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On resolving domination number of special family of graphs

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Abstract. Let G be a simple, finite, and connected graph. A dominating set D is a set of vertices such that each vertex of G is either in D or has at least one neighbor in D. The minimum cardinality of such a set is called the domination number of G, denoted by $\gamma(G)$. For an ordered set $W = \{w_1, w_2, ..., w_k\}$ of vertices and a vertex v in a connected graph G, the metric representation of v with respect to W is the k-vector $r(v|W) = (d(v, w_1), d(v, w_2), ..., d(v, w_k))$, where $d(x, y)$ represents the distance between the vertices x and y. The set W is a resolving set for G if distinct vertices of G have distinct representations with respect to W. A resolving set of minimum cardinality is called a minimum resolving set or a basis and the cardinality of a basis for G, denoted by $dim(G)$. A resolving domination number, denoted by $\gamma_r(G)$, is the minimum cardinality of the resolving dominating set. In this paper, we study the existence of resolving domination number of special graph and its line graph $L(G)$, middle graph $M(G)$, total graph $T(G)$, and central graph $C(G)$ of Star graph and fan graph. We have found the minimum cardinality of those special graphs.

1. Introduction

Graph G is defined as a pair of sets $(V(G), E(G))$, where $V(G)$ is a finite set that is not empty and its members are called vertex and $E(G)$ are sets that may be empty of unsorted pair (u, v) of the points $(u, v) \in V(G)$ called the edge [16]. Thus, a graph G does not have to have an edge but must have at least a vertex [1, 8].

The domination number denoted by $\gamma(G)$ firs introduced by [12]. Domination number is the minimum cardinality of a dominating set. The numerical value of domination always $\gamma(G) \cup V(G)$. Regarding the upper limit of the numbers that dominates is the number of nodes of the graph. When at least one point required for a set of graphs that dominates, then $1 \leq \gamma(G) \leq n$ for each stock that has a sequence of n. let $D \subseteq V$, if every vertex of G without a set of the graph that dominates the G bounded by at least one vertex of a set of graph that dominates the G, then D is said to be a set of graphs which dominates the G [12, 20]. The theorem regarding the upper bounds and lower bounds of the dominating set is as follows by [13]. Let G into a graph any upper and lower bounds is

$$\lceil \tfrac{p}{1+\Delta(G)} \rceil \leq \gamma(G) \leq p - \Delta(G)$$

In graph theory, the metric dimension of the graph G is the minimum cardinality of the S subset of vertices so that all other vertices are uniquely determined by their distance to vertices in S. For the given set $W = \{w_1, w_2, ..., w_k\} \subseteq V(G)$ and v are vertices in the graph G, then the representation of v with respect to W is k-tuple, $r(v|W) = (d(v, w_1), d(v, w_2), ..., d(v, w_k))$. The set W is called the set G if each vertex on G has a different representation and the minimum set called base G. The basic cardinality is called the metric dimension G, denoted by $dim(G)$ [2, 3, 7]. The settlement set for the graph was introduced independently by [18] and [11], while the concept of completion set and metric dimension was determined much earlier by [4].

The resolving domination number first introduced by [5] and [14]. Resolving domination number, denoted by $\gamma_r(G)$, is the minimum cardinality of the resolving dominating set. Researches related to resolving domination number is growing quite rapidly, including [9, 17] and many more. The corresponding parameters for all those sets satisfy by definition $max\{dim(G), \gamma(G)\} \leq \gamma_M(G) \leq dim(G) + \gamma(G)$ [5, 9].

In this study, the resolving domination number will be checked on the star and fan graph. Besides, we develop a graph that will become a new graph (line graph, middle graph, total graph, and central graph). A line graph from a simple graph of G denoted by $L(G)$ is obtained by associating vertices with each edge of the graph and connecting two vertices with edges if the corresponding edges of G have the same node [10]. In [19] the middle graph denoted by $M(G)$ of the connected graph G is a graph whose node-set is $V(G) \cup E(G)$ where two vertices are close together if they are edges which border G or one is the node of G and the other is an edge incident with it. The total graph denoted by $T(G)$ of the connected graph G is a graph whose node-set is $V(G) \cup E(G)$ and two adjacent vertices each time that border or events in G [6] and the central graph denoted by $C(G)$ of the connected graph G is obtained by dividing each edge of G exactly once and joining all nodes that are not next to each other from G in $C(G)$ [15].

2. Result

In this section, we determined the $\gamma_r(G)$ of special graph and its line graph ($L(G)$), middle graph ($M(G)$), total graph ($T(G)$), and central graph ($C(G)$) of star graph (S_n) and fan graph (F_n).

Theorem 1. *Let S_n be a star graph with $n \geq 4$. The resolving domination number of S_n is $\gamma_r(S_n) = n$.*

Proof. The star graph is a graph which is connected to the vertex of $V(S_n) = \{A\} \cup \{x_i; 1 \leq i \leq n\}$ and the edge of $E(S_n) = \{Ax_i; 1 \leq i \leq n\}$. Vertex cardinality of $|V(S_n)| = n + 1$ and edge cardinality of $|E(S_n)| = n$. Maximum degree is $\Delta(S_n) = n$ and $\delta(S_n) = 1$. The proof order of resolving domination number S_n consists of upper bound $\gamma_r(S_n) \leq n$ and lower bound $\gamma_r(S_n) \geq n$.

In this section, we show the upper bound and the lower bound of resolving domination number S_n. First, we will prove the upper bound of resolving domination number S_n which is $\gamma_r(S_n) \leq n$. We choose $S = \{x_i; 1 \leq i \leq n\}$ so that we have vertex representation in S_n which is different. For the detail, we can see in Table 1. S is also a dominating set because the vertex in S dominates the vertex which is not in S, so that S is resolving dominating set. Therefore, it can be concluded that $\gamma_r(S_n) \leq n$.

Furthermore, we will prove the lower bound of resolving domination number S_n which is $\gamma_r(S_n) \geq n$. Assume that resolving domination number of $\gamma_r(S_n) < n$. Take $|W| = n - 1$. The star graph has $n + 1$ vertex covering n vertex as pendant and 1 vertex as center vertex. Because we assume $n - 1$ vertex are in S so that there are several placement conditions of graph point S_n as follows:

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(i) If $n-1$ vertex are located in pedant vertex, there will be one vertex in pedant which is not dominated by $n-1$ vertex in S because the vertex in the pendant is not adjacent. Thus, the set of S is not dominating set.

(ii) If $n-2$ vertex are located in pendant vertex and 1 vertex in center vertex, there will be two in pendant not S element so that both vertex have the same representation because 2 vertex in pendant have the same distance to several vertex in S_n. Thus, the set of S is not resolving set.

