Resolving domination number of helm graph and it's operation

A N Hayyu [1,2], Dafik[1,2], I M Tirta[3], R Adawiyah [1,2], R M Prihandini [1,2]

[1]CGANT. University of Jember, Indonesia
[2]Departement of Mathematics Education. University of Jember, Indonesia
[3]Departement of Mathematics. University of Jember, Indonesia

E-mail: d.dafik@unej.ac.id, aulianandar73@gmail.com

Abstract. Let G be a connected graph. Dominating set is a set of vertices which each vertex D has at least one neighbor in G. The minimum cardinality of D is called the domination number G ($\gamma(G)$). The metric dimension of G is the minimum cardinality of a series of vertices so that each vertex G is uniquely. It is determined by the distance of vector to the selected vertices. A dominating metric dimension set is a set of vertices has a dominating set D which has condition of metric dimension. The minimum cardinality is called the resolving domination number of G, ($Dom_{Dim}(G)$). We analyze the resolving domination number of helm graph and it's operation. We study combine the existence concept of dominating set and metric dimension. We have obtained the minimum cardinality of dominating number.

1. Introduction

Let G be a graph with vertex set $V\ (G)$ and edge set $E(G)$. Let the vertex set of G be $W = \{s_1, s_2, \ldots, s_k\}$. Representation of metric dimension is $r(v|W) = (d(v, s_1), d(v, s_2), ..., d(v, s_k))$ The k-vector $r(v|W) = (d(v, s_1), d(v, s_2), ..., d(v, s_k))$. The minimum cardinality of resolving set W of G is denoted by $dim(G)$. It is called the metric dimension of G [13]. The metric dimension, $dim(G)$ of a graph G is the minimum cardinality of a set of vertices such that every vertex of G is uniquely. It is determined by the distances of vector to the every choosen vertex. A dominating set D is a set of vertices such that each vertex of G has at least one neighbor in G. The minimum cardinality of the domination number of G is ($\gamma(G)$). Based on these two concept dominating set and metric dimension, we initiate to study the combination of those two studies namely ($Dom_{Dim}(G)$). We analyze the ($Dom_{Dim}(G)$) of helm graph. These are some definition and preposition which is usefull to proof some theorems. We are develop a graph that will become a new graph (line graph, middle graph and total graph). A line graph from a simple graph namely of G denoted by L(G) is obtained by associating vertices with each edge of the graph and connecting two vertices with edges if the corresponding edges of G have the same node [11]. In [18] the middle graph denoted by M(G) of the connected graph G is a graph whose node-set is $V(G) \cup E(G)$ where two vertices are close together if they are edges which border G or one is the node of G and the other is an edge incident with node. The total graph denoted by $T(G)$ of the connected graph G is a graph whose node-set is $V(G) \cup E(G)$ and two adjacent vertices each time that border or events in G [7].

Definition 1.1. *The Helm graph (H_m) is a simple graph obtained from the n-wheel W_m graph next to the edge of the pendant at each vertex of the C_m cycle.*

Preposition 1. *Let H_m be a helm graph, the domination number of graph H_m is m, $\gamma(H_m) = m$*

We will explained about the operation resolving domination number of line graph, middle graph, and total graph. The minimum cardinality is called the resolving domination number G,($Dom_{Dim}(G)$). In this study, resolving domination number will be examined on fan graphs, line graphs, star graphs, and complete graphs. In addition, the development of the graph will become a new graphs (line graph, middle graph and total graph). Line graph L (G) (also called adjoin, cover, derivative, derived, edge, edge-to-vertex point, interchange, representative, or θ-obrazom graph) of a simple graph G is obtained by associating a vertex with each edge of the graph and connecting two vertices with an edge if the corresponding edges of G have a vertex in common[17]. The total graph T (G) from the graph G is a graph whose set of vertices is V (G) $\cup$ E (G) and two vertices side by side each time it borders or occurs in G. The middle graph of a connected graph G denoted by $M(G)$ is the graph whose vertex set is V $(G) \cup E(G)$ where two vertices are adjacent if adjecent edges of G or one is a vertex of G and the other is an edge incident with it.[19]

