The locating dominating set (LDS) of generalized of corona product of path graph and any graphs

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Abstract. Graph G is usually written by $G = (V, E)$ is graph G where $V(G)$ is vertex set on graph G and $E(G)$ is edge set on graph G. Graph G used in this study is only on simple and undirected graphs. Dominating set (DS) is graph G which have a vertex set D, where each vertex in D can dominate the neighboring vertices, in other words every vertex from $u \in V(G) - D$ is adjacent to verted $v \in D$. The minimum cardinality of dominating set is called by domination number, symbolized by $\gamma(G)$. Locating dominating set (LDS) is dominating set with additional condition. A graph $G = (V, E)$ is said to be locating dominating set if the set of vertex dominator D satisfies every vertex that is not D, that is $V - D$ has a different intersection set with D. The minimum cardinality of locating dominating set is called by locating domination number, symbolized by $\gamma_L(G)$. In this paper we will determine the LDS on edge corona product. The edge corona product of graph is development of corona product graph. The edge corona of two graphs G and H is obtained by taking one copy of G and $|E(G)|$ copies of H and joining each end vertices of i-th edge of G to every vertex in the i-th copy of H, symbolized by $G \diamond H$. The results in this study are shown that there is a relation between the locating dominating set on the basic graph and its operation.

1. Introduction

Graph G is usually written with $G = (V, E)$ is graph G where $V(G)$ is vertex set on graph G written $V(G) = \{v_1, v_2, ..., v_n\}$ and $E(G)$ is edge set on graph G and not empty set written $E(G) = \{e_1, e_2, ..., e_n\}$. The number of vertex of graph G is order G written $|V(G)| = p$ and the number edge of graph G is size of G written $|E(G)| = q$ Definition of graph G are researched by [3, 1].

Wardani [10] In 2019 determined the relation among DS of corona product graph $\gamma(G \odot H)$ and DS of generalized of corona product of graph $\gamma(G \diamond H)$. They claimed that the edge corona product is generalized of corona product. The corona product, copy of H located in $V(G)$ and the edge corona product, copy of H located in $E(G)$. Definition of corona product of graph are researched by [6, 8], while the edge corona product of graph are researched by [10]. The difference among $G \odot H$ and $G \diamond H$ can be seen in Figure 1.

The history of dominating set (DS) begins when European chess fans study the problem of queen domination. Initially the problem of dominance was used to determine the number of queens so that each queen could dominate or attack every position with one move on a $8x8$ chessboard. In graph theory, the queen is represented as a vertex and the path of movement

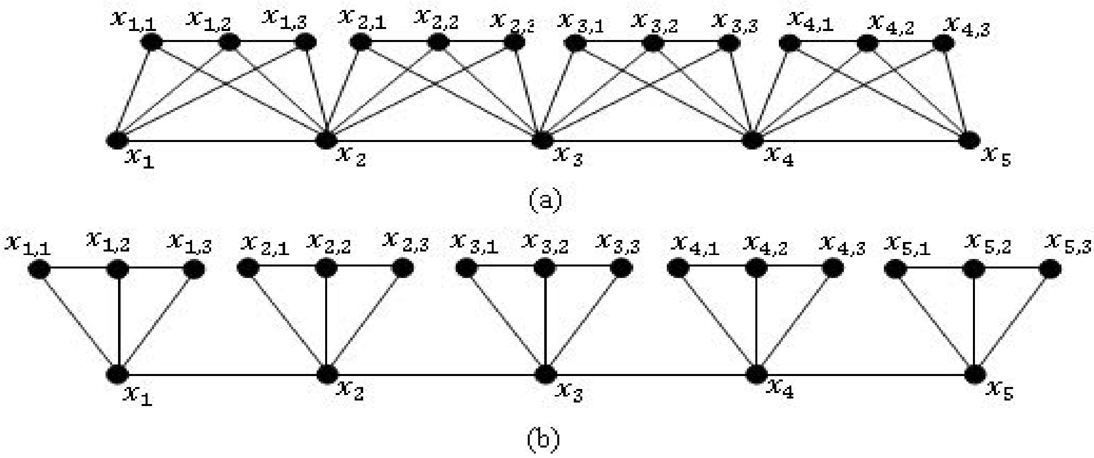

Figure 1. a. Corona Product $P_n \odot P_m$ and b. edge corona product $P_n \diamond P_m$

between boxes on the chessboard is considered as the edge. The minimum number of queens that are possible not to collide with another queen with one step is similar to the domination number of a DS on graph G [4, 5].