Based on (i) and (ii) cases S is not resolving dominating set. Thus, the cardinality of resolving dominating set of S_n is n. The lower bound of resolving set number of S_n is $\gamma_r(S_n) \geq n$. Gives the result, we must have n vertex dominating in $x_i \in S_n$. Because we have proven that $\gamma_r(S_n) \leq n$ and $\gamma_r(S_n) \geq n$, it can be concluded that $\gamma_r(S_n) = n$. For an example, resolving domination number of star graph S_n can be seen in Figure 1

Table 1. Representation of $v \in V(S_n)$ respect to S

v	$r(v\|S)$	Condition
x_1	$(0, \underbrace{2, ..., 2}_{n-1})$	$n \geq 4$
x_i	$(\underbrace{2, ..., 2}_{i-1}, 0, \underbrace{2, ..., 2}_{n-i-1})$	$i \geq 2$
x_n	$(\underbrace{2, ..., 2}_{n-1})$	$n \geq 4$
A	$(\underbrace{1, ..., 1}_{n-1})$	$n \geq 4$

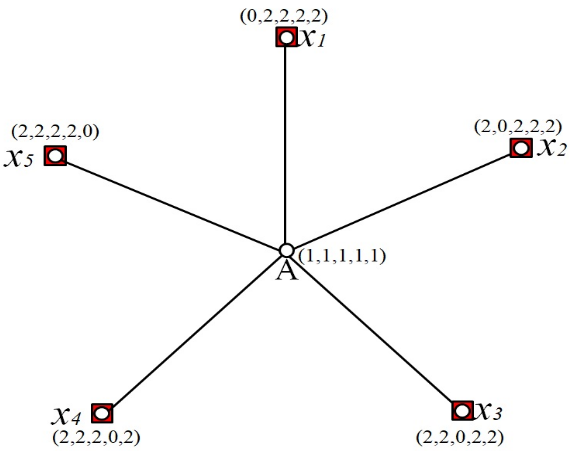

Figure 1. $\gamma_r(S_5) = 5$

Theorem 2. *Let $L(S_n)$ be a line star graph with $n \geq 4$. The resolving domination number of $L(S_n)$ is $\gamma_r(L(S_n)) = n-1$.*

Proof. The line star graph is a graph which is connected to the vertex of $V(L(S_n)) = \{x_i; 1 \leq i \leq n\}$ and the edge of $E(L(S_n)) = \{x_i x_{i+k}; 1 \leq i \leq n-1, 1 \leq k \leq n-i\}$. Vertex cardinality of $|V(L(S_n))| = n$ and edge cardinality of $|E(L(S_n))| = \frac{n^2-n}{2}$. Maximum degree is $\Delta(L(S_n)) = n-1$ and $\delta(L(S_n)) = n-1$. The first order of resolving domination number $L(S_n)$ consists of upper bound $\gamma_r(L(S_n)) \leq n-1$ and lower bound $\gamma_r(L(S_n)) \geq n-1$.

In this section, we show the upper bound and the lower bound of resolving domination number $L(S_n)$. First, we will prove the upper bound of resolving domination number $L(S_n)$ which is $\gamma_r(L(S_n)) \leq n-1$. We choose $S = \{x_i; 1 \leq i \leq n-1\}$ so that we have vertex representation in $L(S_n)$ which is different. For the detail, we can see in Table 2. S is also a dominating set because the vertex in S dominates all vertex in $L(S_n)$, so that S is resolving dominating set. Therefore, it can be concluded that $\gamma_r(L(S_n)) \leq n-1$.

Furthermore, we will prove the lower bound of resolving domination number $L(S_n)$ which is $\gamma_r(L(S_n)) \geq n-1$. Assume that resolving domination number of $\gamma_r(L(S_n)) < n-1$. Take $|W| = n-2$. The line star graph has n vertex. Because we assume $n-2$ points are in S so that there are several placement conditions of graph vertex $L(S_n)$ as follows:

(i) If $n-2$ vertex are located in $L(S_n)$, there will be two vertex dominated by $n-2$ vertex in S. Thus, the set of S is dominating set.

(ii) If $n-2$ vertex are located in $L(S_n)$, there will be two vertex which are not S element so that both vertex gave the same representation. Thus, the set of S is not resolving set.

Based on (i) and (ii) cases S is not resolving dominating set. Thus, the cardinality of resolving dominating set of $L(S_n)$ is $n-1$. The lower bound of resolving set number of $L(S_n)$ is $\gamma_r(L(S_n)) \geq n-1$. Gives the result, we must have $n-1$ vertex dominating in $x_i \in L(S_n)$. Because we have proven that $\gamma_r(L(S_n)) \leq n-1$ and $\gamma_r(L(S_n)) \geq n-1$, it can be concluded that $\gamma_r(L(S_n)) = n-1$. For an example, resolving domination number of line star graph $L(S_n)$ can be seen in Figure 2.

Table 2. Representation of $v \in V(L(S_n))$ respect to S

v	$r(v\|S)$	Condition
x_1	$(0, \underbrace{1, ..., 1}_{n-1})$	
x_i	$(\underbrace{1, ..., 1}_{i-1}, 0, \underbrace{1, ..., 1}_{n-i-1})$	$i \geq 2$
x_n	$(\underbrace{1, ..., 1}_{n-1})$	$n \geq 4$

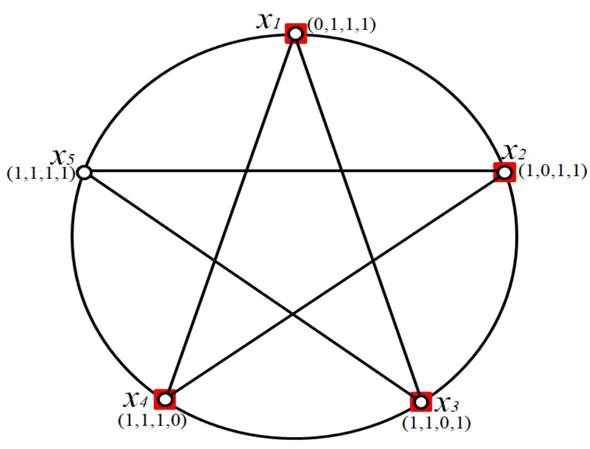

Figure 2. $\gamma_r(L(S_5)) = 4$

Theorem 3. *Let $M(S_n)$ be a middle star graph with $n \geq 4$. The resolving domination number of $M(S_n)$ is $\gamma_r(M(S_n)) = n$.*

Proof. The middle star graph is a graph which is connected to the vertex of $V(M(S_n)) = \{A\} \cup \{x_i, y_j; 1 \leq i, j \leq n\}$ and the edge of $E(M(S_n)) = \{Ay_j, x_iy_j; 1 \leq i, j \leq n\} \cup \{y_jy_{j+k}; 1 \leq j \leq n-1, 1 \leq k \leq n-i\}$. Vertex cardinality of $|V(M(S_n))| = 2n+1$ and edge cardinality of $|E(M(S_n))| = \frac{n^2+3n}{2}$. Maximum degree is $\Delta(M(S_n)) = n+1$ and $\delta(M(S_n)) = 1$. The proof order of resolving domination number $M(S_n)$ consists of upper bound $\gamma_r(M(S_n)) \leq n$ and lower bound $\gamma_r(M(S_n)) \geq n$.

In this section, we show the upper bound and the lower bound of resolving domination number $M(S_n)$. First, we will prove the upper bound of resolving domination number $M(S_n)$ which is $\gamma_r(M(S_n)) \leq n$. We choose $S = \{y_j; 1 \leq j \leq n\}$ so that we have vertex representation in $M(S_n)$ which is different. For the detail, we can see in Table 3. S is also a dominating set because the vertex in S dominates the vertex which is not in S, so that S is resolving dominating set. Therefore, it can be concluded that $\gamma_r(M(S_n)) \leq n$.

Furthermore, we will prove the lower bound of resolving domination number $M(S_n)$ which is $\gamma_r(M(S_n)) \geq n$. Assume that resolving domination number of $\gamma_r(M(S_n)) < n$. Take $|W| = n-1$. The middle star graph has $2n+1$ vertex. Because we assume $n-1$ vertex are in S so that there are several placement conditions of graph vertex $M(S_n)$ as follows:

(i) If $n-1$ vertex are located in y_j vertex, there will be one point in x_i which is not dominated by $n-1$ vertex in S, because the vertex in the x_i is not adjacent. Thus, the set of S is not dominating set.

(ii) If $n-2$ vertex are located in y_j vertex and 1 point in center vertex, there will be two in y_j not S element so that both points have the same representation because 2 points in y_j have the same distance to several points in $M(S_n)$. Thus, the set of S is not resolving set.