2. Resolving Domination Number

The metric dimension of graph G is the minimum cardinality of a set of vertices so that each G is uniquely determined by its distance vector to the selected vertex. The set of vertices which is both and dominating is called resolving dominating set. Let we denoted the resolving dominating set as D_r. The minimum cardinality of resolving dominating set is called the resolving domination number denoted by ($Dom_{Dim}(G)$). Let we take an example in H_4. We can verify a resolving dominating sets such as $D_r = \{x_1, x_2, x_3, x_4\}$. In order to proof D_r is resolving dominating set, let we see the neighborhood $V - D_r$. The set of vertices which is not included in dominator resolving set is $V - D_r = \{y_1\}, \{y_2\}, \{y_3\}, \{y_4\}, \{y_5\}$. We can see that x_1 dominated (y_1, x_2, y_5, x_4), x_2 dominated (x_1, y_2, y_5, x_3), x_3 dominated (x_2, y_3, x_4, y_5), x_4 dominated (x_1, x_3, y_4, y_5). The representation of $v \in V(H_4)$ connect to D_r are: r($y_1|D_r$) $= \{1,2,3,2\}, r(y_2|D_r) = \{2,1,2,3\}, r(y_3|D_r) = \{3,2,1,2\}, r(y_4|D_r = \{2,3,3,1\}, r(y_5|D_r = \{1,1,1,1\}, r(x_1|D_r = \{0,1,1,2\}, r(x_2|D_r = \{1,0,1,2\}, r(x_3|D_r = \{2,1,0,1\}, r(x_4|D_r = \{1,2,1,0\}$

Based on the results, the resolving dominating sets dominated all vertices in H_4 and also the representation of vertices in H_4 respect to D_r are distinct. We can conclude that D_r is resolving dominating set. 4 is the minimum cardinality of resolving dominating set. It can be concluded that $\gamma_r(H_4) = 4$. The illustration of resolving dominating set of H_4 can be seen in figure 1.

3. Results

We are show the results of resolving domination number of H_m

Theorem 3.1. *Given that H_m be a helm graph with $m \geq 3$, resolving domination number of H_m is $\gamma_r(H_m) = m$*

Proof. The H_m is a graph on $2m + 1$ vertex and $3m$ edge with vertex set $V(H_m)$= $\{a_i$; $1 \leq$ i $\leq$ m$\}$ $\cup$ $\{$b$_i$; $1 \leq$ i $\leq$ m$\}$ $\cup\{C\}$ and edge set $E(H_m) = \{$a$_i$ b$_i$; $1 \leq$ i $\leq$ m$\}$ $\cup$ $\{$b$_i$ b$_{i+1}$; $1 \leq$ i $\leq$ m-1 $\}$ $\cup$ $\{$b$_m b_1\}$ $\cup$ $\{$b$_i$C ; $1 \leq$ i $\leq$ m$\}$.

For to prove the resolving domination number of helm graph H_m, we will prove the lower bound of resolving domination number is m thus $\gamma_r(H_m) \geq m$ and the upper bound of resolving domination number is m which is $\gamma_r(H_m) \leq m$.

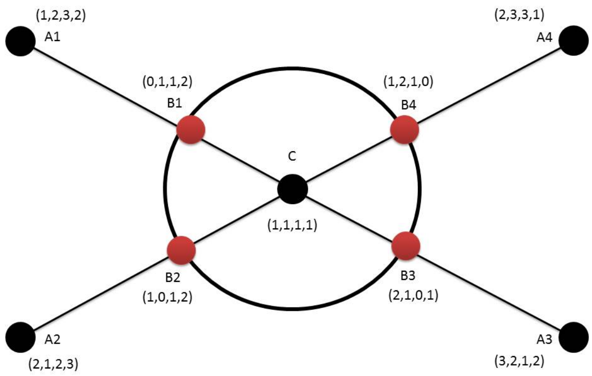

Figure 1. Resolving Dominating Set of H_4

Table 1. The Representation of Resolving Domination Number

v	$r(v\|D_r)$	condition
x_i	$(\underbrace{3,\ldots,3}_{i-2}),1,\underbrace{3,\ldots,3}_{m-i-1})$	$3 \leq i \leq m-1$
x_i	$(1,2,\underbrace{3,\ldots,3}_{m-i-1}),2$	$i=1$
x_i	$(2,1,2,\underbrace{3,\ldots,3}_{m-i-1}),2$	$i=2$
x_i	$(2,\underbrace{3,\ldots,3}_{m-i-1}),2,1$	$i=3$

