Critical thinking dispositions in solving recreational mathematics problem: opposite corners

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Abstract. The purposes of this research were to determine the critical thinking dispositions in solving recreational mathematics. The subjects were students of first year of bachelor's degree mathematics education study program. This study used a descriptive qualitative method. 4 participants selected based on 4 levels of combinatorial thinking after solving the recreational math problem that is the opposite corner. The test results were analyzed based on disposition toward critical thinking and combinatorial thinking processes. The indicators used were truth-seeking, open-mindedness, analyticity, systematics, confidence in reasoning, inquisitiveness, the maturity of judgment. The data analysis showed the thinking process in solving problems showed that students were able to bring up indicators of critical thinking disposition through each stage of completion.

1. Introduction

Higher education is the education, training, and guidance of research that takes place after the level of post-secondary education. In this century, higher education faces more challenges than ever before in its history. Colleges and universities are challenged to prepare a variety of students, from those who are academically gifted to those who are less talented. All of them are prepared for college-level work. It is expected that the future students can be more aware and proactive because of the high level of exposure and guidance available in various fields of life, and it requires an ability to think, namely critical thinking.

According to some experts, critical thinking is thinking skillful, responsible, and conducive in decision making because it is sensitive to context, depends on criteria, and corrects itself. There are various other opinions including critical thinking. Critical thinking is thinking about our thoughts when we think, to make our thinking better. Whereas Norris argues that critical thinking is "Rationally deciding what to do or believe". Moreover, according to Sternberg, critical thinking is a mental process, strategy, and representations that people use to solve problems, make decisions, and learn new concepts [9]. Chukwuyenum says that Critical Thinking Skills was an effective means of enhancing students' understanding of Mathematics concepts because the skills has helped in interpreting, analyzing, evaluating, and presenting data in a logical and sequence manner [4].

From the various meanings above, it can be concluded that critical thinking is thinking that occurs in the students' cognitive systems by using various knowledge and skills that have been previously possessed to assemble various implications in solving problems; so that, they can determine the best decision making. The competencies and abilities of educators in generating various abilities, skills, and creativity of students are very necessary for preparing students' mental and academic conditions towards global competition. Critical Thinking should be made compulsory and integrated into all secondary school curriculum, scheme of work, lesson note, lesson plan and in the classroom when teaching and learning take place because the concepts serves as a learning and teaching aids and also makes the students understand the concept better [4]. Therefore, educators must be able to take the best strategy to instill critical thinking disposition to students.

Critical thinking dispositions refer to a person's character or affective that influences someone's desire to bring up critical thinking. Raising the critical thinking disposition of each student requires a level of ability and professionalism from the educator. The strategies, learning models, and learning methods are chosen by educators must be following with the concept of critical thinking disposition. Many strategies can be used by educators to bring up students' critical thinking disposition; one of them is recreational mathematics.

Recreational mathematics is mathematics that is fun and popular that is, the problems should be understandable to the interested layperson, though the solutions may be harder [11]. Recreational problems has been a source of entertainment and interest for hundreds of years. Recreational problems are often the basis of serious mathematics models whose role is to develop and to foster important abilities in the present. Students can think of various implications in decision making before finally deciding what is best to do. This high level of ability is needed in the 21st century. According to some literature and researches, there are also fields of study that are believed to be able to bring up students' critical thinking disposition, especially in the field of mathematics.

There are two, somewhat overlapping, definitions that cover most of what is meant by recreational mathematics [2]. First, recreational mathematics is fun mathematics. The problems posed are easily understood by layman persons. The problems are very interesting also, though the solutions may be hard. Secondly, recreational mathematics is mathematics that is fun and used as either as a diversion from serious mathematics or as a way of making serious mathematics understandable or palatable. These are the pedagogic uses of recreational mathematics. They are already present in the oldest known mathematics and continue to the present day.