Based on (i) and (ii) cases S is not resolving dominating set. Thus, the cardinality of resolving dominating set of $M(S_n)$ is n. The lower bound of resolving set number of $M(S_n)$ is $\gamma_r(M(S_n)) \geq n$. Gives the result, we must have n points dominating in $y_j \in M(S_n)$. Because we have proven that $\gamma_r(M(S_n)) \leq n$ and $\gamma_r(M(S_n)) \geq n$, it can be concluded that $\gamma_r(M(S_n)) = n$. For an example, resolving domination number of middle star graph $M(S_n)$ can be seen in Figure 3.

Table 3. Representation of $v \in V(M(S_n))$ respect to S

v	$r(v\|S)$	Condition
x_1	$(1, \underbrace{2, ..., 2}_{n-1})$	
x_i	$(\underbrace{2, ..., 2}_{i-1}, 1, \underbrace{2, ..., 2}_{n-i})$	$i \geq 2$
x_n	$(\underbrace{2, ..., 2}_{n-1}, 1)$	$n \geq 4$
y_1	$(0, \underbrace{1, ..., 1}_{n-1})$	
y_j	$(\underbrace{1, ..., 1}_{j-1}, 0, \underbrace{1, ..., 1}_{n-j})$	$j \geq 2$
y_n	$(\underbrace{1, ..., 1}_{n-1}, 0)$	$n \geq 4$
A	$(\underbrace{1, ..., 1}_{n})$	$n \geq 4$

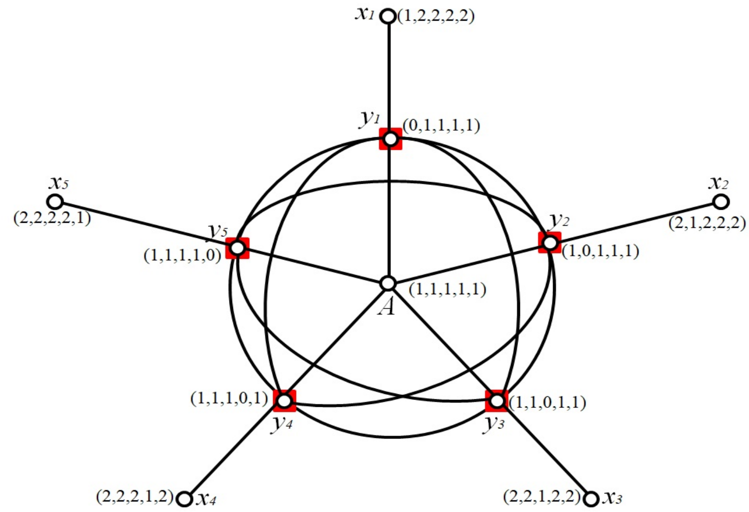

Figure 3. $\gamma_r(M(S_5)) = 5$

Theorem 4. *Let $T(S_n)$ be a total star graph with $n \geq 4$. The resolving domination number of $T(S_n)$ is $\gamma_r(T(S_n)) = n$.*

Proof. The total star graph is a graph which is connected to the vertex of $V(T(S_n)) = \{A\} \cup \{x_i, y_j; 1 \leq i, j \leq n\}$ and the edge of $E(T(S_n)) = \{Ay_j, Ax_i, x_iy_j; 1 \leq i, j \leq n\} \cup \{y_jy_{j+k}; 1 \leq j \leq n-1, 1 \leq k \leq n-j\}$. Vertex cardinality of $|V(T(S_n))| = 2n+1$ and edge cardinality of $|E(T(S_n))| = \frac{n^2+5n}{2}$. Maximum degree is $\Delta(T(S_n)) = n+1$ and $\delta(T(S_n)) = 2$. The proof order of resolving domination number $T(S_n)$ consists of upper bound $\gamma_r(T(S_n)) \leq n$ and lower bound $\gamma_r(T(S_n)) \geq n$.

In this section, we show the upper bound and the lower bound of resolving domination number $T(S_n)$. First, we will prove the upper bound of resolving domination number $T(S_n)$ which is $\gamma_r(T(S_n)) \leq n$. We choose $S = \{y_j; 1 \leq j \leq n\}$ so that we have vertex representation in $T(S_n)$ which is different. For the detail, we can see in Table 4. S is also a dominating set because the vertex in S dominates the vertex which is not in S, so that S is resolving dominating set. Therefore, it can be concluded that $\gamma_r(T(S_n)) \leq n$.

Furthermore, we will prove the lower bound of resolving domination number $T(S_n)$ which is $\gamma_r(T(S_n)) \geq n$. Assume that resolving domination number of $\gamma_r(T(S_n)) < n$. Take $|W| = n-1$. The total star graph has $2n+1$ vertex. Because we assume $n-1$ vertex are in S so that there are several placement conditions of graph vertex $T(S_n)$ as follows:

(i) If $n-1$ vertex are located in y_j vertex, there will be one point in x_i which is not dominated by $n-1$ vertex in S, because the vertex in the x_i is not adjacent. Thus, the set of S is not dominating set.

(ii) If $n-2$ vertex are located in y_j vertex and 1 point in center vertex, there will be two in y_j not S element so that both points have the same representation because 2 points in y_j have the same distance to several points in $T(S_n)$. Thus, the set of S is not resolving set.

Based on (i) and (ii) cases S is not resolving dominating set. Thus, the cardinality of resolving dominating set of $T(S_n)$ is n. The lower bound of resolving set number of $T(S_n)$ is $\gamma_r(T(S_n)) \geq n$. Gives the result, we must have n points dominating in $y_j \in T(S_n)$. Because we have proven that $\gamma_r(T(S_n)) \leq n$ and $\gamma_r(T(S_n)) \geq n$, it can be concluded that $\gamma_r(T(S_n)) = n$. For an example, resolving domination number of total star graph $T(S_n)$ can be seen in Figure 4.

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Table 4. Representation of $v \in V(T(S_n))$ respect to S

v	$r(v\|S)$	Condition
x_1	$(1, \underbrace{2, ..., 2}_{n-1})$	
x_i	$(\underbrace{2, ..., 2}_{i-1}, 1, \underbrace{2, ..., 2}_{n-i})$	$i \geq 2$
x_n	$(\underbrace{2, ..., 2}_{n-1}, 1)$	$n \geq 4$
y_1	$(0, \underbrace{1, ..., 1}_{n-1})$	
y_j	$(\underbrace{1, ..., 1}_{j-1}, 0, \underbrace{1, ..., 1}_{n-j})$	$j \geq 2$
y_n	$(\underbrace{1, ..., 1}_{n-1}, 0)$	$n \geq 4$
A	$(\underbrace{1, ..., 1}_{n})$	$n \geq 4$

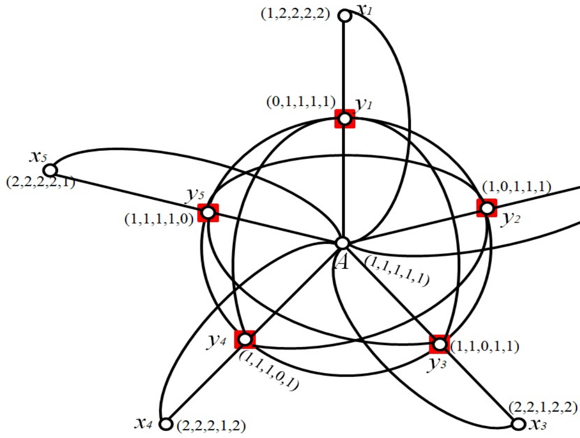

Figure 4. $\gamma_r(T(S_5)) = 5$

Theorem 5. *Let $C(S_n)$ be a central star graph with $n \geq 4$. The resolving domination number of $C(S_n)$ is $\gamma_r(C(S_n)) = n$.*

Proof. The central star graph is a graph which is connected to the vertex of $V(C(S_n)) = \{A\} \cup \{x_i, y_j; 1 \leq i, j \leq n\}$ and the edge of $E(C(S_n)) = \{Ay_j, x_iy_j; 1 \leq i, j \leq n\} \cup \{x_ix_{i+k}; 1 \leq i \leq n-1, 1 \leq k \leq n-i\}$. Vertex cardinality of $|V(C(S_n))| = 2n+1$ and edge cardinality of $|E(C(S_n))| = \frac{n^2+3n}{2}$. Maximum degree is $\Delta(C(S_n)) = n$ and $\delta(C(S_n)) = 2$. The proof order of resolving domination number $C(S_n)$ consists of upper bound $\gamma_r(C(S_n)) \leq n$ and lower bound $\gamma_r(C(S_n)) \geq n$.