DS is divided into edge dominance and vertex dominance. The DS (D) in graph G is a subset of $V(G)$ such that each vertex G not in D is connected and distance one to D. The minimum cardinality of the DS on graph G is called the domination number of graph G and symbolized by $\gamma(G)$ [9].

Locating dominating set (LDS) is a set of DS with additional conditions. A graph G is said to be LDS if the set of dominator D satisfies every vertex not D, that is $V - D$ has a different intersection set with D, so $\{u, v \in V \setminus D\}$ then applies:

1 $N(u) \cap D \neq 0$ and $N(v) \cap D \neq 0$

2 If $u \neq v$ then $N(u) \cap D \neq N(v) \cap D$

Locating domination number is the minimum cardinality of the LDS symbolized by $\gamma_L(G)$ [7]. In this research, we had determined the LDS on edge corona product of graph, denoted by $G \diamond H$ with one of the graphs is path graph P_n. G and H are two simple and undirected graphs. The edge corona $G \diamond H$ of G and H is defined as the graph obtained by taking copy of H to every vertex in the i-th copy of $E(G)$.

2. Result

In this research, we determined the locating domination number of edge corona product ($G \diamond H$) of path graph and any graphs $\gamma_L(G \diamond P_2)$, $\gamma_L(G \diamond P_m)$, and $\gamma_L(P_2 \diamond H)$ with any graph G and H is Path graph, Cycle graph, and Star graph. We can see on [2] to determined the cardinality of edge set and vertex set.

Lemma 1. $\gamma_L(G \diamond P_2) \geq 2q(G)$.

Proof. The vertex and edge cardinality of graph $G \diamond P_2$ are $|V(G \diamond P_2)| = p(G) + 2$ and $|E(G \diamond P_2)| = q(G) + 2p(G) + 1$. Thus, $\Delta(G \diamond P_2) = \Delta(G) + 4$ and $\delta(P_2 \diamond P_m) = 3$.

Based on [8], Graph G order $n \geq 3$ and graph H order $m \geq 3$. The dominator of $G \diamond H$ located in graph H. Based on Definition of $G \diamond H$ every vertex of graph H connected to two vertices (u, v) of graph G where $uv \epsilon E(G)$. In this case, the graph is $G \diamond P_2$, we can see that the graph H is P_2 so, the dominator located in P_2. If we select one of two vertices in P_2 then the definition of LDS is not satisfied on this graph $(G \diamond P_2)$. It can be said that the dominator located in every vertex of P_2. □

Lemma 2. $\gamma_L(G \diamond P_m) \geq \gamma_L(P_m)q(G)$.

Proof. The vertex and edge cardinality of graph $G \diamond P_m$ are $|V(G \diamond P_m)| = p(G) + q(G)m$ and $|E(G \diamond P_m)| = q(G) + (m-1)q(G) + 2m(q(G))$. Thus, $\Delta(G \diamond P_m) = \Delta(G) + 2m$ and $\delta(G \diamond P_m) = 3$.

Based on Lemma 1, we know that the dominator located in graph P_2. Likewise with this Lemma. The dominator located in H, in this case the graph H is P_m for every positive integer $m \geq 3$. Based on Definition of $G \diamond H$ every vertex of graph H connected to two vertices (u, v) of graph G where $uv \epsilon E(G)$. We can put the dominator in graph P_m. Every dominator of graph P_m, $\gamma(P_m)$ can be dominated two vertex of graph G where $u, v \in V(G)$ then $uv \in E(G)$. It can be say that the dominator located in P_m □

Lemma 3. $\gamma_L(P_2 \diamond H) \geq \gamma_L(H) + 1$.

Proof. The vertex and edge cardinality of graph $P_2 \diamond H$ are $|V(P_2 \diamond H)| = p(H) + 2$ and $|E(P_2 \diamond H)| = q(H) + 2p(H) + 1$. Thus, $\Delta(P_2 \diamond H) = \Delta(H) + 2$ and $\delta(P_2 \diamond H) = 3$.