In this section we proposed the proof of lower and upper bound of resolving domination number on helm graph. We will prove that the lower bound of of resolving domination number is m, which is $\gamma_r(H_m) \geq m$. It has been explained that the resolving dominating set of helm graph fulfill the requirement of dominating set and resolving set. Assume that $\gamma_r(H_m) < m$. We take $|D_r| = m-1$ where $w_i \in D_r$. Based on preposition 1, we know that $\gamma_(H_m) = m$. If we have $|D_r| = m-1$, it means that there will be a set of vertices which is not dominated by D_r. Thus, it is a contradiction. It contradict with the definition of resolving domination number which should fulfill the requirement of dominating set and resolving set. Furthermore, we should have m vertices to dominate all of the vertices in helm graph H_m. Thus, we should have minimum m vertices to be the resolving domination number. Then, we should check whether the representation of all vertices respect to D_r are distinct or not to check the upper bound.

Furthermore, we will prove that the upper bound of resolving dominating set of H_m is m, which is $\gamma_r(H_m) \leq m$. Choose the edge metric generator $S = \{x_i, 1 \leq i \leq m\}$ so the representation of all vertices $v \in V(H_m)$ respect to D_r can be seen in the Table 1. Based on Table 1, all vertices representation of H_m with respect to D_r are distinct, so D_r is the dominating metric generator of H_m with the cardinality of D_r namely is $|D_r| = m$. So, the upper bound of the resolving domination number of H_m is $\gamma_r(H_m) \leq m$. It has been proved that the lower bound of resolving domination number is m and the upper bound of resolving domination number is n. It can be concluded that $\gamma_r(H_m) = m$.

Theorem 3.2. *Let $L(H_m)$ be a line (H_m) with $m \geq 3$, resolving domination number of $L(H_m)$ is $\gamma_r(L(H_m)) = m$*

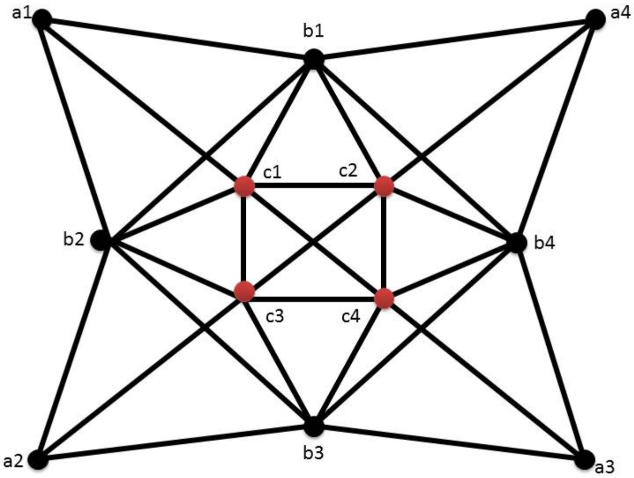

Figure 2. Line Helm Graph of H_4

Proof. The (H_m) is a graph on $3m$ vertices and $5m+6$ edges with vertex set $V(H_n) = \{\mathrm{a}_i\ ;\ 1 \leq \mathrm{i} \leq \mathrm{m}\} \cup \{\mathrm{b}_i\ ;\ 1 \leq \mathrm{i} \leq \mathrm{m}\} \cup \{\mathrm{C}_i C_{i+1}\}$ and edge set $E(LH_m) = \{\mathrm{a}_i\ \mathrm{b}_i\ ;\ 1 \leq \mathrm{i} \leq \mathrm{m}\} \cup \{\mathrm{b}_i\ \mathrm{b}_{i+1}\ ;\ 1 \leq \mathrm{i} \leq \mathrm{m}\text{-}1\ \} \cup \{\mathrm{b}_m b_1\} \cup \{\mathrm{b}_i\mathrm{C}\ ;\ 1 \leq \mathrm{i} \leq \mathrm{m}\}$.