In this section, we show the upper bound and the lower bound of resolving domination number $C(S_n)$. First, we will prove the upper bound of resolving domination number $C(S_n)$ which is $\gamma_r(C(S_n)) \leq n$. We choose $S = \{y_j; 1 \leq j \leq n\}$ so that we have vertex representation in $C(S_n)$ which is different. For the detail, we can see in Table 5. S is also a dominating set because the vertex in S dominates the vertex which is not in S, so that S is resolving dominating set. Therefore, it can be concluded that $\gamma_r(C(S_n)) \leq n$.

Furthermore, we will prove the lower bound of resolving domination number $C(S_n)$ which is

$\gamma_r(C(S_n)) \geq n$. Assume that resolving domination number of $\gamma_r(C(S_n)) < n$. Take $|W| = n-1$. The central star graph has $2n+1$ vertex. Because we assume $n-1$ vertex are in S so that there are several placement conditions of graph vertex $C(S_n)$ as follows:

(i) If $n-1$ vertex are located in y_j vertex, there will be one point in x_i which is not dominated by $n-1$ vertex in S, because the vertex in the x_i is not adjacent. Thus, the set of S is not dominating set.

(ii) If $n-2$ vertex are located in y_j vertex and 1 point in center vertex, there will be two in y_j not S element so that both points have the same representation because 2 points in y_j have the same distance to several points in $C(S_n)$. Thus, the set of S is not resolving set.

Based on (i) and (ii) cases S is not resolving dominating set. Thus, the cardinality of resolving dominating set of $C(S_n)$ is n. The lower bound of resolving set number of $C(S_n)$ is $\gamma_r(C(S_n)) \geq n$. Gives the result, we must have n points dominating in $y_j \in C(S_n)$. Because we have proven that $\gamma_r(C(S_n)) \leq n$ and $\gamma_r(C(S_n)) \geq n$, it can be concluded that $\gamma_r(C(S_n)) = n$. For an example, resolving domination number of central star graph $C(S_n)$ can be seen in Figure 5.

Table 5. Representation of $v \in V(C(S_n))$ respect to S

v	$r(v\|S)$	Condition
x_1	$(1, \underbrace{2, ..., 2}_{n-1})$	
x_i	$(\underbrace{2, ..., 2}_{i-1}, 1, \underbrace{2, ..., 2}_{n-i})$	$i \geq 2$
x_n	$(\underbrace{2, ..., 2}_{n-1}, 1)$	$n \geq 4$
y_1	$(0, \underbrace{2, ..., 2}_{n-1})$	
y_j	$(\underbrace{2, ..., 2}_{j-1}, 0, \underbrace{2, ..., 2}_{n-j})$	$j \geq 2$
y_n	$(\underbrace{2, ..., 2}_{n-1}, 0)$	$n \geq 4$
A	$(\underbrace{1, ..., 1}_{n})$	$n \geq 4$

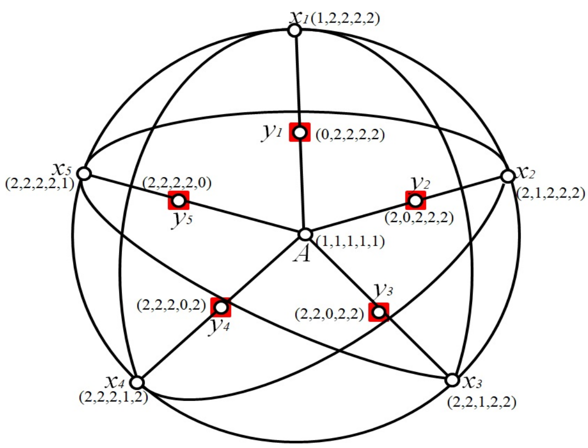

Figure 5. $\gamma_r(C(S_5)) = 5$

Theorem 6. *Let F_n be a fan graph with $n \geq 4$. The resolving domination number of F_n is $\gamma_r(F_n) = \lceil \frac{n}{3} \rceil + 1$.*

Proof. The fan graph is a graph which is connected to the vertex of $V(F_n) = \{A\} \cup \{x_i; 1 \leq i \leq n\}$ and the edge of $E(F_n) = \{Ax_i; 1 \leq i \leq n\} \cup \{x_i x_{i+1}; 1 \leq i \leq n-1\}$. Vertex cardinality of $|V(F_n)| = n+1$ and edge cardinality of $|E(F_n)| = 2n-1$. Maximum degree is $\Delta(F_n) = n$ and $\delta(F_n) = 2$. The proof order of resolving domination number F_n consists of upper bound $\gamma_r(F_n) \leq \lceil \frac{n}{3} \rceil + 1$ and lower bound $\gamma_r(F_n) \geq \lceil \frac{n}{3} \rceil + 1$.

In this section, we show the upper bound and the lower bound of resolving domination number F_n. First, we will prove the upper bound of resolving domination number F_n which is $\gamma_r(F_n) \leq \lceil \frac{n}{3} \rceil + 1$. We choose $S = \{x_i; 1 \leq i \leq n\}$ so that we have vertex representation in F_n which is different. For the detail, we can see in Table 6. S is also a dominating set because the vertex in S dominates the vertex which is not in S, so that S is resolving dominating set. Therefore, it can be concluded that $\gamma_r(F_n) \leq \lceil \frac{n}{3} \rceil + 1$.

Furthermore, we will prove the lower bound of resolving domination number F_n which is $\gamma_r(F_n) \geq \lceil \frac{n}{3} \rceil + 1$. Assume that resolving domination number of $\gamma_r(F_n) < \lceil \frac{n}{3} \rceil + 1$. Take $|W| = \lceil \frac{n}{3} \rceil$. The fan graph has $n+1$ vertex. Because we assume $\lceil \frac{n}{3} \rceil$ vertex are in S so that there are several placement conditions of graph vertex F_n as follows:

(i) If $\lceil \frac{n}{3} \rceil$ vertex are located in x_i vertex, there will be one point in x_i which is not dominated by $\lceil \frac{n}{3} \rceil$ vertex in S, because the vertex in the x_i is not adjacent. Thus, the set of S is not dominating set.

(ii) If $\lceil \frac{n}{3} \rceil$ vertex are located in x_i vertex and 1 point in center vertex, there will be two in x_i not S element so that both points have the same representation because 2 points in x_i have the same distance to several points in F_n. Thus, the set of S is not resolving set.

