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On rainbow antimagic coloring of some special graph

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Abstract. Let $G = (V, E)$ be a connected and simple graphs with vertex set V and edge set E. A coloring of graph G is rainbow connected if there is a rainbow path that connects each two vertices of graph G. The minimum k such that G has a rainbow-connected using k colors of the edges of G is the rainbow connection number $rc(G)$ of G. A graph with a bijective mapping $f : E \longrightarrow \{1, 2, ..., |E|\}$. The sums of each paired vertex has distinct value, defined as $\sum_{e\in E(v)} f(e)$. Thus, the function of G clearly an antimagic labeling if the sums of each paired vertex has distinct value. It is clear that rainbow antimagic connection number is the smallest number of colors which are needed to make G rainbow connected, denoted by $rc_A(G)$. A bijection function $f : E \longrightarrow \{1, 2, ..., |E|\}$ is called a rainbow antimagic labeling if there is a rainbow path between every pair of vertices and for each edge $e = uv \in E(G)$, the weight $w(e) = f(u) + f(v)$. A graph G is rainbow antimagic if G has a rainbow antimagic labeling. In this paper, we will analyze the rainbow antimagic coloring of related book graph.

1. Introduction

We study about the rainbow antimagic coloring. Every graph considered in this paper are connected and simple graph. Let $G = (V, E)$ be a connected and simple graphs with vertex set V and edge set E.

Suppose G is a nontrivial connected graph on which an edge coloring $c : (E(G) \rightarrow \{1, 2, 3, ..., s\}, s \in N$, where adjacent edges may have the same color. We could say rainbow path if there are no two edges with the same coloring on the $u - v$ path in G. A coloring of graph G is rainbow connected if there is a rainbow path that connects each two vertices of graph G. The minimum k such that G has a rainbow-connected using k colors of the edges of G is the rainbow connection number which is denoted by $rc(G)$ of G. The complete concept of rainbow connection can see in Chartrand *et al* [3]. Furthermore, there are further research about rainbow can be found in Agustin *et al* [1], Dafik *et al* [4], Fauziah *et al* [5], and Hasan *et al* [7].

There is study in graph about magic labeling. The opponent of magic labeling, it is known as an antimagic label. Hartsfield and Ringel [6] was proposed the antimagic labeling in graphs with two conjecture. Then, Jackanich [8] defined it. Furthermore, another studies about antimagic are as follows [9], [10], [12]. Let $G = (V(G), E(G))$ be a graph with a bijective mapping $f : E \longrightarrow \{1, 2, ..., |E|\}$ so that the sums of each paired vertex has distinct value, which is the sums of each paired vertex is defined as $\sum_{e\in E(v)} f(e)$. Thus, the function of G clearly an antimagic labeling if the sums of each paired vertex has distinct value.

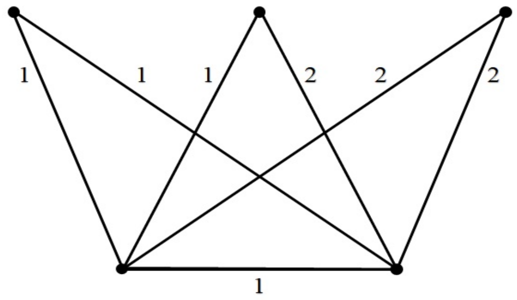

Figure 1. Triangular Book with 2 rainbow connection

Now, we study the rainbow antimagic coloring in this paper. It is clear that rainbow antimagic connection number is the smallest number of colors which are needed to make G rainbow connected, denoted by $rc_A(G)$. A bijection function $f : E \longrightarrow \{1, 2, ..., |E|\}$ is called a rainbow antimagic labeling if there is a rainbow path between every pair of vertices and for each edge $e = uv \in E(G)$, the weight $w(e) = f(u) + f(v)$. A graph G is rainbow antimagic if G has a rainbow antimagic labeling, then G is rainbow antimagic.

2. Result

In this paper, we determine the rainbow antimagic coloring of book (B_n), triangular book (Tb_n), and generalized of book.

