 3 \leq j \leq m \\ \frac{nj}{2} + 2nm - p + 1, & \text{if } e = x_p x_{p,j},\ 1 \leq p \leq n \text{ and } j = 2, 4, 6, \text{...n} \\ & 2 \leq j \leq m,\ 2 \leq j \leq m \\ \frac{5nm+nj+p}{2}, & \text{if } e = x_p x_{p,j},\ p = 1, 3, 5, \text{...n},\ 1 \leq p \leq n \text{ and} \\ & j = 1, 3, 5, \text{...n},\ 1 \leq j \leq m \\ \frac{5nm+nj-n+p}{2}, & \text{if } e = x_p x_{p,j},\ p = 2, 4, 6, \text{...n},\ 1 \leq p \leq n \text{ and} \\ & j = 1, 3, 5, \text{...n},\ 1 \leq j \leq m \\ 3nm + n - p - 1, & \text{if } e = x_p x_{p+},\ p = 1, 3, 5, \text{...n},\ 1 \leq p \leq n-1 \\ 3nm + n - p + 1, & \text{if } e = x_p x_{p+1},\ p = 2, 4, 6, \text{...n},\ 2 \leq p \leq n-1 \end{cases}$$

Determine the total side weight by adding two point labels and one side label that is

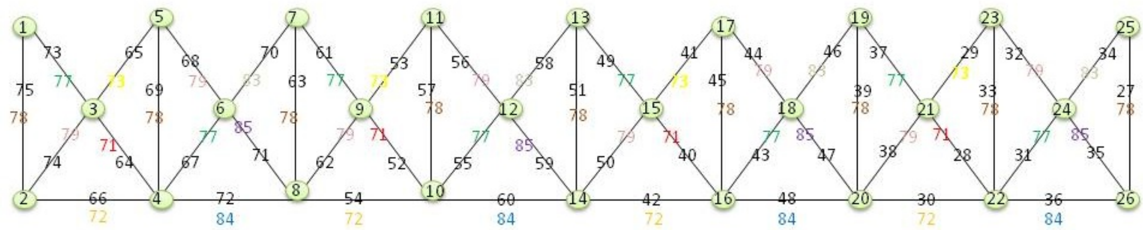

Figure 3. Three circular Ladder Graphs

$w_t(e) = f(u) + f(uv) + f(v)$, so we get the total edge weights as follows:

$$w(e) = \begin{cases} \frac{6nm+2nj+3n+3}{2}, & \text{if } e = x_px_{p,j},\ 1 \le p \le n \text{ and } j = 1,\ 3,\ 5,..\text{n},\ 1 \le j \le m \\ \frac{4nm+2nj+n+3}{2}, & \text{if } e = x_px_{p,j},\ 1 \le p \le n \text{ and } j = 2,\ 4,\ 6,...\text{n},\ 2 \le j \le m \\ \frac{5nm+5n+3}{2}, & \text{if } e = x_{p,j}x_{p,j+1},\ j = 1 \\ \frac{5nm+3n+3}{2}, & \text{if } e = x_{p,j}x_{p,j+1},\ 1 \le p \le n \text{ and } j = 2,\ 4\ ,6...\text{n},\ 2 \le j \le m \\ \frac{5nm+7n+3}{2}, & \text{if } e = x_{p,j}x_{p,j+1},\ 1 \le p \le n \text{ and } j = 1,\ 3,\ 5,...\text{n},\ 3 \le j \le m \\ \frac{6nm+3n+1}{2}, & \text{if } e = x_px_{p+1},\ p = 1,\ 3,\ 5,...\text{n},\ 1 \le p \le n-1 \\ \frac{6nm+3n+5}{2}, & \text{if } e = x_px_{p+1},\ p = 2,\ 4,\ 6,...\text{n},\ 1 \le p \le n-1 \end{cases}$$

Based on the total edge weight function, then $\gamma_{elat}(P_n \odot P_m) \le m+5$. In the total edge weight function, it can be seen that the edge weights on the neighboring edges have different side weights, this shows that the colors on the neighboring edges are also different. Because $\gamma_{elat}(P_n \odot P_m) \ge m+5$ and $\gamma_{elat}(P_n \odot P_m) \le m+5$ it can be concluded that for n odd and m odd is $\gamma_{elat}(P_n \odot P_m) = m+5$. □