For to prove the resolving domination number of helm graph H_m is $\gamma_r(H_m) = m$, we will prove the lower bound of resolving domination number is m thus $\gamma_r(H_m) \geq m$ and the upper bound of resolving domination number is m which is $\gamma_r(H_m) \leq m$.

In this section we proposed the proof of lower and upper bound of resolving domination number on helm graph. We will prove that the lower bound of of resolving domination number is m, which is $\gamma_r(H_m) \geq m$. It has been explained that the resolving dominating set of helm graph fulfill the requirement of dominating set and resolving set. Assume that $\gamma_r(H_m) < m$. We take $|D_r| = m-1$ where $w_i \in D_r$. Based on preposition 1, we know that $\gamma_(H_m) = m$. If we have $|D_r| = m-1$, it means that there will be a set of vertices which is not dominated by D_r. Thus, it is a contradiction. It contradict with the definition of resolving domination number which should fulfill the requirement of dominating set and resolving set. Furthermore, we should have m vertices to dominate all of the vertices in helm graph H_m. Thus, we should have minimum m vertices to be the resolving domination number. Then, we should check whether the representation of all vertices respect to D_r are distinct or not to check the upper bound.

Furthermore, we will prove that the upper bound of resolving dominating set of H_m is m, which is $\gamma_r(H_m) \leq m$. Choose the edge metric generator $S = \{x_i, 1 \leq i \leq m\}$ so the representation of all vertices $v \in V(H_m)$ respect to D_r can be seen in the Table 2. Based on Table 2, all vertices representation of H_m with respect to D_r are distinct, so D_r is the dominating metric generator of H_m with the cardinality of D_r namely is $|D_r| = m$. So, the upper bound of the resolving domination number of H_m is $\gamma_r(H_m) \leq m$. It has been proved that the lower bound of resolving domination number is m and the upper bound of resolving domination number is m. It can be concluded that $\gamma_r(H_m) = m$.

Theorem 3.3. *Let $M(H_m)$ be a middle (H_m) with $m \geq 3$, resolving domination number of $M(H_m)$ is $\gamma_r(M(H_m)) = m+1$*

Proof. The middle (H_m) is a graph on $5m+1$ vertex and $3m$ edge. For to prove resolving

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Table 2. The Representation of Resolving Domination Number

v	$r(v\|D_r)$	condition
x_i	$(\underbrace{3,\ldots,3}_{i-2}),1,\underbrace{3,\ldots,3}_{m-i-1})$	$3 \leq i \leq m-1$
x_i	$(1,2,\underbrace{3,\ldots,3}_{m-i-1}),2$	$i=1$
x_i	$(2,1,2,\underbrace{3,\ldots,3}_{m-i-1}),2$	$i=2$
x_i	$(2,\underbrace{3,\ldots,3}_{m-i-1}),2,1$	$i=3$

domination number of helm graph $M(H_m)$ is $\gamma_r(M(H_m)) = m+1$, In here will prove that lower bound of resolving domination number is $m+1$ so $\gamma_r(M(H_m)) \geq n+1$ and upper bound of dominating metric dimension number is m which is $\gamma_r(M(H_m)) \leq m+1$.

For to prove the resolving domination number of helm graph H_m is $\gamma_r(H_m) = m+1$, we will prove the lower bound of resolving domination number is $m+1$ thus $\gamma_r(H_m) \geq m+1$ and the upper bound of resolving domination number is m which is $\gamma_r(H_m) \leq m+1$.

In this section we proposed the proof of lower and upper bound of resolving domination number on helm graph. We will prove that the lower bound of of resolving domination number is $m+1$, which is $\gamma_r(H_m) \geq m+1$. It has been explained that the resolving dominating set of helm graph fulfill the requirement of dominating set and resolving set. Assume that $\gamma_r(H_m) < m+1$. We take $|D_r| = m-1$ where $w_i \in D_r$. Based on preposition 1, we know that $\gamma_(H_m) = m+1$. If we have $|D_r| = m-1$, it means that there will be a set of vertices which is not dominated by D_r. Thus, it is a contradiction. It contradict with the definition of resolving domination number which should fulfill the requirement of dominating set and resolving set. Furthermore, we should have $m+1$ vertices to dominate all of the vertices in helm graph H_m. Thus, we should have minimum $m+1$ vertices to be the resolving domination number. Then, we should check whether the representation of all vertices respect to D_r are distinct or not to check the upper bound.