Based on Lemma 1 and Lemma 2 the dominator located in H, in this case the graph G is P_2 and any graph H order $m \geq 3$. Based on Definition of $G \diamond H$ every vertex of graph H connected to two vertices (u, v) of graph G where $uv \epsilon E(G)$. If we put the dominator in graph H then we can see that x_1 and x_2 of graph P_2 have the same dominator and the definition of LDS is not satisfied on this graph $(P_2 \diamond H)$. So we must add another vertex in P_2. It can be say that the dominator located in H and one vertex in P_2. □

Theorem 1. $\gamma_L(P_2 \diamond P_m) = \gamma_L(P_m) + 1$.

Proof. The vertex and edge set of $P_2 \diamond P_m$ are $V(P_2 \diamond P_m) = \{x_1, x_2\} \cup \{y_i; i = 1, 2, 3, ..., m\}$ and $E(P_2 \diamond P_m) = \{x_1x_2\} \cup \{x_1y_i; i = 1, 2, 3, ..., m\} \cup \{x_2y_i; i = 1, 2, 3, ..., m\} \cup \{y_iy_{i+1}; i = 1, 2, 3, ..., m-1\}$. The vertex and edge cardinality of $P_2 \diamond P_m$ are $|V(P_2 \diamond P_m)| = m + 2, |E(P_2 \diamond P_m)| = 3m$ respectively. Hence, $\Delta(P_2 \diamond P_m) = m + 1$ and $\delta(P_2 \diamond P_m) = 3$.

Based on Lemma 2 the dominator located in P_m. Select $D = \{y_i; i \equiv 0 \bmod 2\}$ as the LDS of $P_2 \diamond P_m; m \geq 3$, thus $|D| = \lceil \frac{n}{2} \rceil$, we can see that x_1 and x_2 have the same dominator and the definition of LDS is not satisfied on this graph $(P_2 \diamond P_m)$. Based on Lemma 3 we must select another vertex in P_2, select $D = \{x_1\} \cup \{y_i; i \equiv 0 \bmod 2\}$ as the LDS of $P_2 \diamond P_m; m \geq 3$, hence $|D| = \lfloor \frac{n}{2} \rfloor + 1$. Therefore $\gamma_L(P_2 \diamond P_m) = \lfloor \frac{n}{2} \rfloor + 1$. □

Figure 2 is example of LDS on $P_2 \diamond P_m$ with $m = 4$, then $P_2 \diamond P_4$.

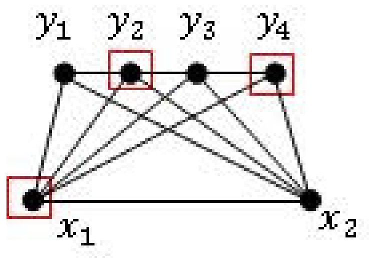

Figure 2. LDS of $P_2 \diamond P_4$

Theorem 2. $\gamma_L(P_n \diamond P_m) = \gamma_L(P_m)q(P_n)$.

Proof. The vertex and edge set of $P_n \diamond P_m$ are $V(P_n \diamond P_m) = \{x_i; i = 1,2,3,...,n\} \cup \{x_{i,j}; i = 1,2,3,...,n-1; j = 1,2,3,...,m\}$ and $E(P_n \diamond P_m) = \{x_ix_i + 1; i = 1,2,3,...,n-1\} \cup \{x_ix_{i,j}; i = 1,2,3,...,n-1; j = 1,2,3,...,m\} \cup \{x_{i+1}x_{i,j}; i = 1,2,3,...,n-1; j = 1,2,3,...,m\} \cup \{x_{i,j}x_{i,j+1}; i = 1,2,3,...,n-1; j = 1,2,3,...,m-1\}$. The vertex and edge cardinality of $P_n \diamond P_m$ are $|V(P_n \diamond P_m)| = n+nm-m, |E(P_n \diamond P_m)| = 3nm-3m$ respectively. Hence, $\Delta(P_n \diamond P_m) = 2(m+1)$ and $\delta(P_n \diamond P_m) = 3$.

Based on Lemma 2 the dominator located in P_m. Select $D = \{x_{i,j}; i = 1,2,3,...,n-1; j \equiv 0 \bmod 2\}$ as the LDS of $P_n \diamond P_m; n, m \geq 3$, hence $|D| = (\lfloor \frac{n}{2} \rfloor)q(P_n)$. Therefore $\gamma_L(P_n \diamond P_m) = \gamma_L(P_m)q(P_n)$. □

Figure 3 is example of LDS on $P_n \diamond P_m$ with $n = 5$ and $m = 3$, then $P_2 \diamond P_4$.