Theorem 1. *Suppose that G is a triangular book graph with $n \geq 2$. The rainbow connection number*

$$rc = \begin{cases} 2, & \text{if } 2 \leq n \leq 4 \\ 3, & \text{if } n \geq 5 \end{cases}$$

Proof. The proof of this theorem proofed by Alfarisi in [2]

Lemma 2. *Suppose that $a, b, c_i, i = 1, 2, ..., n$ with $n =$ poitive integers and $a \neq b \neq c_i$, $\forall i$. $W_1 = \{a + c_i | i = 1, 2, 3, ..., n\}$; $W_2 = \{b + c_i | i = 1, 2, 3, ..., n\}$; so, $|W_1 \cup W_2| \geq n + 1$.*

Proof. Suppose $|W_1 \cup W_2| \leq n$, because $|W_1| = |W_2| = n$, so $|W_1 \cup W_2| = n$. it consequent $W_1 = W_2$, or $a + c_i = b + c_i$, the result is $a = b$, which is contradiction with $a \neq b$ Thus, $|W_1 \cup W_2| \geq n + 1$.

Theorem 3. *Let Tb_n be a triangular book graph with $n \geq 3$. The rainbow antimagic connection number $rc_A(Tb_n) = n + 2$.*

Proof. Triangular book has vertex set is $\{\mathrm{x}_1 x_2\} \cup \{y_j, 1 \leq j \leq n\}$ and the edge set is $\{\mathrm{x}_1 x_2\} \cup \{y_j x_1, y_j x_2, 1 \leq j \leq n\}$. Cardinality of vertex is $|V(Tb_n)| = n + 2$ and the cardinality of edge is $|E(Tb_n)| = 2n + 1$.

Based on Lemma 2, it is clear that the lower bound has been proved. Afterwards, we prove the upper bound in this graph. Based on definition of antimagic labeling, define a bijective mapping $f : E \longrightarrow \{1, 2, ..., |E|\}$, we define $f(x_1) = 2$; $f(x_2) = 1$; $f(y_j) = j + 2, 1 \leq j \leq n$. The weight (W) of triangular book in Figure 2 are as follows:

$$w(e) = \begin{cases} w(x_1 x_2) = 3 & \\ w(x_1 y_j) = j + 4, & \text{if } 1 \leq j \leq n \\ w(x_2 y_j) = j + 3, & \text{if } 1 \leq j \leq n \end{cases}$$

Theorem 4. *Suppose that B_n with $n \geq 3$, clearly $rc(B_n) = 4$.*

Table 1. Rainbow path antimagic in Triangular Book

Edge	Color of Edge
x_1x_2	3
x_1y_1	5
x_1y_2	6
x_1y_3	7
...	...
x_2y_j	$j+4$
x_2y_1	4
x_1y_2	5
x_1y_3	6
...	...
x_2y_j	$j+3$

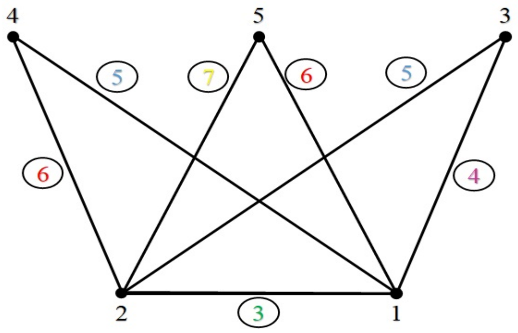

Figure 2. Triangular Book with 5 rainbow antimagic coloring

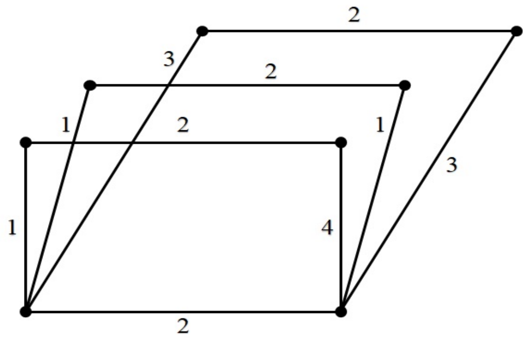

Figure 3. Book with 3 rainbow connection

Proof. This theorem has been proofed by Syafrizal in [11].

