On r-dynamic vertex coloring of line, middle, total of lobster graph

L O Harjito[1,2], Dafik[1,2], A I Kristiana[1,3], R Alfarisi[1,4], R M Prihandini[1,3]

[1]CGANT University of Jember, Indonesia
[2]Department of Mathematics Education Post Graduate University of Jember, Indonesia
[3]Department of Mathematics Education University of Jember, Indonesia
[4]Department of Primary Education University of Jember, Indonesia

E-mail: lelitaoktafianti@gmail.com; d.dafik@unej.ac.id

Abstract. Let G be a simple, connected and undirected graph that has a set of vertex and edge. The degree of $v \in V(G)$ is denoted by $d(v)$. The maximum and minimum degree of G respectively are $\Delta(G)$ and $\delta(G)$. The r-dynamic color of the graph G is calculated as a map c from V to a color set such that if $u, v \in V(G)$ is adjacent, then $c(u) \neq c(v)$, and for each $v \in V(G), |c(N(v))| \geq min\{r, d(v)\}$. The number of r-dynamic coloring of G denoted by $\chi_r(G)$ is minimum color k in G. In this paper, we have obtained the r-dynamic vertex coloring of line, middle, total of lobster graph $\mathcal{L}_n(2,1)$.

1. Introduction

Suppose $G = (V, E)$ is a simple graph. The vertex and edge set G are given as $V(G)$ and $E(G)$, respectively. The maximum degree denoted by $\Delta(G)$ and the minimum degree denoted by $\delta(G)$. For each $v \in V(G)$, $d(v)$ represents the degree of v, $N(v)$ represents the relative set of V, and $c(v)$ represents the color of v. The vertex coloring of graph G by k color is a surjective fuction of $c : V(G) \to \{1, 2, ..., k\}$ with this character: if $u, v \in V(G)$ and $u, v \in E(G)$, then $c(u)$ and $c(v)$ are different. The r-dynamic coloring of graph G, introduced by Montgomery [13] exactly k implementation G coloring of graph for each of v expected only $\min\{r, d(v)\}$ different color. The chromatic number of r-dynamic, $\chi_r(G)$ is the minimum k so the graph G has r-dynamic k colors. The following observation is useful for our study, proposed by Montgomery [13]:

Observation 1. *Let $\Delta(G)$ be the maximum degree of G. It holds $\chi_r(G) \geq min\{\Delta(G), r\} + 1$.*

Montgomery [13] explains the r-dynamic color of the graph G is calculated as a map c from V to a color set such that if $u, v \in V(G)$ is adjacent, then $c(u) \neq c(v)$, and for each $v \in V(G), |c(N(v))| \geq min\{r, d(v)\}$. The lobster graph is a caterpillar graph without pendant vertices from tree graph, it is denoted by $\mathcal{L}_n(l, m)$ [12].

A line graph of G denoted by $L(G)$ is obtained by associating vertices with each edge of G and connecting two vertices with edges if the corresponding edges of G have the same node [6]. In [16] the middle graph denoted by $M(G)$ of the connected graph G is a graph whose node-set is $V(G) \cup E(G)$ where two vertices are close together if they are edges which border G or one is the node of G and the other is an edge incident with it. The total graph denoted by $T(G)$ of the connected graph G is a graph whose node-set is $V(G) \cup E(G)$ and two adjacent vertices each time that border or events in G [4].

Montgomery [13] is uncovered the lower bound of the r-dynamic chromatic number, $\chi_r(G) \geq min\{r, \Delta(G)\}+1$. Kang et.al [8] are discovered the r-dynamic chromatic number of the m-by-n grid for all r, m, n. Kristiana et.al [9] are uncovered the lower bound of the r-dynamic chromatic number of the coronation of path as well as the several graphs. The r-dynamic chromatic number has been inquired by several authors, it can be seen in [1, 2, 3, 5, 7, 10, 11, 14, 15].

An important application of r-dynamic vertex coloring is coloring the map. The coloring of map is not arbitrary, the type of color is used must be minimal. In this study, the r-dynamic vertex coloring will be examined on lobster graphs. In addition, the development of the graph will become a new graphs (line graph, middle graph, and total graph).

2. Result

In this research, we have obtained the exact value of r-dynamic vertex coloring of line, middle, and total of lobster graphs $\mathcal{L}_n(2,1)$.