One field of study in recreational mathematics that allows students to bring up critical thinking disposition is "opposite corners problem". Students must develop problem-solving techniques [11]. A fun and interesting way to learn about problem-solving is to provide recreational problems, especially opposite corners problems. Problem-solving can be a stimulating and fun hobby. Sometimes a person can see a solution method immediately. Other times, problems can cause hours of frustration. But the satisfaction that comes with the final solution of a difficult problem, by itself, is a gift that makes all efforts worthwhile [1]. Students not only feel happy and challenged but must also use basic mathematical concepts such as combinatorics.

2. Method

This research used a qualitative method research approach. Qualitative data were in the form of words or images that emphasized more on the process obtained from observations during interviews and when students presented the results of their work. The subject taking technique used in this study is purposive sampling with the condition that the subject meets all levels of combinatorial thinking. The subject in this study was 1 student of the second year (3rd semester) of a bachelor's degree in mathematics education study program who had taken Discrete Mathematics courses. In this study, representative of the subject was coded with S1. The type of this research was descriptive, and the purposes were to describe critical thinking dispositions in solving the problem of the opposite corner. The results were analyzed based on disposition toward critical thinking and combinatorial thinking processes. The critical thinking dispositions indicators used in this research are below:

Table 1. The indicators of combinatorial thinking dispositions

Indicator	Description
Truth-seeking	If the student used to analyze in-depth, looking for various possibilities generalization of the formula
Open-mindedness	If the student dared to express an opinion or accept input from other students related to generalizing formulas.
Analyticity	If students were able to provide reasons through evidence related to the generalization of the formula
Systematics	If students were able to explain each step of solving problems logically and clearly
Confidence in reasoning	If students conveyed decisions/reasoning, and they felt comfortable and confident
Inquisitiveness	If students could use several ways to determiner generalization of the formula
Maturity of judgment	Students had the understanding that the generalization formula he found applies to the size of c x c column and square size n x n, and be careful in determining the generalizing the formula. Having thoughts of developing the problem and questioning whether the formula found is valid for the problem he developed

The instruments used in this research were the opposite corner problem and interview. The following are the opposite corner of problems which already provided:

Here the numbers are arranged in 10 columns

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

In the 2×2 square
$7\times 18 = 126$
$8\times 17 = 136$
The difference between them is 10

7	8
17	18

In the 3×3 square
$22\times 34 = 408$
$14\times 32 = 448$
The difference between them is 40

12	13	14
22	23	24
32	33	34

Investigate to see if you can find any rules or patterns connecting the size of square chosen and the difference. If you find a rule, use it to *predict* the difference for larger squares.

Test your rule by looking at squares like 8×8 or 9×9.
Can you generalise the rule? Can you prove the rule?

3. Results and Discussion

Critical thinking is examining ideas, evaluating ideas from what is already known and making decisions about those ideas. Critical thinking can also help make your arguments so that you will be able to present and justify every claim made based on the evidence that has been evaluated. Critical thinking aims is to try to maintain an ‘objective’ position [6]. When think critically, you weigh up all sides of an argument and evaluate its strengths and weaknesses. However, just developing critical thinking skills is not enough, Rather it is critical thinking dispositions that need to be questioned and promoted [7]. A thinking disposition, as we define it, is a rational impulse toward a particular thinking pattern or thinking quality, which encourages active involvement in thought processes such as combinatoric thinking.

Combinatorics is an essential component of mathematics and, as such, it has an important role to play in university students because combinatorics is rich in potential to teach the problem-solving processes [7]. Besides, problems in opposite corners also facilitate the development of enumeration processes, as well as conjectures, generalizations, and systematic thinking. According to Rezaie M. & Gooya Z [10], there are 4 levels of combinatorial thinking understanding in which each level contains indicators of critical thinking dispositions.