Lemma 5. *Suppose that* $a_i, b_i, i = 1, 2, ..., n$ *with* n = *positive integers and* $a_i \neq b_i$, $\forall i$. $W_1 = \{a + a_i | i = 1, 2, 3, ..., n\}$; $W_2 = \{b + b_i | i = 1, 2, 3, ..., n\}$; $W_3 = \{a_i + b_i | i = 1, 2, 3, ..., n\}$ *so,* $|W_1 \cup W_2 \cup W_3| \geq n + 1$.

Proof. Suppose $|W_1 \cup W_2 \cup W_3| \leq n$, because $|W_1| = |W_2| = |W_3| = n$, so $|W_1 \cup W_2 \cup W_3| = n$. it consequent $W_1 = W_2 = W_3$, or $a+a_i = b+b_i = a_i+b_i$, the result is $a = b$, which is contradiction with $a \neq b$ Thus, $|W_1 \cup W_2| \geq n + 1$.

Theorem 6. *Suppose that* B_n *with* $3 \leq n \leq 7$. *The rainbow antimagic connection number*

$$rc_A(B_n) = \begin{cases} n+2; & n{=}odd \\ n+1; & n{=}even \end{cases}$$

Proof. Book graph has vertex set $\{ab\}\cup\{a_i, 1 \leq i \leq n\} \cup \{b_i, 1 \leq i \leq n\}$ and the edge set is $\{ab\}\cup\{aa_i, 1 \leq i \leq n\} \cup \{bb_i, 1 \leq i \leq n\} \cup \{a_ib_i, 1 \leq i \leq n\}$. The cardinality of vertex is $|V(B_n)| = 2n + 2$ and the cardinality of edge is $|E(B_n)| = 3n + 1$.

It is easy to see that the lower bound has been proved in lemma 5. Then, we are going to proof the upper bound. Based on definition of antimagic labeling, define a bijective mapping $f : E \longrightarrow \{1, 2, ..., |E|\}$, we define the function of x is defined as follows

$$f(x_i) = \begin{cases} \frac{\Sigma v}{2}; & \text{i=1} \\ \frac{\Sigma v}{2} + 1; & \text{i=2} \end{cases}$$

For book graph $n = 3$, the function of y is defined as follows:

$$f(y_i) = \begin{cases} i + (\Sigma v - 1); & \text{i=1} \\ i + \frac{\Sigma v}{2}; & \text{i=2} \\ i - 1; & \text{i=n} \end{cases}$$

For book graph $n = 4$, the function of y is defined as follows:

$$f(y_i) = \begin{cases} i + (\Sigma v - 1); & \text{i=1} \\ i + (\frac{\Sigma v}{2} + 1); & \text{i=2} \\ i + 1; & \text{i=n-1} \\ i - 2; & \text{i=n} \end{cases}$$

For book graph $n = 5$, the function of y is defined as follows:

$$f(y_i) = \begin{cases} i + (\Sigma v - 1); & \text{i=1} \\ i + (\frac{\Sigma v}{2} + 2); & \text{i=2} \\ i + (\frac{\Sigma v}{2} - 1); & \text{i=3} \\ i; & \text{i=n-1} \\ i - 1; & \text{i=n} \end{cases}$$

For book graph $n = 6$, the function of y is defined as follows:

$$f(y_i) = \begin{cases} i + (\Sigma v - 1); & \text{i=1} \\ i + (\frac{\Sigma v}{2} + 3); & \text{i=2} \\ i + (\frac{\Sigma v}{2}); & \text{i=3} \\ i + 2; & \text{i=4} \\ i - 1; & \text{i=n-1} \\ i - 4; & \text{i=n} \end{cases}$$

For book graph $n = 7$, the function of y is defined as follows:

$$f(y_i) = \begin{cases} i + (\Sigma v - 1); & \text{i=1} \\ i + (\frac{\Sigma v}{2} + 4); & \text{i=2} \\ i + (\frac{\Sigma v}{2} + 1); & \text{i=3} \\ i + (\frac{\Sigma v}{2} - 2); & \text{i=4} \\ i + 1; & \text{i=5} \\ i - 2; & \text{i=n-1} \\ i - 5; & \text{i=n} \end{cases}$$