Theorem 2.3. *Let TCL_n be Three Circular Ladder graph for $n \ge 4$, the local edge antimagic total chromatic number of TCL_n is* $5 \le \chi_{elat}(TCL_n) \le 9$

Proof. The graph TCL_n is a graph that is connected to the vertexs set $B(TCL_n) = \{x_p, y_p; 1 \le p \le n+1\} \cup \{z_p; 1 \le p \le n\}$=2{n+1} + n =2n+2+n and edge set $E(TCL_n) = \{x_py_p; 1 \le p \le n+1\} \cup \{x_pz_p; 1 \le p \le n\} \cup \{z_py_p; 1 \le p \le n\} \cup \{z_px_{p+1}; 1 \le p \le n\} \cup \{z_py_{p+1}; 1 \le p \le n\} \cup \{y_py_{p+1}; 1 \le p \le n\}$. Hence we have cardinality of graph (TCL_n) are $|B(TCL_n)| = 3n+2$ and $|E(TCL_n)| = 6n+1$.

To proof the edge local antimagic total chromatic number of (TCL_n) is $5 \le \chi_{elat}(TCL_n) \le 9$, we will show $\chi_{elat}(TCL_n) \ge 5$ and $\chi_{elat}(TCL_n) \le 9$. To show $\chi_{elat}(TCL_n) \ge 5$, we identify the maximum degree of graph TCL_n. Then we have $\Delta(TCL_n) = 5$, so there must be at least have 5 colors. all of these are examples of the lower bound of the edge local of total antimagic coloring (TCL_n) is $\gamma_{elat}(TCL_n) \ge 5$.

Next, we will show that the upper bound of the edge local antimagic total coloring of (TCL_n) is$\gamma_{elat}(TCL_n) \le m+1$. Define a bijection $f : B(TCL_n) \cup E(TCL_n) \longrightarrow \{1, 2, 3, ..., |B(TCL_n)| + |E(TCL_n)|\}$ by the following.

$$f(v) = \begin{cases} 6p-5 & \text{if } v = x_p \ for \ p = 1,3,5,...n \\ 6p-1 & \text{if } v = x_p \ for p = 2,4,6,...n \\ 6p-4 & \text{if } v = y_p \ for, p = 1,3,5,...n \\ 6p-2 & \text{if } v = y_p \ for, p = 2,4,6,...n \\ 6p-3 & \text{if } v = z_p \ for, p = 1,3,5,...n \\ 6p & \text{if } v = z_p \ forp = 2,4,6,...n \end{cases}$$

$$f(e) = \begin{cases} 9n-6p+9 & \text{if } e = x_p, y_p \ for \ p = 1,3,5,...n \\ 9n-6p+9 & \text{if } e = x_p, y_p \ for \ p = 2,4,6,...n \\ 9n-6p+7 & \text{if } e = x_p, z_p \ for \ p = 1,3,5,...n \\ 9n-6p+8 & \text{if } e = x_p, z_p \ for \ p = 2,4,6,...n \\ 9n-6p+8 & \text{if } e = z_p, y_p \ for \ p = 1,3,5,...n \\ 9n-6p+7 & \text{if } e = z_p, y_p \ for \ p = 2,3,5,...n \\ 9n-6p-1 & \text{if } e = z_p, x_{p+1} \ for \ p = 1,3,5,...n \\ 9n-6p+10 & \text{if } e = z_p, x_{p+1} \ for \ p = 2,4,6,...n \\ 9n-6p-2 & \text{if } e = z_p, y_{p+1} \ for \ p = 1,3,5,...n \\ 9n-6p+11 & \text{if } e = z_p, y_{p+1} \ for \ p = 2,4,6,...n \\ 9n-6p & \text{if } e = y_p, y_{p+1} \ for \ p = 1,3,5,...n \\ 9n-6p+12 & \text{if } e = y_p, y_{p+1} \ for \ p = 2,4,6,...n \end{cases}$$