Based on (i) and (ii) cases S is not resolving dominating set. Thus, the cardinality of resolving dominating set of F_n is $\lceil \frac{n}{3} \rceil + 1$. The lower bound of resolving set number of F_n is $\gamma_r(F_n) \geq \lceil \frac{n}{3} \rceil + 1$. Gives the result, we must have $\lceil \frac{n}{3} \rceil + 1$ points dominating in $x_i \in F_n$. Because we have proven that $\gamma_r(F_n) \leq \lceil \frac{n}{3} \rceil + 1$ and $\gamma_r(F_n) \geq \lceil \frac{n}{3} \rceil + 1$, it can be concluded that $\gamma_r(F_n) = \lceil \frac{n}{3} \rceil + 1$. For an example, resolving domination number of fan graph F_n can be seen in Figure 6.

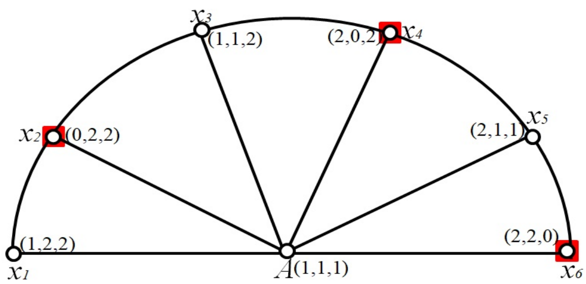

Figure 6. $\gamma_r(F_6) = 3$

Theorem 7. *Let $L(F_n)$ be a line fan graph with $n \geq 4$. The resolving domination number of $L(F_n)$ is $\gamma_r(L(F_n)) = \lceil \frac{n}{3} \rceil$.*

Proof. The line fan graph is a graph which is connected to the vertex of $V(L(F_n)) = \{x_i; 1 \leq i \leq n\} \cup \{y_j; 1 \leq j \leq n+1\}$ and the edge of $E(L(F_n)) = \{x_i x_{i+1}; 1 \leq i \leq n-1\} \cup \{x_i y_j, x_i y_{j+1}, y_j y_{j+1}; 1 \leq i, j \leq n\} \cup \{y_j y_{j+k}; 1 \leq j \leq n-1, 2 \leq k \leq n-i\}$. Vertex cardinality of $|V(L(F_n))| = 2n+1$ and edge cardinality of $|E(L(F_n))| = \frac{n^2+7n-2}{2}$. Maximum degree is $\Delta(L(F_n)) = n+2$ and $\delta(L(F_n)) = 3$. The proof order of resolving domination number $L(F_n)$ consists of upper bound $\gamma_r(L(F_n)) \leq \lceil \frac{n}{3} \rceil$ and lower bound $\gamma_r(L(F_n)) \geq \lceil \frac{n}{3} \rceil$.

Table 6. Representation of $v \in V(F_n)$ respect to S

v	$r(v\|S)$	Condition
x_1	$(1, \underbrace{2, ..., 2}_{\lfloor \frac{n}{2} \rfloor - 1})$	$n \geq 4$
x_i	$(\underbrace{2, ..., 2}_{i-2}, 0, \underbrace{2, ..., 2}_{\|\lceil \frac{n}{3} \rceil - i + 1\|})$	i even
x_i	$(\underbrace{2, ..., 2}_{i-3}, 1, 1, \underbrace{2, ..., 2}_{\lceil \frac{n}{2} \rceil - i + 1})$	i odd
x_{n-1}	$(\underbrace{2, ..., 2}_{\frac{n}{2} - 2}, 1, 1)$	n even
x_{n-1}	$(\underbrace{2, ..., 2}_{\lceil \frac{n}{2} \rceil - 2}, 0)$	n odd
x_n	$(\underbrace{2, ..., 2}_{\frac{n}{2} - 2}, 0)$	n even
x_n	$(\underbrace{2, ..., 2}_{\lceil \frac{n}{2} \rceil - 2}, 1)$	n odd
A	$(\underbrace{1, ..., 1}_{\lceil \frac{n}{3} \rceil})$	$n \geq 4$

In this section, we show the upper bound and the lower bound of resolving domination number $L(F_n)$. First, we will prove the upper bound of resolving domination number $L(F_n)$ which is $\gamma_r(L(F_n)) \leq \lceil \frac{n}{3} \rceil$. We choose $S = \{y_j; 1 \leq i \leq n\}$ so that we have vertex representation in $L(F_n)$ which is different. For the detail, we can see in Table 7. S is also a dominating set because the vertex in S dominates the vertex which is not in S, so that S is resolving dominating set. Therefore, it can be concluded that $\gamma_r(L(F_n)) \leq \lceil \frac{n}{3} \rceil$.

Furthermore, we will prove the lower bound of resolving domination number $L(F_n)$ which is $\gamma_r(L(F_n)) \geq \lceil \frac{n}{3} \rceil$. Assume that resolving domination number of $\gamma_r(L(F_n)) < \lceil \frac{n}{3} \rceil$. Take $|W| = \lceil \frac{n}{3} \rceil - 1$. The line fan graph has $2n + 1$ vertex. Because we assume $\lceil \frac{n}{3} \rceil - 1$ vertex are in S so that there are several placement conditions of graph vertex $L(F_n)$ as follows:

(i) If $\lceil \frac{n}{3} \rceil - 1$ vertex are located in y_j vertex, there will be one point in x_i which is not dominated by $\lceil \frac{n}{3} \rceil - 1$ vertex in S, because the vertex in the x_i is not adjacent. Thus, the set of S is not dominating set.

(ii) If $\lceil \frac{n}{3} \rceil - 1$ vertex are located in y_j vertex, there will be two in y_j not S element so that both points have the same representation because 2 points in y_j have the same distance to several points in $L(F_n)$. Thus, the set of S is not resolving set.

Based on (i) and (ii) cases S is not resolving dominating set. Thus, the cardinality of resolving dominating set of $L(F_n)$ is $\lceil \frac{n}{3} \rceil$. The lower bound of resolving set number of $L(F_n)$ is $\gamma_r(F_n) \geq \lceil \frac{n}{3} \rceil$. Gives the result, we must have $\lceil \frac{n}{3} \rceil$ points dominating in $y_i \in L(F_n)$. Because we have proven that $\gamma_r(L(F_n)) \leq \lceil \frac{n}{3} \rceil$ and $\gamma_r(L(F_n)) \geq \lceil \frac{n}{3} \rceil$, it can be concluded that $\gamma_r(L(F_n)) = \lceil \frac{n}{3} \rceil$. For an example, resolving domination number of line fan graph $L(F_n)$ can be seen in Figure 7.

Theorem 8. *Let $M(F_n)$ be a middle fan graph with $n \geq 4$. The resolving domination number of $M(F_n)$ is $\gamma_r(M(F_n)) = \lceil \frac{n}{3} \rceil$.*

Proof. The middle fan graph is a graph which is connected to the vertex of $V(M(F_n)) = \{A\} \cup \{x_i, y_j; 1 \leq i, j \leq n\} \cup \{z_l; 1 \leq l \leq n-1\}$ and the edge of $E(M(F_n)) = \{Ax_i, x_iy_j; 1 \leq$

Table 7. Representation of $v \in V(L(F_n))$ respect to S

v	$r(v\|S)$	Condition
x_1	$(1,\ 1,\ \underbrace{2, ..., 2}_{n-2})$	
x_i	$(\underbrace{2, ..., 2}_{i-1},\ 1,\ 1,\ \underbrace{2, ..., 2}_{n-i-1})$	$i \geq 2$
x_n	$(\underbrace{2, ..., 2}_{n-1},\ 1)$	$n \geq 4$
y_1	$(0,\ \underbrace{1, ..., 1}_{n-1})$	
y_j	$(\underbrace{1, ..., 1}_{j-1},\ 0,\ \underbrace{1, ..., 1}_{n-j})$	$j \geq 2$
y_n	$(\underbrace{1, ..., 1}_{n-1},\ 0)$	$n \geq 4$
y_{n+1}	$(\underbrace{1, ..., 1}_{n})$	$n \geq 4$