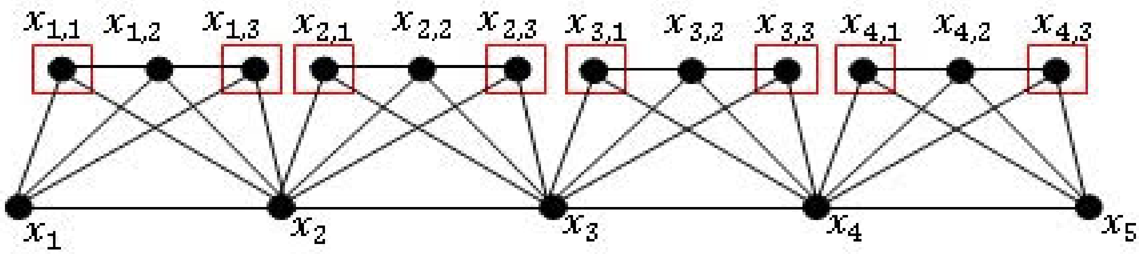

Figure 3. LDS of $P_5 \diamond P_3$

Theorem 3. $\gamma_L(P_n \diamond P_2) = 2q(P_n)$.

Proof. The vertex and edge set of $P_n \diamond P_2$ are $V(P_n \diamond P_2) = \{x_i; i = 1,2,3,...,n\} \cup \{x_{i,j}; i = 1,2,3,...,n-1; j = 1,2\}$ and $E(P_n \diamond P_2) = \{x_ix_{i+1}; i = 1,2,3,...,n-1\} \cup \{x_ix_{i,j}; i = 1,2,3,...,n-1; j = 1,2\} \cup \{x_{i+1}x_{i,j}; i = 1,2,3,...,n-1; j = 1,2\} \cup \{x_{i,j}x_{i,j+1}; i = 1,2,3,...,n-1; j = 1\}$. The vertex and edge cardinality of $P_n \diamond P_2$ are $|V(P_n \diamond P_2)| = 3n-2, |E(P_n \diamond P_2)| = 6n-6$ respectively. Hence, $\Delta(P_n \diamond P_2) = 2(m+1)$ and $\delta(P_n \diamond P_2) = 3$.

Based on Lemma 1 the dominator located in P_2. Select $D = \{x_{i,j}; i = 1,2,3,...,n-1; j = 1\}$ as the LDS of $P_n \diamond P_2; n \geq 3$, thus $|D| = n-1$, we can see every vertex in one piece $uv \in E(P_n)$ have the same dominator and the definition of LDS is not satisfied on this graph $(P_n \diamond P_2)$. Based on Lemma 3 we must select another vertex in P_2, select $D = \{x_{i,j}; i = 1,2,3,...,n-1; j = 1,2\}$ as the LDS of $P_n \diamond P_2; n \geq 3$, hence $|D| = 2(n-1)$. Therefore $\gamma_L(P_n \diamond P_2) = 2q(P_n)$. □

Figure 4 is example of LDS on $P_n \diamond P_2$ with $n = 5$, then $P_2 \diamond P_4$.

Theorem 4. $\gamma_L(C_n \diamond P_2) = 2q(C_n)$.

Proof. The vertex and edge set of $C_n \diamond P_2$ are $V(C_n \diamond P_2) = \{x_i; i = 1,2,3,...,n\} \cup \{x_{i,j}; i = 1,2,3,...,n; j = 1,2\}$ and $E(C_n \diamond P_2) = \{x_ix_{i+1}; i = 1,2,3,...,n\} \cup \{x_ix_{i,j}; i = 1,2,3,...,n; j = 1,2\} \cup \{x_{i+1}x_{i,j}; i = 1,2,3,...,n; j = 1,2\} \cup \{x_{i,j}x_{i,j+1}; i = 1,2,3,...,n; j = 1\}$. The vertex and edge cardinality of $C_n \diamond P_2$ are $|V(C_n \diamond P_2)| = 3n, |E(C_n \diamond P_2)| = 6n$ respectively. Hence, $\Delta(C_n \diamond P_2) = 2(m+1)$ and $\delta(P_n \diamond P_2) = 3$.