For book graph $n = 3$, the function of z is defined as follows:

$$f(z_i) = \begin{cases} 1; & \text{i=1} \\ i + 1; & \text{i=2} \\ \Sigma v - 1; & \text{i=n} \end{cases}$$

For book graph $n = 4$, the function of z is defined as follows:

$$f(z_i) = \begin{cases} 1; & \text{i=1} \\ i+1; & \text{i=2} \\ \frac{\Sigma v}{2}+2; & \text{i=n-1} \\ \Sigma v - 1; & \text{i=n} \end{cases}$$

For book graph $n = 5$, the function of z is defined as follows:

$$f(z_i) = \begin{cases} 1; & \text{i=1} \\ i+1; & \text{i=2} \\ \frac{\Sigma v}{2}-1; & \text{i=3} \\ \frac{\Sigma v}{2}+3; & \text{i=n-1} \\ \Sigma v - 1; & \text{i=n} \end{cases}$$

For book graph $n = 6$, the function of z is defined as follows:

$$f(z_i) = \begin{cases} 1; & \text{i=1} \\ i+1; & \text{i=2} \\ \frac{\Sigma v}{2}-2; & \text{i=3} \\ \frac{\Sigma v}{2}+2; & \text{i=4} \\ \frac{\Sigma v}{2}+4; & \text{i=n-1} \\ \Sigma v - 1; & \text{i=n} \end{cases}$$

For book graph $n = 7$, the function of z is defined as follows:

$$f(z_i) = \begin{cases} 1; & \text{i=1} \\ i+1; & \text{i=2} \\ \frac{\Sigma v}{2}-3; & \text{i=3} \\ \frac{\Sigma v}{2}-1; & \text{i=4} \\ \frac{\Sigma v}{2}+3; & \text{i=5} \\ \frac{\Sigma v}{2}+5; & \text{i=n-1} \\ \Sigma v - 1; & \text{i=n} \end{cases}$$

We define the weight (w) of book in Figure 4 that for book graph $n = 3$, the weight is defined as follows:

$$w(e) = \Sigma v + 1 \begin{cases} e = x_1 x_2 \\ e = y_i z_i \end{cases}$$

$$w(e) = \frac{3}{2}\Sigma v \begin{cases} e = x_1 y_1 \\ e = x_2 z_n \end{cases}$$

$$w(e) = \Sigma v + 2; \quad \text{e=x}_1 y_2$$

$$w(e) = \frac{\Sigma v}{2} + 4; \quad \text{e=x}_2 z_2$$

$$w(e) = \frac{\Sigma v}{2} + 2 \begin{cases} e = x_1 y_n \\ e = x_2 z_1 \end{cases}$$

We define the weight (w) of book in Figure 4 that for book graph $n = 4$, the weight is defined as follows:

$$w(e) = \Sigma v + 1 \begin{cases} e = x_1x_2 \\ e = y_iz_i \end{cases}$$

$$w(e) = \frac{3}{2}\Sigma v \begin{cases} e = x_1y_1 \\ e = x_2z_n \end{cases}$$

$$w(e) = \Sigma v + 3 \begin{cases} e = x_1y_2 \\ e = x_2z_3 \end{cases}$$

$$w(e) = \frac{\Sigma v}{2} + 4 \begin{cases} e = x_1y_3 \\ e = x_2z_2 \end{cases}$$

$$w(e) = \frac{\Sigma v}{2} + 2 \begin{cases} e = x_1y_n \\ e = x_2z_1 \end{cases}$$

We define the weight (w) of book in Figure 4 that for book graph $n = 5$, the weight is defined as follows:

$$w(e) = \Sigma v + 1 \begin{cases} e = x_1x_2 \\ e = y_iz_i \end{cases}$$

$$w(e) = \frac{3}{2}\Sigma v \begin{cases} e = x_1y_1 \\ e = x_2z_n \end{cases}$$

$$w(e) = \Sigma v + 4 \begin{cases} e = x_1y_2 \\ e = x_2z_4 \end{cases}$$

$$w(e) = \Sigma v + 2; \quad \text{e=x}_1y_3$$

$$w(e) = \Sigma v; \quad \text{e=x}_2z_3$$

$$w(e) = \frac{\Sigma v}{2} + 4 \begin{cases} e = x_1y_4 \\ e = x_2z_2 \end{cases}$$

$$w(e) = \frac{\Sigma v}{2} + 2 \begin{cases} e = x_1y_n \\ e = x_2z_1 \end{cases}$$