From the vertex label and edge label above, we have the total edge weight as follows

$$w_t(e) = \begin{cases} 9n+6 & \text{if } e = x_p, y_p \\ 9n+5 & \text{if } e = x_p, z_p \ for \ p = 1,3,5,...n \\ 9n+5 & \text{if } e = y_p, z_p \ for \ p = 2,4,6,...n \\ 9n+7 & \text{if } e = x_p, z_p \ for \ p = 1,3,5,...n \\ 9n+7 & \text{if } e = y_p, z_p \ for \ p = 2,4,6,...n \\ 9n & \text{if } e = y_p, y_{p+1} \ for \ p = 1,3,5,...n \\ 9n+18 & \text{if } e = y_p, y_{p+1} \ for \ p = 2,4,6,...n \\ 9n+1 & \text{if } e = z_p, y_{p+1} \ for \ p = 1,3,5,...n \\ 9n+13 & \text{if } e = z_p, y_{p+1} \ for \ p = 2,4,6,...n \\ 9n+1 & \text{if } e = z_p, x_{p+1} \ for \ p = 1,3,5,...n \\ 9n+11 & \text{if } e = z_p, x_{p+1} \ for \ p = 2,4,6,...n \end{cases}$$

Therefore, we can see how the description of total edge weights above f will induce the right edge coloring of a graph TCL_n and it gives $\chi_{elat}(TCL_n) \leq 9$. Based on the lower bound is $\gamma_{elat}(TCL_n) \geq 5$. It concludes that $5 \leq \gamma_{elat}(TCL_n) \leq 9$. □

Figure 3 shows an example edge local antimagic total coloring of TCL_n

Theorem 2.4. *Let $shack(F_2, v, n)$ be a Shackles of fan graph with $n \geq 3$ and n odd, the super edge local antimagic total chromatic number of $shack(F_2, v, n)$ is $4 \leq \chi_{elat}$ shack(F_2, v, n) ≤ 5*

Proof. The graph $shack(F_2, v, n)$ are $|B$ shack(F_2, v, n) are $|B$ shack(F_2, v, n) are $|B$ shack$(\mathrm{F}_2, v, n)| = 3n + 1$ and $|E$ shack$(\mathrm{F}_2, v, n)| = 5n.| = 3n + 1$ and $|E$ shack$(\mathrm{F}_2, v, n)| = 5n.| = 3n + 1$ and $|E$ shack$(\mathrm{F}_2, v, n)| = 5n.$ is a connected graph

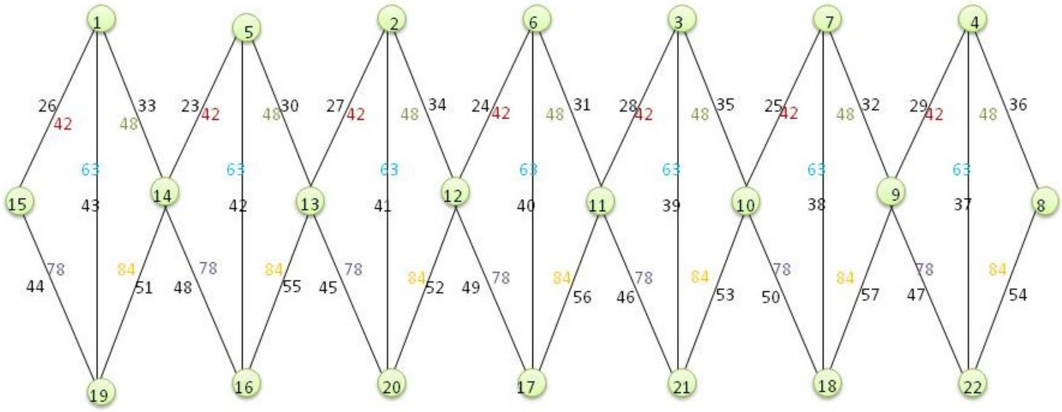

Figure 4. Shackles Graph

with vertex set B shack$(\mathrm{F}_2, v, n) = \{x_p, z_p; 1 \leq p \leq n\} \cup \{y_p; 1 \leq p \leq n+1\}$ and edge set E shack$(\mathrm{F}_2, v, n) = \{x_p y_p; 1 \leq p \leq n\} \cup \{y_p z_p; 1 \leq p \leq n\} \cup \{x_p z_p; 1 \leq p \leq n\} \cup \{x_p y_{p+1}; 1 \leq p \leq n\} \cup \{y_{p+1} z_p; 1 \leq p \leq n\}$. Hence we have cardinality of graph $shack(F_2, v, n)$ are $|B$ shack$(\mathrm{F}_2, v, n)| = 3n+1$ and $|E$ shack$(\mathrm{F}_2, v, n)| = 5n$.