Based on Lemma 1 the dominator located in P_2. Select $D = \{x_{i,j}; i = 1,2,3,...,n; j = 1\}$ as the LDS of $C_n \diamond P_2; n \geq 3$, hence $|D| = n$, we can see every vertex in one piece $uv \in E(C_n)$ have the same dominator and the definition of LDS is not satisfied of this graph $(C_n \diamond P_2)$. Based on

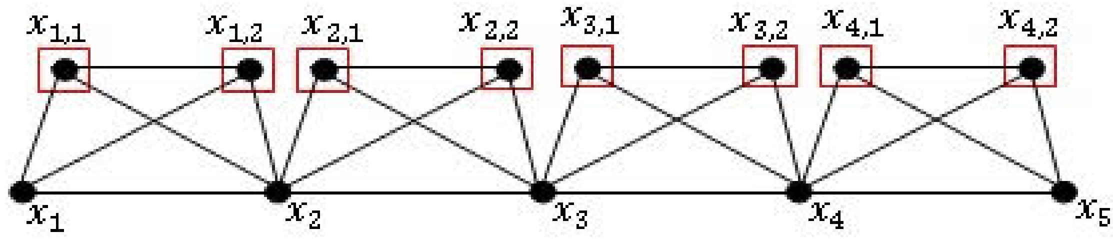

Figure 4. LDS of $P_5 \diamond P_2$

Lemma 3 we must select another vertex in P_2, select $D = \{x_{i,j}; i = 1, 2, 3, ..., n; j = 1, 2\}$ as the LDS of $C_n \diamond P_2; n \geq 3$, hence $|D| = 2n$. Therefore $\gamma_L(C_n \diamond P_2) = 2q(C_n)$. □

Figure 5 is example of LDS on $C_4 \diamond P_2$ with $n = 4$, then $C_4 \diamond P_2$.

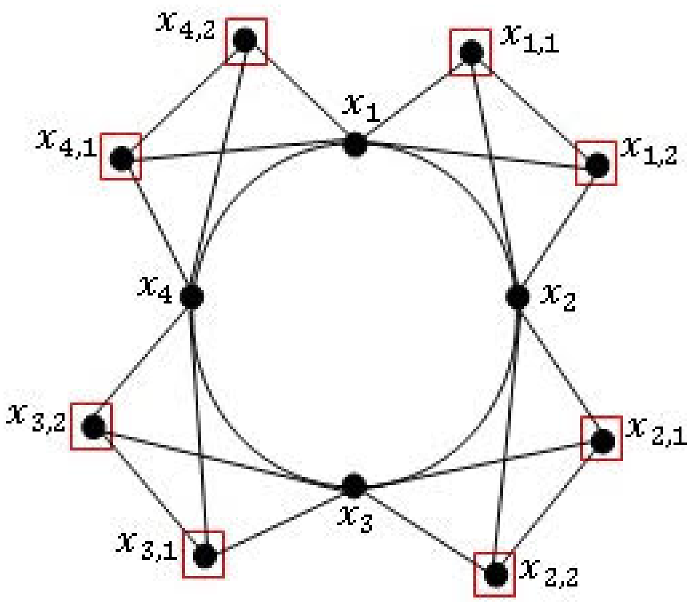

Figure 5. LDS of $C_4 \diamond P_2$

Theorem 5. $\gamma_L(S_n \diamond P_2) = 2q(S_n)$.

Proof. The vertex and edge set of $S_n \diamond P_2)$ are $V(S_n \diamond P_2) = \{A\} \cup \{x_i; i = 1, 2, 3, ..., n\} \cup \{x_{i,j}; i = 1, 2, 3, ..., n; j = 1, 2\}$ and $E(S_n \diamond P_2) = \{Ax_i; i = 1, 2, 3, ..., n\} \cup \{x_i x_{i,j}; i = 1, 2, 3, ..., n; j = 1, 2\} \cup \{Ax_{i,j}; i = 1, 2, 3, ..., n; j = 1, 2\} \cup \{x_{i,j} x_{i,j+1}; i = 1, 2, 3, ..., n; j = 1\}$. The vertex and edge cardinality of $S_n \diamond P_2$ are $|V(S_n \diamond P_2)| = 3n + 1, |E(S_n \diamond P_2)| = 6n$ respectively. Hence, $\Delta(S_n \diamond P_2) = 3n$ and $\delta(P_n \diamond P_2) = 3$.