3.1. Level 1: Investigating “Some Cases”

The first attempt made by students in dealing with this problem was to find "several different cases that might be studied to generalize formulas" by looking that the problems they were thinking could be expanded regularly. Based on the results and interviews, it shows that S had the indicators related to critical thinking disposition, namely truth-seeking and inquisitiveness. The following are the results of the interview with the subject:

A: How do you get the formula for a column to c x c and square size n x n?
S: The way that I used to obtain the formula is the same as I explained in number 3
A: How do you generalize the formula to the problem you have?
S: Starting from the smallest part, observing patterns formed, looking for relationships between patterns and things that are known, generalizing formulas
A: Are you sure with the formula that you first found is right? Do you still want to correct and look for the other possibilities?
S: No, I'm not sure about the formula that I found for the first time. Yes, I am still correcting the formula that I have found and then looking for the other possibility.
A: Are you still looking for other learning resources aside from lecturer explanations or learning resources that are commonly used by students in your class? If so, how far?
S: No, I'm not looking for other learning resources other than explanations from the lecturer
A: Did you ever think that the generalization of the formula that you made could not be used for a c- column expand?
S: Yes, that's why I continue to check the formula that I found until it can be used in general
A: How often do you consult with your supervisor about the results of your work?
S: Once
A: Have you ever looked for a formula of many different columns to compare with the formula of many previous columns?
S: Yes, I found the formula by trying to use several columns to find out the pattern formed so that the formula that applies to all columns is obtained

S was able to search for more than one different cases to find out the pattern formed pattern and ordered, so it was able to be generalized. Next, S realized that there were so many cases that he thought from the simple to the complex. It shows that S1 was very open in processing their imagination and thinking in finding various cases and pattern as his choice. In addition, S was also very careful in making decisions regarding which pattern to choose for later generalizing the formula of the c columns and n x n square.

3.2. Level 2:"How am I sure that I have counted all the cases?"

At this level, students expanded their chosen graph starting from the $c = 3$ until $c = 5$ with confidence that pattern that could be expanded to c columns with n x n square. S was sure that after seeing the pattern of resolving sets from $c = 1$ until $c = 5$, the following patterns had regularities that were aligned with patterns 1 through 5. The following are the results of the interview with the subject:

A: If you have had different thoughts from your friends about generalizing formulas, can you convey those thoughts honestly and openly in front of the class?
S: Yes, because by presenting the results of our thoughts we can exchange our knowledge
A: Can you respond properly if there is a refutation from other students about the process of finding a formula generalization?
S: Yes, because everyone has a different way to find a solution so we also have to allow others to express their opinions
A: Are you willing to accept suggestions as a form of improvement of your work if there are suggestions about generalizing formulas?
S: Yes, because understanding the concept can affect the results found.

Based on the results and interviews, it shows that S had the indicators related to critical thinking disposition, namely open-mindedness. S was able to search and analyze more deeply related to the possibility of a pattern of cases chosen, either by consulting the lecturer or looking for other learning resources. S tried to find various possibilities by drawing cases from small expansion to highest. This is related to the opinion of Edi Syahputra that combinatorial problems can be overcome by enumeration starting from a simple case which is then gradually followed by enumeration to a more complex stage [12].

3.3. Level 3:Systematically Generating All Cases

This level was the level at which students searched, investigated, and analyzed the generalizations of patterns in various ways according to their thinking, creativity and reasoning abilities. Some students felt that this stage was more complex and more abstract situation because they found it difficult to determine the appropriate formula. This is foloowing the opinion which states that in further research, students often struggle hard enough to solve combinatorial problems [5].

Based on the research, when the students were instructed to come in front of the class to present the results of their work, the majority of them would try their best to convey their thoughts logically and clearly related to the causes of how they found the generalization formula. Besides, students would also learn to be able to open their minds and try to understand and accept various suggestions and input from other students. The following are the results of the interview with the subject:

1) ***Analyticity:***

A: Can you show and give reasons that the generalization of the formula you have found is correct?

S: Yes, I can use the generalization formula that I found to find the difference in the size of n x n square in a particular column and explain how I got it
A: How do you find the generalization of the formula until the c x c column expansion with the size of n x n square?
S: There are two ways that I use to find generalization formulas.
The first way:

- Look for the difference in the size of nxn square in column 3 with n = 2 and n = 3
  Here the numbers are arranged in 3 columns