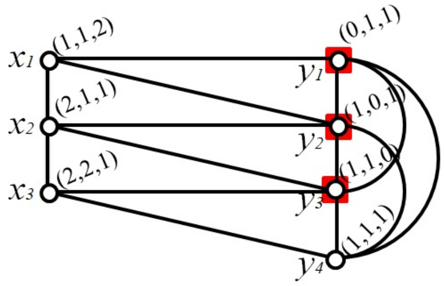

Figure 7. $\gamma_r(L(F_4)) = 3$

$i, j \leq n\} \cup \{x_i x_{i+1}, x_i z_l, x_{i+1} z_j, y_j z_l, z_l y_{j+1}; 1 \leq i, j, l \leq n-1\} \cup \{z_l z_{l+1}, x_i x_{i+2}; 1 \leq i, l \leq n-2\}$. Vertex cardinality of $|V(M(F_n))| = 3n$ and edge cardinality of $|E(M(F_n))| = 9n - 9$. Maximum degree is

$$\Delta(M(F_n)) = \begin{cases} 7; & \text{for } n \leq 7 \\ n; & \text{for } n > 7 \end{cases}$$

and $\delta(M(F_n)) = 2$. The proof order of resolving domination number $M(F_n)$ consists of upper bound $\gamma_r(M(F_n)) \leq \lceil \frac{n}{3} \rceil$ and lower bound $\gamma_r(M(F_n)) \geq \lceil \frac{n}{3} \rceil$.

In this section, we show the upper bound and the lower bound of resolving domination number $M(F_n)$. First, we will prove the upper bound of resolving domination number $M(F_n)$ which is $\gamma_r(M(F_n)) \leq \lceil \frac{n}{3} \rceil$. We choose $S = \{x_i; 1 \leq i \leq n\}$ so that we have vertex representation in $M(F_n)$ which is different. For the detail, we can see in Table 8. S is also a dominating set because the vertex in S dominates the vertex which is not in S, so that S is resolving dominating set. Therefore, it can be concluded that $\gamma_r(M(F_n)) \leq \lceil \frac{n}{3} \rceil$.

Furthermore, we will prove the lower bound of resolving domination number $M(F_n)$ which is $\gamma_r(M(F_n)) \geq \lceil \frac{n}{3} \rceil$. Assume that resolving domination number of $\gamma_r(M(F_n)) < \lceil \frac{n}{3} \rceil$. Take $|W| = \lceil \frac{n}{3} \rceil - 1$. The middle fan graph has $3n$ vertex. Because we assume $\lceil \frac{n}{3} \rceil - 1$ vertex are in S so that there are several placement conditions of graph vertex $M(F_n)$ as follows:

(i) If $\lceil \frac{n}{3} \rceil - 1$ vertex are located in x_i vertex, there will be one point in y_j which is not dominated by $\lceil \frac{n}{3} \rceil - 1$ vertex in S, because the vertex in the y_j is not adjacent. Thus, the set of S is not dominating set.

(ii) If $\lceil \frac{n}{3} \rceil - 2$ vertex are located in x_i vertex, there will be two in x_i not S element so that both points have the same representation because 2 points in x_i have the same distance to several points in $M(F_n)$. Thus, the set of S is not resolving set.

Based on (i) and (ii) cases S is not resolving dominating set. Thus, the cardinality of resolving dominating set of $M(F_n)$ is $\lceil \frac{n}{3} \rceil$. The lower bound of resolving set number of $M(F_n)$ is $\gamma_r(M(F_n)) \geq \lceil \frac{n}{3} \rceil$. Gives the result, we must have $\lceil \frac{n}{3} \rceil$ points dominating in $x_i \in M(F_n)$. Because we have proven that $\gamma_r(M(F_n)) \leq \lceil \frac{n}{3} \rceil$ and $\gamma_r(M(F_n)) \geq \lceil \frac{n}{3} \rceil$, it can be concluded that $\gamma_r(M(F_n)) = \lceil \frac{n}{3} \rceil$. For an example, resolving domination number of middle fan graph $M(F_n)$ can be seen in Figure 8.

Table 8. Representation of $v \in V(M(F_n))$ respect to S

v	$r(v\|S)$	Condition
x_1	$(0, \underbrace{1, ..., 1}_{n-1})$	
x_i	$(\underbrace{1, ..., 1}_{i-1}, 0, \underbrace{1, ..., 1}_{n-i})$	$i \geq 2$
x_n	$(\underbrace{1, ..., 1}_{n-1}, 0)$	
y_1	$(1, \underbrace{2, ..., 2}_{n-1})$	
y_j	$(\underbrace{2, ..., 2}_{j-1}, 1, \underbrace{2, ..., 2}_{n-j})$	$j \geq 2$
y_n	$(\underbrace{2, ..., 2}_{n-1}, 1)$	
z_1	$(1, 1, \underbrace{2, ..., 2}_{n-2})$	
z_l	$(\underbrace{2, ..., 2}_{l-1}, 1, 1, \underbrace{2, ..., 2}_{n-l-1})$	$l \geq 2$
z_n	$(\underbrace{2, ..., 2}_{n-2}, 1, 1)$	
A	$(\underbrace{1, ..., 1}_{n})$	$n \geq 4$

Theorem 9. *Let $T(F_n)$ be a total fan graph with $n \geq 4$. The resolving domination number of $T(F_n)$ is $\gamma_r(T(F_n)) = \lceil \frac{n}{3} \rceil$.*

Proof. The total fan graph is a graph which is connected to the vertex of $V(T(F_n)) = \{A\} \cup \{x_i, y_j; 1 \leq i, j \leq n\} \cup \{z_l; 1 \leq l \leq n-1\}$ and the edge of $E(T(F_n)) = \{Ax_i, x_iy_j; 1 \leq i, j \leq n\} \cup \{x_ix_{i+2}, z_lz_{l+1}; 1 \leq i, l \leq n-2\} \cup \{x_ix_{i+1}, z_ly_{j+1}, y_jy_{j+1}, y_jz_l, x_iz_l, x_{i+1}z_l; 1 \leq i, j, l \leq n-1\} \cup \{Ay_j, Ay_n\}$. Vertex cardinality of $|V(T(F_n))| = 3n$ and edge cardinality of $|E(T(F_n))| = 10n - 8$. Maximum degree is $\Delta(T(F_n)) = n + 2$ and $\delta(T(F_n)) = 4$. The proof order of resolving domination number $T(F_n)$ consists of upper bound $\gamma_r(T(F_n)) \leq \lceil \frac{n}{3} \rceil$ and lower bound $\gamma_r(T(F_n)) \geq \lceil \frac{n}{3} \rceil$.

In this section, we show the upper bound and the lower bound of resolving domination number $T(F_n)$. First, we will prove the upper bound of resolving domination number $T(F_n)$ which is $\gamma_r(T(F_n)) \leq \lceil \frac{n}{3} \rceil$. We choose $S = \{x_i; 1 \leq i \leq n\}$ so that we have vertex representation in $T(F_n)$ which is different. For the detail, we can see in Table 9. S is also a dominating set because

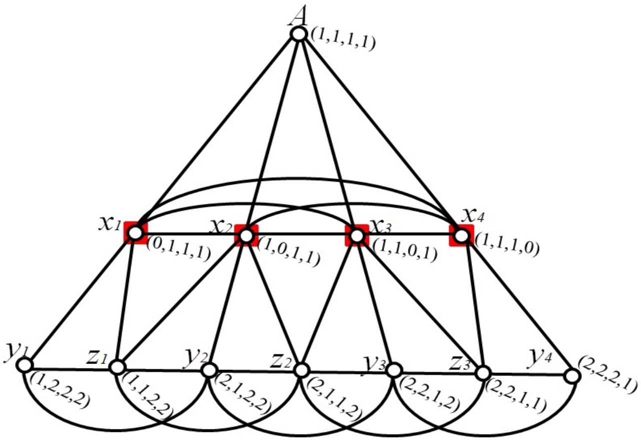

Figure 8. $\gamma_r(M(F_4)) = 4$

the vertex in S dominates the vertex which is not in S, so that S is resolving dominating set. Therefore, it can be concluded that $\gamma_r(T(F_n)) \leq \lceil \frac{n}{3} \rceil$.