Based on Lemma 1 the dominator located in P_2. Select $D = \{x_{i,j}; i = 1, 2, 3, ..., n; j = 1\}$ as the LDS of $S_n \diamond P_2; n \geq 3$, hence $|D| = n$, we can see every vertices in one piece $uv \in E(S_n)$ have the same dominator and the definition of LDS is not atisfied on $S_n \diamond P_2$. Based on Lemma 3 we must select another vertex in P_2, select $D = \{x_{i,j}; i = 1, 2, 3, ..., n; j = 1, 2\}$ as the LDS of

$S_n \diamond P_2; n \geq 3$, hence $|D| = 2n$. Therefore $\gamma_L(S_n \diamond P_2) = 2q(S_n)$. □

Figure 6 is example of LDS on $S_n \diamond P_2$ with $n = 3$, then $S_3 \diamond P_2$.

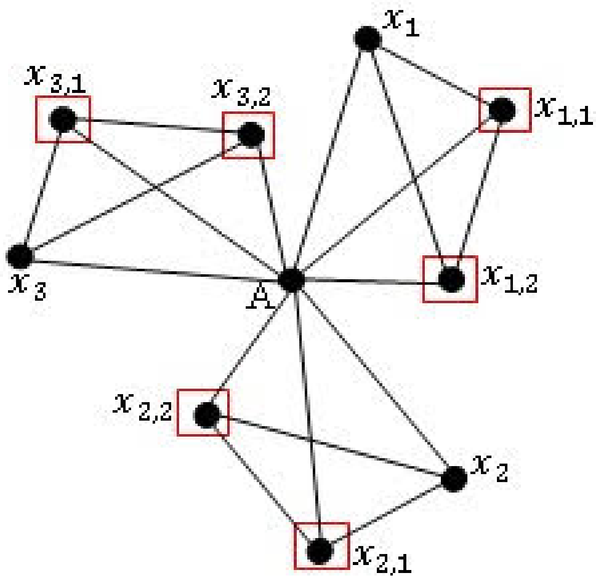

Figure 6. LDS of $S_3 \diamond P_2$

Theorem 6. $\gamma_L(C_n \diamond P_m) = \gamma_L(P_m)q(C_n)$.

Proof. The vertex and edge set of $C_n \diamond P_m$ are $V(C_n \diamond P_m) = \{x_i; i = 1,2,3,...,n\} \cup \{x_{i,j}; i = 1,2,3,...,n; j = 1,2,3,...,m\}$ and $E(C_n \diamond P_m) = \{x_i x_i + 1; i = 1,2,3,n\} \cup \{x_i x_{i,j}; i = 1,2,3,...,n; j = 1,2,3,...,m\} \cup \{x_{i+1} x_{i,j}; i = 1,2,3,...,n; j = 1,2,3,...,m\} \cup \{x_{i,j} x_{i,j+1}; i = 1,2,3,...,n; j = 1,2,3,...,m-1\}$. The vertex and edge cardinality of $C_n \diamond P_m$ are $|V(C_n \diamond P_m)| = n + nm$, $|E(C_n \diamond P_m)| = 3nm$ respectively. Hence, $\Delta(C_n \diamond P_m) = 2(m+1)$ and $\delta(P_n \diamond P_2) = 3$.

Based on Lemma 2 the dominator located in P_m. Select $D = \{x_{i,j}; i = 1,2,3,...,n; j \equiv 0 \bmod 2\}$ as the LDS of $C_n \diamond P_m; n, m \geq 3$, hence $|D| = n(\lfloor \frac{m}{2} \rfloor)$. Therefore $\gamma_L(C_n \diamond P_m) = \gamma_L(P_m)q(C_n)$. □

Figure 7 is example of LDS on $C_n \diamond P_m$ with $n = 4$ and $m = 3$, then $C_4 \diamond P_3$.

Theorem 7. $\gamma_L(S_n \diamond P_m) = \gamma_L(P_m)q(S_n)$.

Proof. The vertex and edge set of $S_n \diamond P_m$ are $V(S_n \diamond P_m) = \{A\} \cup \{x_i; i = 1,2,3,...,n\} \cup \{x_{i,j}; i = 1,2,3,...,n; j = 1,2,3,...,m\}$ and $E(S_n \diamond P_m) = \{Ax_i; i = 1,2,3,...,n\} \cup \{x_i x_{i,j}; i = 1,2,3,...,n; j = 1,2,3,...,m\} \cup \{Ax_{i,j}; i = 1,2,3,...,n; j = 1,2,3,...,m\} \cup \{x_{i,j} x_{i,j+1}; i = 1,2,3,...,n; j = 1,2,3,...,m-1\}$. The vertex and edge cardinality of $S_n \diamond P_m$ are $|V(S_n \diamond P_m)| = nm + n + 1$, $|E(S_n \diamond P_m)| = 3nm$ respectively. Hence, $\Delta(S_n \diamond P_m) = nm + n$ and $\delta(P_n \diamond P_m) = 3$.