1	2	3
4	5	6
7	8	9

a. In the 2×2 square

$1\times 5=5$
$2\times 4=8$
The difference between them is 3

1	2
4	5

$4\times 8=32$
$5\times 7=35$
The difference between them is 3

4	5
7	8

$2\times 6=12$
$3\times 5=15$
The difference between them is 3

2	3
5	6

$5\times 9=45$
$6\times 8=48$
The difference between them is 3

5	6
8	9

So, the difference for a 2×2 square on 3 columns is 3

b. In the 3×3 square

$1\times 9=9$
$3\times 7=21$
The difference between them is 12

1	2	3
4	5	6
7	8	9

So, the difference for a 3×3 square on 3 columns is 12

- Find the difference in the size of the square nxn in column 4 with n = 2, n = 3, and n = 4
  Here the numbers are arranged in 4 columns

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

a. In the 2×2 square

$1\times 6=6$
$2\times 5=10$
The difference between them is 4

1	2
5	6

$3\times 8=24$
$4\times 7=28$
The difference between them is 4

3	4
7	8

$2\times7=14$
$3\times6=18$
The difference between them is 4

2	3
6	7

$5\times10=50$
$6\times9=54$
The difference between them is 4

5	6
9	10

So, the difference for a 2×2 square on 4 columns is 4

b. In the 3×3 square

$1\times11=11$
$3\times9=27$
The difference between them is 16

1	2	3
5	6	7
9	10	11

$5\times15=75$
$7\times13=91$
The difference between them is 16

5	6	7
9	10	11
13	14	15

$2\times12=24$
$4\times10=40$
The difference between them is 16

2	3	4
6	7	8
10	11	12

$6\times16=96$
$8\times14=112$
The difference between them is 16

6	7	8
10	11	12
14	15	16

So, the difference for a 3×3 square on 4 columns is 16

c. In the 4×4 square

$1\times16=16$
$4\times13=52$
The difference between them is 36

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

So, the difference for a 4×4 square on 4 columns is 36

- Find the difference in the size of the square nxn in column 5 with n = 2, n = 3, n = 4, and n = 5
  Here the numbers are arranged in 5 columns

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

a. In the 2×2 square

$1\times7=7$
$2\times6=12$
The difference between them is 5

1	2
6	7

$3\times9=27$
$4\times8=32$
The difference between them is 5

3	4
8	9

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$2\times 8=16$
$3\times 7=21$
The difference between them is 5

2	3
7	8

$4\times 10=40$
$5\times 9=45$
The difference between them is 5

4	5
9	10

So, the difference for a 2×2 square on 5 columns is 5

b. In the 3×3 square

$1\times 13=13$
$3\times 11=33$
The difference between them is 20

1	2	3
6	7	8
11	12	13

$3\times 15=45$
$5\times 13=65$
The difference between them is 20

3	4	5
8	9	10
13	14	15

$2\times 14=28$
$4\times 12=48$
The difference between them is 20

2	3	4
7	8	9
12	13	14

$6\times 18=108$
$8\times 16=128$
The difference between them is 20

6	7	8
11	12	13
16	17	18

So, the difference for a 3×3 square on 5 columns is 20

c. In the 4×4 square

$1\times 19=19$
$4\times 16=64$
The difference between them is 45

1	2	3	4
6	7	8	9
11	12	13	14
16	17	18	19

$6\times 24=144$
$9\times 21=189$
The difference between them is 45

6	7	8	9
11	12	13	14
16	17	18	19
21	22	23	24

$1\times 20=20$
$5\times 17=85$
The difference between them is 45

2	3	4	5
7	8	9	10
12	13	14	15
17	18	19	20

$7\times 25=175$
$10\times 22=220$
The difference between them is 45

7	8	9	10
12	13	14	15
17	18	19	20
22	23	24	25

So, the difference for a 4×4 square on 5 columns is 45

d. In the 5×5 square

$1\times 19=19$
$4\times 16=64$
The difference between them is 45

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

So, the difference a 5×5 square on 5 columns is 80

- Make a table of the results obtained to find the relationship between the size of the square with the specified column