Furthermore, we will prove the lower bound of resolving domination number $T(F_n)$ which is $\gamma_r(T(F_n)) \geq \lceil \frac{n}{3} \rceil$. Assume that resolving domination number of $\gamma_r(T(F_n)) < \lceil \frac{n}{3} \rceil$. Take $|W| = \lceil \frac{n}{3} \rceil - 1$. The total fan graph has $3n$ vertex. Because we assume $\lceil \frac{n}{3} \rceil - 1$ vertex are in S so that there are several placement conditions of graph vertex $T(F_n)$ as follows:

(i) If $\lceil \frac{n}{3} \rceil - 1$ vertex are located in x_i vertex, there will be one point in y_j which is not dominated by $\lceil \frac{n}{3} \rceil - 1$ vertex in S, because the vertex in the y_j is not adjacent. Thus, the set of S is not dominating set.

(ii) If $\lceil \frac{n}{3} \rceil - 2$ vertex are located in x_i vertex, there will be two in x_i not S element so that both points have the same representation because 2 points in x_i have the same distance to several points in $T(F_n)$. Thus, the set of S is not resolving set.

Based on (i) and (ii) cases S is not resolving dominating set. Thus, the cardinality of resolving dominating set of $T(F_n)$ is $\lceil \frac{n}{3} \rceil$. The lower bound of resolving set number of $T(F_n)$ is $\gamma_r(T(F_n)) \geq \lceil \frac{n}{3} \rceil$. Gives the result, we must have $\lceil \frac{n}{3} \rceil$ points dominating in $x_i \in M(F_n)$. Because we have proven that $\gamma_r(T(F_n)) \leq \lceil \frac{n}{3} \rceil$ and $\gamma_r(T(F_n)) \geq \lceil \frac{n}{3} \rceil$, it can be concluded that $\gamma_r(T(F_n)) = \lceil \frac{n}{3} \rceil$. For an example, resolving domination number of total fan graph $T(F_n)$ can be seen in Figure 9.

Theorem 10. *Let $C(F_n)$ be a central fan graph with $n \geq 4$. The resolving domination number of $C(F_n)$ is $\gamma_r(C(F_n)) = \lceil \frac{n}{3} \rceil$.*

Proof. The central fan graph is a graph which is connected to the vertex of $V(C(F_n)) = \{A\} \cup \{x_i, y_j; 1 \leq i, j \leq n\} \cup \{z_l; 1 \leq l \leq n-1\}$ and the edge of $E(C(F_n)) = \{Ax_i, x_iy_j; 1 \leq i, j \leq n\} \cup \{y_jz_l, z_ly_{j+1}; 1 \leq j, l \leq n-1\} \cup \{y_jy_{j+k} : 1 \leq j \leq n-2, 2 \leq k \leq n-i\}$. Vertex cardinality of $|V(T(F_n))| = 3n$ and edge cardinality of $|E(C(F_n))| = \frac{n^2+5n-2}{2}$. Maximum degree is $\Delta(C(F_n)) = n$ and $\delta(C(F_n)) = 2$. The proof order of resolving domination number $C(F_n)$ consists of upper bound $\gamma_r(C(F_n)) \leq \lceil \frac{n}{3} \rceil$ and lower bound $\gamma_r(C(F_n)) \geq \lceil \frac{n}{3} \rceil$.

In this section, we show the upper bound and the lower bound of resolving domination number $C(F_n)$. First, we will prove the upper bound of resolving domination number $C(F_n)$ which is $\gamma_r(C(F_n)) \leq \lceil \frac{n}{3} \rceil$. We choose $S = \{y_j; 1 \leq i \leq n\} \cup \{A\}$ so that we have vertex representation in $T(F_n)$ which is different. For the detail, we can see in Table 10. S is also a dominating set because the vertex in S dominates the vertex which is not in S, so that S is resolving dominating set. Therefore, it can be concluded that $\gamma_r(C(F_n)) \leq \lceil \frac{n}{3} \rceil$.

Table 9. Representation of $v \in V(T(F_n))$ respect to S

v	$r(v\|S)$	Condition
x_1	$(0, \underbrace{1, ..., 1}_{n-1})$	$n \geq 4$
x_i	$(\underbrace{1, ..., 1}_{i-1}, 0, \underbrace{1, ..., 1}_{n-i})$	$i \geq 2$
x_n	$(\underbrace{1, ..., 1}_{n-1}, 0)$	
y_1	$(1, \underbrace{2, ..., 2}_{n-1})$	$n \geq 4$
y_j	$(\underbrace{2, ..., 2}_{j-1}, 1, \underbrace{2, ..., 2}_{n-j})$	$j \geq 2$
y_n	$(\underbrace{2, ..., 2}_{n-1}, 1)$	
z_1	$(1, 1, \underbrace{2, ..., 2}_{n-2})$	
z_l	$(\underbrace{2, ..., 2}_{l-1}, 1, 1, \underbrace{2, ..., 2}_{n-l-1})$	$l \geq 2$
z_n	$(\underbrace{2, ..., 2}_{n-2}, 1, 1)$	
A	$(\underbrace{1, ..., 1}_{n})$	$n \geq 4$

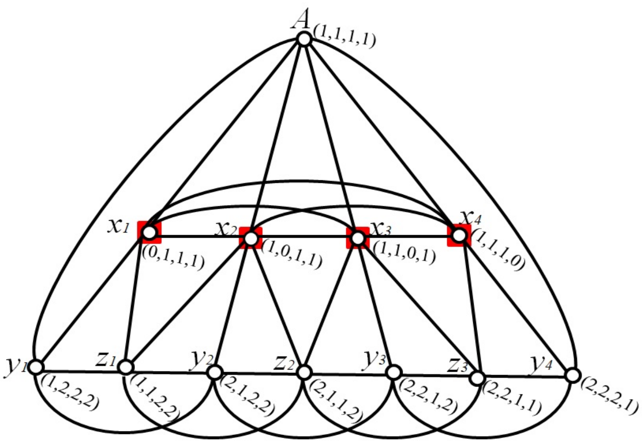

Figure 9. $\gamma_r(T(F_4)) = 4$

Furthermore, we will prove the lower bound of resolving domination number $C(F_n)$ which is $\gamma_r(C(F_n)) \geq \lceil \frac{n}{3} \rceil$. Assume that resolving domination number of $\gamma_r(C(F_n)) < \lceil \frac{n}{3} \rceil$. Take $|W| = \lceil \frac{n}{3} \rceil - 1$. The central fan graph has $3n$ vertex. Because we assume $\lceil \frac{n}{3} \rceil - 1$ vertex are in S so that there are several placement conditions of graph vertex $TC(F_n)$ as follows:

(i) If $\lceil \frac{n}{3} \rceil - 1$ vertex are located in y_j vertex, there will be one point in x_i which is not dominated by $\lceil \frac{n}{3} \rceil - 1$ vertex in S, because the vertex in the x_i is not adjacent. Thus, the set of S is not dominating set.