Then, we will prove that the upper bound of resolving dominating set of H_m is $m+1$, which is $\gamma_r(H_m) \leq m+1$. Choose the edge metric generator $S = \{x_i, 1 \leq i \leq m\}$ so the representation of all vertices $v \in V(H_m)$ respect to D_r can be seen in the Table 3. Based on Table 3, all vertices representation of H_m with respect to D_r are distinct, so D_r is the dominating metric generator of H_m with the cardinality of D_r namely is $|D_r| = m+1$. So, the upper bound of the resolving domination number of H_m is $\gamma_r(H_m) \leq m+1$. It has been proved that the lower bound of resolving domination number is $m+1$ and the upper bound of resolving domination number is n. It can be concluded that $\gamma_r(H_m) = m+1$.

Theorem 3.4. *Let $T(H_m)$ be a total (H_m) with $m \geq 3$, resolving domination number $T(H_m)$ is $\gamma_r(T(H_m)) = m+1$*

Proof. The middle (H_m) is a graph on $5m+1$ vertex and $3m$ edge. For to prove resolving domination number of helm graph $T(H_m)$ is $\gamma_r(T(H_m)) = m+1$, We will prove that lower bound of resolving domination number is $m+1$ so $\gamma_r(T(H_m)) \geq n+1$ and upper bound of resolving domination number is n which is $\gamma_r(T(H_m)) \leq m+1$.

For to prove the resolving domination number of helm graph H_m is $\gamma_r(H_m) = m+1$, we will prove the lower bound of resolving domination number is $m+1$ thus $\gamma_r(H_m) \geq m+1$ and the upper bound of resolving domination number is n which is $\gamma_r(H_m) \leq m+1$.

We are proposed the proof of lower and upper bound of resolving domination number on helm graph. We will prove that the lower bound of of resolving domination number is $m+1$,

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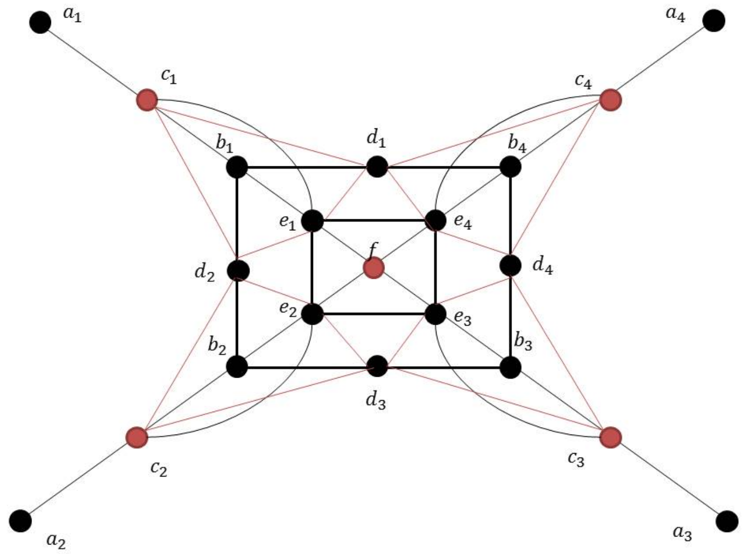