Columns (c)	$(n \times n)$	The difference for a $(n \times n)$ square on column	Connection
3	2×2	3	$3(1) = 3(2-1)^2 = 3$
	3×3	12	$3(4) = 3(3-1)^2 = 12$
4	2×2	4	$4(1) = 4(2-1)^2 = 4$
	3×3	16	$4(4) = 4(3-1)^2 = 16$
	4×4	36	$4(9) = 4(4-1)^2 = 36$
5	2×2	5	$5(1) = 5(2-1)^2 = 5$
	3×3	20	$5(4) = 5(3-1)^2 = 20$
	4×4	45	$5(9) = 5(4-1)^2 = 45$
	5×5	80	$5(16) = 5(5-1)^2 = 80$

- Observe the pattern formed
- Find the relationship between square and column sizes
- Find the generalization of formulas

  Based on the table above, it can known that $2 \le n \le c$ and the difference for a $(n \times n)$ square in (c) columns can be formulated by $c(n-1)^2$

The second way

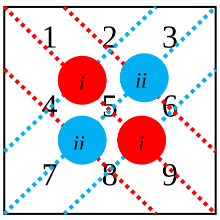
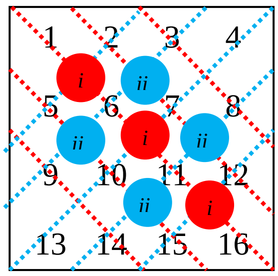
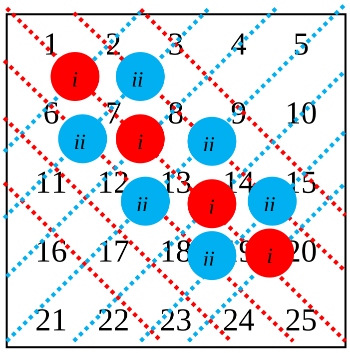
- Make columns 3, 4, and 5

1	2	3
4	5	6
7	8	9

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25

- Look for differences in adjacent numbers on the diagonal, both from the right to left diagonal and from the left to right diagonal

- Find the relationship between the pattern of differences between adjacent numbers on the diagonal with the number of columns, so we get the formula of the difference between the adjacent numbers on the diagonal

The difference between adjacent numbers on a diagonal have same value. These conditions can be formulated as follows

$i = c + 1$

$ii = c - 1$

- Look for the difference in the size of nxn square in column 3 with n = 2 and n = 3
- Look for the difference in the size of nxn square in column 4 with n = 2, n = 3, and n = 4
- Look for the difference in the size of nxn square in column 5 with n = 2, n = 3, n = 4, and n = 5
- Make a table of the results obtained in columns 3, 4, and 5 to see the patterns formed

$(n \times n)$	Columns (c)		
	3	4	5
2×2	$\lvert(1\times 5)-(2\times 4)\rvert = 3$ $\lvert(2\times 6)-(3\times 5)\rvert = 3$ $\lvert(4\times 8)-(5\times 7)\rvert = 3$ $\left\lvert\underbrace{(5\times 9)}_{i}-\underbrace{(6\times 8)}_{ii}\right\rvert = \underbrace{3}_{iii}$	$\lvert(1\times 6)-(2\times 5)\rvert = 4$ $\lvert(2\times 7)-(3\times 6)\rvert = 4$ $\lvert(3\times 8)-(4\times 7)\rvert = 4$ $\left\lvert\underbrace{(5\times 10)}_{i}-\underbrace{(6\times 9)}_{ii}\right\rvert = \underbrace{4}_{iii}$ $\vdots$	$\lvert(1\times 7)-(2\times 6)\rvert = 5$ $\lvert(2\times 8)-(3\times 7)\rvert = 5$ $\lvert(3\times 9)-(4\times 8)\rvert = 5$ $\left\lvert\underbrace{(4\times 10)}_{i}-\underbrace{(5\times 9)}_{ii}\right\rvert = \underbrace{5}_{iii}$ $\vdots$
	The difference between the number in:		
	i condition is 4	*i* condition is 5	*i* condition is 6
	ii condition is 2	*ii* condition is 3	*ii* condition is 4
	iii condition: the difference for a 2×2 square on:		
	3 columns is 3	4 columns is 4	5 columns is 5
3×3	$\left\lvert\underbrace{(1\times 9)}_{i}-\underbrace{(3\times 7)}_{ii}\right\rvert = \underbrace{12}_{iii}$	$\lvert(1\times 11)-(3\times 9)\rvert = 16$ $\lvert(2\times 12)-(4\times 10)\rvert = 16$ $\lvert(5\times 15)-(7\times 13)\rvert = 16$ $\left\lvert\underbrace{(6\times 16)}_{i}-\underbrace{(8\times 14)}_{ii}\right\rvert = \underbrace{16}_{iii}$	$\lvert(1\times 13)-(3\times 11)\rvert = 20$ $\lvert(2\times 14)-(4\times 12)\rvert = 20$ $\lvert(3\times 15)-(5\times 13)\rvert = 20$ $\left\lvert\underbrace{(6\times 18)}_{i}-\underbrace{(8\times 16)}_{ii}\right\rvert = \underbrace{20}_{iii}$ $\vdots$
	The difference between the number in:		
	i condition is 8	*i* condition is 10	*i* condition is 12
	ii condition is 4	*ii* condition is 6	*ii* condition is 8
	iii condition: the difference for a 3×3 square on:		
	3 columns is 12	4 columns is 16	5 columns is 20
4×4		$\left\lvert\underbrace{(1\times 16)}_{i}-\underbrace{(4\times 13)}_{ii}\right\rvert = \underbrace{36}_{iii}$	$\lvert(1\times 19)-(4\times 16)\rvert = 45$ $\lvert(2\times 20)-(5\times 17)\rvert = 45$ $\lvert(6\times 24)-(9\times 21)\rvert = 45$ $\left\lvert\underbrace{(7\times 25)}_{i}-\underbrace{(10\times 22)}_{ii}\right\rvert = \underbrace{45}_{iii}$