(ii) If $\lceil \frac{n}{3} \rceil - 2$ vertex are located in y_j vertex, there will be two in y_j not S element so that both points have the same representation because 2 points in y_j have the same distance to several points in $C(F_n)$. Thus, the set of S is not resolving set.

Based on (i) and (ii) cases S is not resolving dominating set. Thus, the cardinality of resolving dominating set of $C(F_n)$ is $\lceil \frac{n}{3} \rceil$. The lower bound of resolving set number of $C(F_n)$ is $\gamma_r(C(F_n)) \geq \lceil \frac{n}{3} \rceil$. Gives the result, we must have $\lceil \frac{n}{3} \rceil$ points dominating in $x_i \in M(F_n)$. Because we have proven that $\gamma_r(C(F_n)) \leq \lceil \frac{n}{3} \rceil$ and $\gamma_r(C(F_n)) \geq \lceil \frac{n}{3} \rceil$, it can be concluded that $\gamma_r(C(F_n)) = \lceil \frac{n}{3} \rceil$. For an example, resolving domination number of central fan graph $C(F_n)$ can be seen in Figure 10.

Table 10. Representation of $v \in V(C(F_n))$ respect to S

v	$r(v\|S)$	Condition
x_1	$(0, \underbrace{1, ..., 1}_{n-1})$	$n \geq 4$
x_i	$(\underbrace{1, ..., 1}_{i-1}, 0, \underbrace{1, ..., 1}_{n-i})$	$i \geq 2$
x_n	$(\underbrace{1, ..., 1}_{n-1}, 0)$	
y_1	$(1, \underbrace{2, ..., 2}_{n-1})$	
y_j	$(\underbrace{2, ..., 2}_{j-1}, 1, \underbrace{2, ..., 2}_{n-j})$	$j \geq 2$
y_n	$(\underbrace{2, ..., 2}_{n-1}, 1)$	
z_1	$(1, 1, \underbrace{2, ..., 2}_{n-2})$	
z_l	$(\underbrace{2, ..., 2}_{l-1}, 1, 1, \underbrace{2, ..., 2}_{n-l-1})$	$l \geq 2$
z_n	$(\underbrace{2, ..., 2}_{n-2}, 1, 1)$	
A	$(0, \underbrace{2, ..., 2}_{n})$	$n \geq 4$

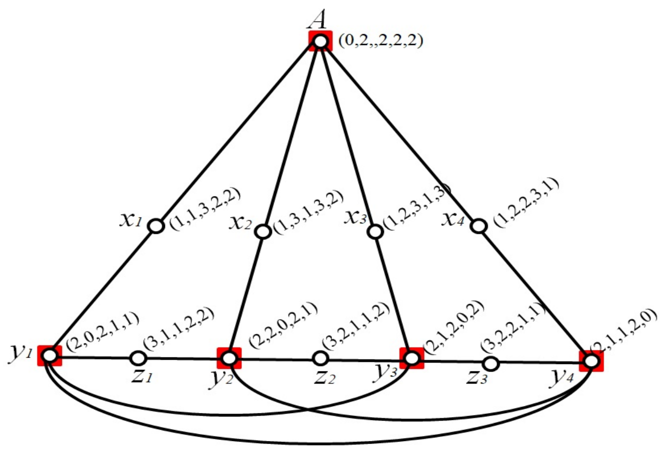

Figure 10. $\gamma_r(C(F_4)) = 5$

3. Conclusion

In this research, we have analyze of resolving domination number of star graph (S_n) and fan graph (F_n). Based on research $\gamma_r(S_n) = n$, $\gamma_r(L(S_n)) = n - 1$, $\gamma_r(M(S_n)) = n$, $\gamma_r(T(S_n)) = n$, $\gamma_r(C(S_n)) = n$, $\gamma_r(F_n) = \lceil \frac{n}{3} \rceil + 1$, $\gamma_r(L(F_n)) = \lceil \frac{n}{3} \rceil$, $\gamma_r(M(F_n)) = \lceil \frac{n}{3} \rceil$, $\gamma_r(T(F_n)) = \lceil \frac{n}{3} \rceil$, and $\gamma_r(C(F_n)) = \lceil \frac{n}{3} \rceil$. The open problem of this research is:

Open Problem 1. *Let G be any connected graph, determine the edge resolving domination number of any graph G.*

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References

[1] Adawiyah R, Agustin I H, Dafik, Slamin and Albirri E R 2018 Related Wheel Graphs and Its Locating Edge Domination Number *Journal of Physics: Conf* **1022**
[2] Adawiyah R, Dafik, Alfarisi R, Prihandini R M and Agustin I H 2019 Edge Metric Dimension on Some Families of Tree *Journal of Physics: Conference Series* **1180** 012005
[3] Albirri E R, Dafik, Agustin I H, Adawiyah R, Alfarisi R and Prihandini R M 2019 The local (adjacency) metric dimension of split related complete graph *Journal of Physics: Conference Series* **1211** 012015
[4] Blumenthal L M 1953 *Theory and Applications of Distance Geometry* (Clarendon: Clarendon Press)
[5] Brigham R C, Chartrand G, Dutton R D and Zhang P 2003 Resolving Domination in Graphs *Mathematica Bohemica* **128** 25-36
[6] Capobianco M and Molluzzo J 1978 *Examples and Counterexamples in Graph Theory* (New York: North-Holland)
[7] Chartrand G, Eroh L, Jhonson M A and Oellermann O R 2000 Resolvability in graphs and the metric dimenson of a graph *Disrete Applied Mathematics* **105** 99-113
[8] Dafik, Agustin I H, Hasan M, Adawiyah R, Alfarisi R and Wardani D A R 2018 On the Locating Edge Domination Number of Comb Product of Graphs *Journal of Physics: Conf. Series* **1022** 012003
[9] Gonzalez A, Hernando C and Mora M 2018 Metric-locating-dominating sets of graphs for constructing related subsets of vertices *Applied Mathematics and Computation* **332** 449-456
[10] Gross J T and Yellen J 2006 *Graph Theory and Its Applications* (Boca Raton, FL: CRC Press)
[11] Harary F and Melter R A 1976 On the metric dimension of a graph *Ars Combinatoria* **2** 191195
[12] Haynes T W 1998 *Fundamental of Dominating in Graphs* (New York: Marcel Dekker, INC)
[13] Haynes T W and Henning M A 2002 Total Domination Good vertices in Graphs *Australasian Journal of Combinatorics* **26** 305-315
[14] Henning M A and Ocllermann O R 2004 Metric-Locating-Dominating Sets in Graphs *Ars Combin* **73** 129-141
[15] Kavitha K and David N G 2012 Dominator Coloring of Central Graphs *International Journal of Computer Applications(0975-8887)* **51(12)** 11-14
[16] Lipschutz S and Lipson M 2007 *Scahaum's Outlines Discrete Mathematics*,Third Edition (New York: The McGraw-Hill Companies)
[17] Naji A M and Soner N D 2015 Resolving Connected Domination in Graphs *International J.Math. Combin* **4** 129-136
[18] Slater P J 1975 Leaves of trees *Proc. 6th Southeastern Conference on Combinatorics, Graph Theory, and Computing* **14** 549-559
[19] Thenmozhi B and Prabha R 2017 Power Dominating of Middle Graph of Path, Cycle and Star *International Journal of Pure and Applied Mathematics* **114(5)** 13-19
[20] Wardani D A R, Dafik, Agustin I H and Kurniawati E Y 2019 The relation of $\gamma(G \bigodot H)$ and $\gamma(G \diamond H)$ *Journal of Physics: Conference Series* **1211**