We define the weight (w) of book in Figure 4 that for book graph $n = 6$, the weight is defined as follows:

$$w(e) = \Sigma v + 1 \begin{cases} e = x_1x_2 \\ e = y_iz_i \end{cases}$$

$$w(e) = \frac{3}{2}\Sigma v \begin{cases} e = x_1y_1 \\ e = x_2z_n \end{cases}$$

$$w(e) = \Sigma v + 5 \begin{cases} e = x_1y_2 \\ e = x_2z_5 \end{cases}$$

$$w(e) = \Sigma v + 3 \begin{cases} e = x_1y_3 \\ e = x_2z_4 \end{cases}$$

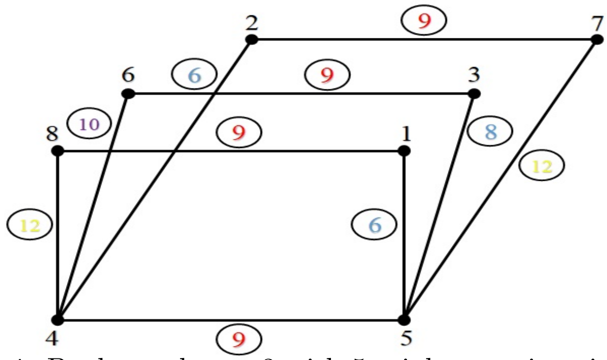

Figure 4. Book graph $n = 3$ with 5 rainbow antimagic coloring

$$w(e) = \Sigma v - 1 \begin{cases} e = x_1 y_4 \\ e = x_2 z_3 \end{cases}$$

$$w(e) = \frac{\Sigma v}{2} + 4 \begin{cases} e = x_1 y_5 \\ e = x_2 z_2 \end{cases}$$

$$w(e) = \frac{\Sigma v}{2} + 2 \begin{cases} e = x_1 y_n \\ e = x_2 z_1 \end{cases}$$

We define the weight (w) of book in Figure 4 that for book graph $n = 7$, the weight is defined as follows:

$$w(e) = \Sigma v + 1 \begin{cases} e = x_1 x_2 \\ e = y_i z_i \end{cases}$$

$$w(e) = \frac{3}{2} \Sigma v \begin{cases} e = x_1 y_1 \\ e = x_2 z_n \end{cases}$$

$$w(e) = \Sigma v + 6 \begin{cases} e = x_1 y_2 \\ e = x_2 z_6 \end{cases}$$

$$w(e) = \Sigma v + 4 \begin{cases} e = x_1 y_3 \\ e = x_2 z_5 \end{cases}$$

$$w(e) = \Sigma v + 2; \quad \text{e=x}_1 y_4$$

$$w(e) = \Sigma v; \quad \text{e=x}_2 z_4$$

$$w(e) = \Sigma v - 2 \begin{cases} e = x_1 y_5 \\ e = x_2 z_3 \end{cases}$$

$$w(e) = \frac{\Sigma v}{2} + 4 \begin{cases} e = x_1 y_6 \\ e = x_2 z_2 \end{cases}$$

$$w(e) = \frac{\Sigma v}{2} + 2 \begin{cases} e = x_1 y_n \\ e = x_2 z_1 \end{cases}$$