$(n\times n)$	Columns (c)		
	3	4	5
		The difference between the number in: *i* condition is 15 *ii* condition is 9 *iii* condition: the difference for a 4×4 square on: 4 columns is 36	 *i* condition is 18 *ii* condition is 12 5 columns is 45
5×5			$\left\lvert \underbrace{(1\times 25)}_{i} - \underbrace{(5\times 21)}_{ii} \right\rvert = \underbrace{80}_{iii}$ The difference between the number in: *i* condition is 24 *ii* condition is 16 *iii* condition: the difference for a 4×4 square on 5 columns is 80

- Connect the difference between adjacent numbers on the size of the square with the difference in the size of the square nxn in the column by adding constants

Columns (c)	$(n\times n)$	The difference
3	2×2	$(4-2)+1=3$
	3×3	$(8-4)+8=12$ or $2(4-2)+8=12$
4	2×2	$(5-3)+2=4$
	3×3	$(10-6)+12=16$ or $2(5-3)+12=16$
	4×4	$(15-9)+30=36$ or $3(5-3)+30=36$

Columns (c)	$(n\times n)$	The difference
5	2×2	$(6-4)+3=5$
	3×3	$(12-8)+16=20$ or $2(6-4)+16=20$
	4×4	$(18-12)+39=45$ or $3(6-4)+39=45$
	5×5	$(24-16)+72=80$ or $4(6-4)+72=80$

- Find the generalization of formulas
  $(n-1)[(c+1)-(c-1)]+\left[(c-2)(n-1)+c((n-1)-1)^2+c((n-1)-1)\right]=c(n-1)^2$
  $(n-1)[(c+1)-(c-1)]+\left[(c-2)(n-1)+c(n-2)^2+c(n-2)\right]=c(n-1)^2$

2) Systematicit :

A: Can you explain the formula that has been logically obtained and can be accepted by other students?
S: Yes, I can
A: Can your friends accept or agree with the work that you presented in front of the class?
S: Yes

A: Are there any objections from your friends about the generalization of the formula that you have made?
S: No, there are not

3) Confidence in reasoning
A: Do you think that the generalization of the formula that you have gotten is correct? Please explain!
S: Yes, the way to find the generalization of the formula is the same as before and it has been proven to find the difference in the size of nxn square in certain columns
A: Do you feel confident when delivering or presenting your work?
S: Yes, I do

Based on the result and interviews, it shows that S had the Indicator related to critical thinking disposition, namely Analyticity, Systematicity, and Confidence in reasoning. S was able to provide explanations and chronology of how he found the formula for several cases he obtained by providing evidence in the form from the first to the fifth expansion. S also said that the generalization formula he made is correct because after consulting and re-checking the formula, there was a compatible formula with the cases. Therefore, S felt challenged to look deeper with clear and logical reasons for each step. This is related to Kapur's opinion that combinatorics is important and must be taught in school because combinatorics has many challenging topics [12].