Figure 3. Middle Helm Graph of H_4

Table 3. The Representation of Resolving Domination Number

v	$r(v\|D_r)$	condition
x_i	$(\underbrace{3,\ldots,3}_{i-2}),1,\underbrace{3,\ldots,3}_{m-i-1})$	$3 \leq i \leq m-1$
x_i	$(1,2,\underbrace{3,\ldots,3}_{m-i-1}),2$	$i=1$
x_i	$(2,1,2,\underbrace{3,\ldots,3}_{m-i-1}),2$	$i=2$
x_i	$(2,\underbrace{3,\ldots,3}_{m-i-1}),2,1$	$i=3$

which is $\gamma_r(H_m) \geq m+1$. It has been explained that the resolving dominating set of helm graph fulfill the requirement of dominating set and resolving set. Assume that $\gamma_r(H_m) < m+1$. We take $|D_r| = m-1$ where $w_i \in D_r$. Based on preposition 1, we know that $\gamma_(H_m) = m+1$. If we have $|D_r| = m-1$, it means that there will be a set of vertices which is not dominated by D_r. Thus, it is a contradiction. It contradict with the definition of resolving domination number which should fulfill the requirement of dominating set and resolving set. Furthermore, we should have $m+1$ vertices to dominate all of the vertices in helm graph H_m. Thus, we will have minimum $m+1$ vertices to be the resolving domination number. Then, we should check whether the representation of all vertices respect to D_r are distinct or not to check the upper bound.

Furthermore, we will prove that the upper bound of resolving dominating set of H_m is $m+1$, which is $\gamma_r(H_m) \leq m+1$. Choose the edge metric generator $S = \{x_i, 1 \leq i \leq m\}$ so the representation of all vertices $v \in V(H_m)$ respect to D_r can be seen in the Table 3. Based on Table 3, all vertices representation of H_m with respect to D_r are distinct, so D_r is the dominating metric generator of H_m with the cardinality of D_r namely is $|D_r| = m+1$. So, the upper bound of the resolving domination number of H_m is $\gamma_r(H_m) \leq m+1$. It has been proved that the lower bound of resolving domination number is $m+1$ and the upper bound of resolving domination number is n. It can be concluded that $\gamma_r(H_m) = m+1$.

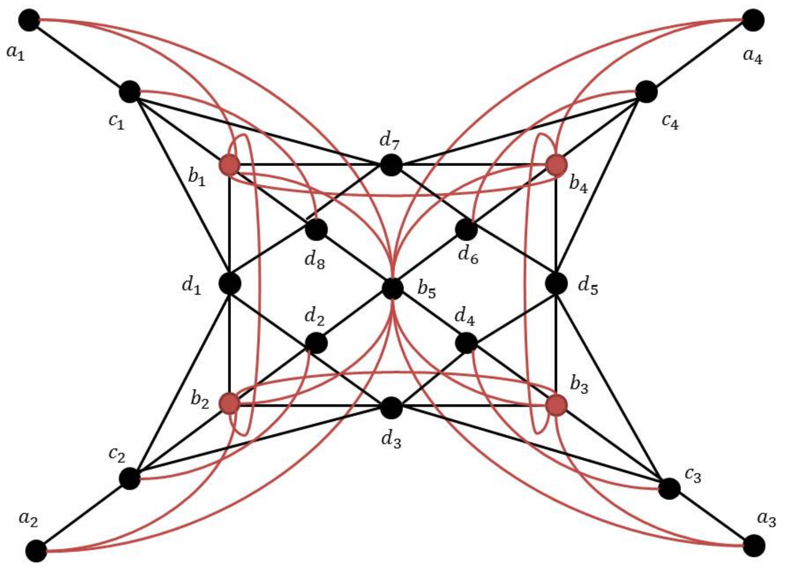

Figure 4. Total Helm Graph of H_4

Table 4. The Representation of Resolving Domination Number

v	$r(v\|D_r)$	condition
x_i	$(\underbrace{3,\ldots,3}_{i-2}),1,\underbrace{3,\ldots,3}_{m-i-1})$	$3 \le i \le m-1$
x_i	$(1,2,\underbrace{3,\ldots,3}_{m-i-1}),2$	$i=1$
x_i	$(2,1,2,\underbrace{3,\ldots,3}_{m-i-1}),2$	$i=2$
x_i	$(2,\underbrace{3,\ldots,3}_{m-i-1}),2,1$	$i=3$

4. Conclusion

We have given results of line graph, middle graph, and total graph on helm graph. Doe to this research is still new, thus many problems must be found.

Open Problem 1. *Given that G be a graph, need to prove the dominating metric dimension any graph and its operations.*

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