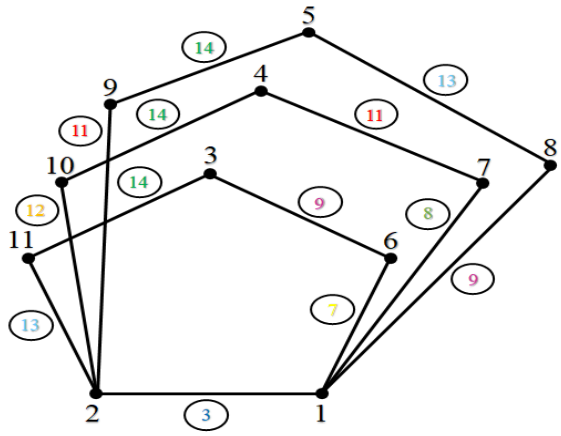

Figure 5. Generalized Book Graph with 6 rainbow antimagic coloring

Lemma 7. *Suppose that* $a, b, x_{i,1}, x_{i,2}, x_{i,3}, i = 1, 2, ..., n$ *with* $n =$ *poitive integers and* $a \neq b \neq x_{i,1} \neq x_{i,2} \neq x_{i,3}$, $\forall i$. $W_1 = \{a + x_{i,1} | i = 1, 2, 3, ..., n\}$; $W_2 = \{x_{i,1} + x_{i,2} | i = 1, 2, 3, ..., n\}$; $W_3 = \{x_{i,2} + x_{i,3} | i = 1, 2, 3, ..., n\}$; $W_4 = \{b + x_{i,3} | i = 1, 2, 3, ..., n\}$; *so,* $|W_1 \cup W_2 \cup W_3 \cup W_4| \geq n+1$.

Proof. Suppose $|W_1 \cup W_2 \cup W_3 \cup W_4| \leq n$, because $|W_1| = |W_2| = |W_3| = |W_4| = n$, so $|W_1 \cup W_2| = n$. It consequent $W_1 = W_2 = W_3 = W_4$, or $a + x_{i,1} = x_{i,1} + x_{i,2} = x_{i,2} + x_{i,3} = b + x_{i,3}$, the result is $a = b$, which is contradiction with $a \neq b$ Thus, $|W_1 \cup W_2 \cup W_3 \cup W_4| \geq n + 1$.

Theorem 8. *Suppose that* G *be a generalized of book graph with* $n \geq 3$. *The rainbow antimagic connection number* $rc_A(G) = n + 5$.

Proof. The graph has vertex set $\{xy\} \cup \{x_{a,1}, 1 \leq a \leq n\} \cup \{x_{a,2}, 1 \leq a \leq n\} \cup \{x_{a,3}, 1 \leq a \leq n\}$ and the edge set is $\{xy\} \cup \{xx_{a,1}, 1 \leq a \leq n\} \cup \{yx_{a,3}, 1 \leq a \leq n\} \cup \{x_{a,1}x_{a,2}, 1 \leq a \leq n\} \cup \{x_{a,2}x_{a,3}, 1 \leq a \leq n\}$. The cardinality of vertex is $|V(G)| = 3n + 2$ and $|E(G)| = 4n + 1$ is cardinality of edge.

In Lemma 7, it is clear that the lower bound has been proofed. Afterwards, we are going to prove the upper bound. Based on definition of antimagic labeling, define a bijective mapping $f : E \longrightarrow \{1, 2, ..., |E|\}$, we define $f(x) = 2$; $f(y) = 1$; $f(x_{a,1}) = 3n + 3 - a, 1 \leq a \leq n$; $f(x_{a,2}) = a + 2, 1 \leq a \leq n$; $f(x_{a,3}) = a + 5, 1 \leq a \leq n$. The weight (W) of this graph in Figure 5 that.

$$w(e) = \begin{cases} w(xy) = 3 & \\ w(xx_{a,1}) = 3n + 5 - a, & \text{if } 1 \leq a \leq n \\ w(yx_{a,3}) = a + 6, & \text{if } 1 \leq a \leq n \\ w(x_{a,1}x_{a,2}) = 3n + 5, & \text{if } 1 \leq a \leq n \\ w(x_{a,2}x_{a,3}) = 2a + 7, & \text{if } 1 \leq a \leq n \end{cases}$$

3. Conclusion

We have obtained the exact value of rainbow antimagic of triangular book graph (Tb_n), book graph (B_n), and generalized of book graph.

Open Problem 1. *Let* G *be a special graph, determine the rainbow antimagic coloring of* G.

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