The proof of the edge local antimagic total chromatic number of $shack(F_2, v, n)$ is $5 \leq \chi_{late} shack(F_2, v, n) \leq 5$, we will show $\chi_{elat} shack(F_2, v, n) \geq 4$ and $\chi_{elat} shack(F_2, v, n) \leq 5$. To show $\chi_{elat} shack(F_2, v, n) \geq 5$, we identify the maximum degree of graph $shack(F_2, v, n)$. Then we have Δ $shack(F_2, v, n) = 4$, so there must be at least have 4 colors. And this is a lower boundary of the edge local of total antimagic coloring $shack(F_2, v, n)$ is $\gamma_{elat} shack(F_2, v, n) \geq 4$.

Next, we will display and present in the paper this upper bound of the edge local is total antimagic coloring $shack(F_2, v, n)$ is$\gamma_{elat} shack(F_2, v, n) \leq m+1$.Define a bijection $f : B$ shack$(\mathrm{F}_2, v, n) \cup E(C_n) \longrightarrow \{1, 2, 3, ..., |B$ shack$(\mathrm{F}_2, v, n)| + |E$ shack$(\mathrm{F}_2, v, n)|\}$ by the following.

$$f(v) = \begin{cases} \frac{p+1}{2}, & \text{if } v = x_p \; for \; p = 1, 3, 5, ...n \\ \frac{p+8}{2}, & \text{if } v = x_p \; for \; p = 2, 4, 6, ...n \\ 2n - p + 2, & \text{if } v = y_p \\ \frac{p+37}{2}, & \text{if } v = z_p \; for \; p = 1, 3, 5, ...n \\ \frac{p+30}{2}, & \text{if } v = z_p \; for \; p = 2, 4, 6, ...n \end{cases}$$

$$f(e) = \begin{cases} \frac{12n+p+3}{2}, & \text{if } e = y_p z_p \ for\ p = 1,3,5,...n \\ \frac{12n+p+10}{2}, & \text{if } e = y_p z_p \ for\ p = 2,4,6,...n \\ \frac{12n+p+17}{2}, & \text{if } e = y_{p+1} z_p \ for\ p = 1,3,5,...n \\ \frac{12n+p+24}{2}, & \text{if } e = y_{p+1} z_p \ for\ p = 2,4,6,...n \\ 12n-p-40, & \text{if } e = x_p z_p \\ \frac{12n+p+2-33}{2}, & \text{if } e = x_p y_p \ for\ p = 1,3,5,...n \\ \frac{12n+p+2-40}{2}, & \text{if } e = x_p y_p \ for\ p = 2,4,6,...n \\ \frac{12n+p+2-19}{2}, & \text{if } e = x_p y_{p+1} \ for\ p = 1,3,5,...n \\ \frac{12n+p+2-26}{2}, & \text{if } e = x_p y_{p+1} \ for\ p = 2,4,6,...n \end{cases}$$

From the vertex label and edge label above, we have the total edge weight as follows

$$w_t(e) = \begin{cases} 12n-40 & \text{if } e = y_p, z_p \ for\ p = 1,3,5,...n \\ 12n-36 & \text{if } e = y_p, z_p \ for\ p = 2,4,6,...n \\ 12n-33 & \text{if } e = y_{p+1}, z_p \ for\ p = 1,3,5,...n \\ 12n-29 & \text{if } e = y_{p+1}, z_p \ for\ p = 2,4,6,...n \\ 12n-41 & \text{if } e = x_p, z_p \\ 12n-58 & \text{if } e = x_p, y_p \ for\ p = 1,3,5,...n \\ 12n-61 & \text{if } e = x_p, y_p \ for\ p = 2,4,6,...n \\ 12n-51 & \text{if } e = x_p, y_{p+1} \ for\ p = 1,3,5,...n \\ 12n-54 & \text{if } e = x_p, y_{p+1} \ for\ p = 2,4,6,...n \end{cases}$$

Therefore, we can easily see the total edge weights above f induces a proper edge coloring of $shack(F_2, v, n)$ and it gives $\chi_{elat} shack(F_2, v, n) \leq 5$. Based on the lower bound is $\gamma_{elat} shack(F_2, v, n) \leq 7$. □

Figure 3 shows an example local edge antimagic total coloring of $shack(F_2, v, n)$

3. Conclusion

In this paper we have determine the chromatic number of local edge antimagic total coloring of DL_n, $P_n \odot P_m$, Three Circular ladder graph, and $shack(F_2, v, n)$.

Open Problem 1. *find the super edge local antimagic total chromatic number of other some graf operation.*

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