Based on Lemma 2 the dominator located in P_m. Select $D = \{x_{i,j}; i = 1,2,3,...,n; j \equiv 0 \bmod 2\}$ as the LDS of $S_n \diamond P_m; n, m \geq 3$, hence $|D| = n(\lfloor \frac{m}{2} \rfloor)$. Therefore $\gamma_L(S_n \diamond P_m) = \gamma_L(P_m)q(S_n)$. □

Figure 8 is example of LDS on $S_n \diamond P_m$ with $n = 3$ and $m = 3$, then $S_3 \diamond P_3$.

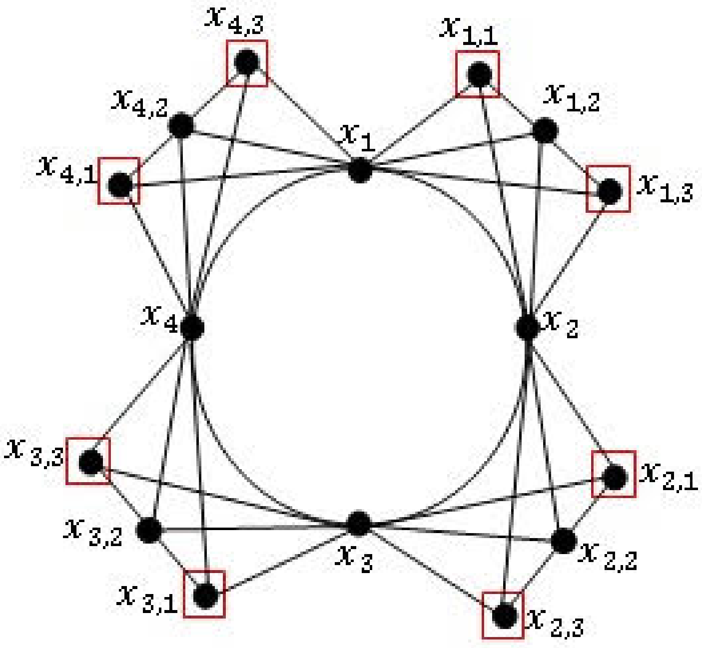

Figure 7. LDS of $C_4 \diamond P_3$

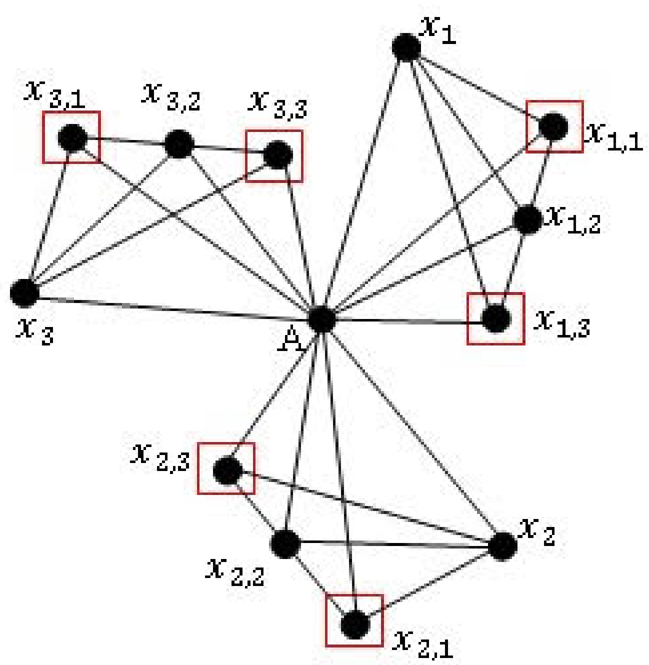

Figure 8. LDS of $S_3 \diamond P_3$

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3. Concluding Remark

In this research, we determined the locating domination number of edge corona product of path graph P_n and any graph $(P_n \diamond H)$ and $(G \diamond P_m)$. The conclusion is every edge corona product of graph depends on to its graph constructor. This research still gives the open proplem:

Open Problem 1. *Analyze the locating domination number of another operation graph, and determine the relation of that operation with the basic graph.*

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