3.4. Level 4: Changing the problem into another problem combinatorial

At this level, students solved problems by finding the relation of size column and square start from the first expansion up to third expansion or certain *n square and c column*. The findings showed that generally the relation found by students was related to arithmetic, odd-even functions, or modulo concepts, it shows that S had the Indicator related to critical thinking disposition, namely Maturity of judgment. The following are the results of the interview with the subject:

4) Maturity of judgment:
A: Do you think there is another generalization formula that is different from what you have found?
S: Yes, because there are many ways to get the same results
A: Can you find other generalization formulas that are different from what you have found?
S: Maybe I can find another formula generalization
A: Do you think that the generalization formula that you have found can already answer that problem?
S: Yes, because using this formula can answer similar problems given
A: Have you thought about what if the size of the column is c x d with the square size n x n?
S: Yes. If the size is different, the formula obtained is not valid

Based on the result and interviews, It is seen that S sought to explore various possibilities in-depth, various possible generalization formulas, either by determining the generalization pattern or by drawing expansion from the first expansion to the fifth expansion. S realized that there was an error in determining the pattern. This indicated that the S had found a link on pattern on expand to 1 up to 3 or n particular, S even dared to change the initial problems he had associated "how to determine the generalization formula for n x n square?", became a matter of "how to determine the generalization formula for n x m square?" or " how to determine the generalization formula for c x d column" here S found the point of his mistake. This is same as the opinion that error is like over-counting, which involves determining a quantity, which is bigger than the wanted, and difficulties in choosing the right operation to occur, and it is predicted that errors in the problems are lack of understanding [5]. But then S dared to correct himself until finally he could give an explanation related to how he got the generalization formula from his problem in analyticity and systematics.

Recreational mathematics provided many problems, ideas and almost every problem can be extended or changed. Hence recreational mathematics is also a treasury of problems for student investigations [11]. The opposite corner problems are open-ended and generalizations are often unsolved, so one has to re-examine the problem and ask new questions to solving problems. Based on the results and analysis it was found that S in solving the problem gave rise to new and logical ideas, so students are required to be creative and are also required to think and be critical in solving the problem. This is following Jennifer's opinion that recreational mathematics can provide a most pleasant introduction to logic and deep results in mathematics [3]. Besides that, recreational mathematics has proved to be ideal for developing problem-solving skills. As for the skills raised by S namely the subject clarifying assumptions to make various conjectures to solve the problem, The subject made some notes or symbol, the subject using methods and new fundamental ideas, subject reviewing the problem and ask some new questions related to conjecture that he raised to solve the problem.

4. Conclusion

It can be concluded that critical thinking disposition can be raised from within each student through problem solving related to one of the topics in the field of mathematics, recreational mathematic especially "opposite corners problems". Combinatorial thinking levels implicitly contain indicators of critical thinking. Besides, the narrowing of the learning approach is also very influential. recreational mathematic problem is an approach that has the potential to increase students' cognitive abilities and abilities, especially in problem-solving by raising critical thinking disposition.

The four levels of combinatorics thinking contained several critical thinking dispositions. At level one, students began to recognize the problems they faced. They have opportunity to bring up indicators of truth seeking and inquisitiveness. At the second level, students began to look for various possible solutions to the problem. It was a level that provided an opportunity to bring up open mindedness indicators. The next level was level three. Students were required to analyze, search and generalize formulas in general. This could bring up indicators of confidence in reasoning, analyticity and systematics. Then, at level four, students not only focused on the problems they faced, but also tried and reasoned for other various problems that might be studied from the problems previously obtained. This could bring up indicators of maturity of judgment.

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