Theorem 1. *Let $\mathcal{L}_n(2,1)$ be a lobster graph for $n \geq 2$, the r-dynamic chromatic number of $\mathcal{L}_n(2,1)$ is*

$$\chi_r(\mathcal{L}_n(2,1)) = \begin{cases} 2, & \text{r} = 1 \\ r+1, & 2 \leq r \leq 3 \\ 5, & \text{r} \geq 4 \end{cases}$$

Proof. The vertex set of $\mathcal{L}_n(2,1)$ is $V(\mathcal{L}_n(2,1)) = \{a_s; 1 \leq s \leq n\} \cup \{x_s; 1 \leq s \leq n\} \cup \{x_{s,1}; 1 \leq s \leq n\} \cup \{y_s; 1 \leq s \leq n\} \cup \{y_{s,1}; 1 \leq s \leq n\}$ and the edge set of $\mathcal{L}_n(2,1)$ is $E(\mathcal{L}_n(2,1)) = \{x_s x_{s,1}; 1 \leq s \leq n\} \cup \{x_s a_s; 1 \leq s \leq n\} \cup \{a_s y_s; 1 \leq s \leq n\} \cup \{y_s y_{s,1}; 1 \leq s \leq n\} \cup \{a_s a_{s+1}; 1 \leq s \leq n-1\}$. The vertex and edge cardinality of $\mathcal{L}_n(2,1)$ are $|V(\mathcal{L}_n(2,1))| = 5n$, $|E(\mathcal{L}_n(2,1))| = 5n-1$ respectively. We define three cases, namely for $r = 1$, $2 \leq r \leq 3$, and $r \geq 4$.

Figure 1 is the illustration of r-dynamic vertex coloring of $\mathcal{L}_n(2,1)$

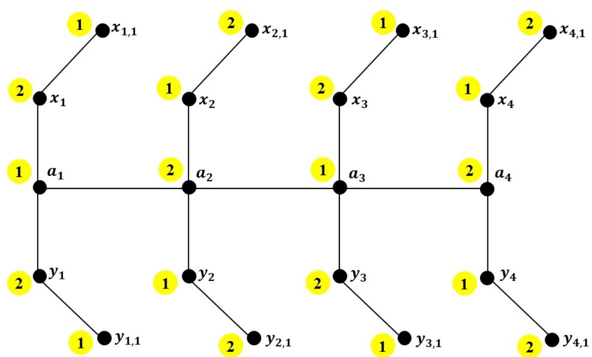

Figure 1. The r-dynamic vertex coloring of $\mathcal{L}_n(2,1)$ for $r = 1$

Case 1: For $r = 1$

Based on Observation 1, since $\chi_r(G) \geq min\{r, \Delta\}+1 = min\{1, 4\}+1 = 1+1 = 2$, then the lower bound of $\mathcal{L}_n(2,1)$ is $\chi_r(G) \geq 2$. We need to prove the upper bound of r-dynamic vertex coloring of $\mathcal{L}_n(2,1)$ is $\chi_r(\mathcal{L}_n(2,1)) \leq 2$. Define the function $c : V(G) \to \{1, 2, ..., k\}$ as follows:

$$c(a_s) = \begin{cases} 1; & s \equiv 1 \ mod \ 2 \\ 2; & s \equiv 0 \ mod \ 2 \end{cases}$$

$$c(x_s) = \begin{cases} 1; & s \equiv 0 \ mod \ 2 \\ 2; & s \equiv 1 \ mod \ 2 \end{cases}$$

$$c(x_{s,1}) = \begin{cases} 1; & s \equiv 1 \ mod \ 2 \\ 2; & s \equiv 0 \ mod \ 2 \end{cases}$$

$$c(y_s) = \begin{cases} 1; & s \equiv 0 \ mod \ 2 \\ 2; & s \equiv 1 \ mod \ 2 \end{cases}$$

$$c(y_{s,1}) = \begin{cases} 1; & s \equiv 1 \ mod \ 2 \\ 2; & s \equiv 0 \ mod \ 2 \end{cases}$$

It is clear that $\chi_r(\mathcal{L}_n(2,1)) \leq 2$. We can conclude that $\chi_r(\mathcal{L}_n(2,1)) = 2$ for $r = 1$.

Case 2: For $2 \leq r \leq 3$
Based on Observation 1, since $\chi_r(G) \geq min\{r, \Delta\} + 1 = min\{r, 4\} + 1 = r + 1$, then the lower bound of $\mathcal{L}_n(2,1)$ is $\chi_r(G) \geq r + 1$. We need to prove the upper bound of r-dynamic vertex coloring of $\mathcal{L}_n(2,1)$ is $\chi_r(\mathcal{L}_n(2,1)) \leq r + 1$. Define the function $c : V(G) \to \{1, 2, ..., k\}$ as follows:

$$c(a_s) = \begin{cases} 1; & s \equiv 1 \ mod \ 2; \ 2 \leq r \leq 3 \\ 2; & s \equiv 0 \ mod \ 2; \ 2 \leq r \leq 3 \end{cases}$$

$$c(x_s) = \begin{cases} 1; & s \equiv 0 \ mod \ 2; \ r = 2 \\ 2; & s \equiv 1 \ mod \ 2; \ r = 2 \\ 4; & 1 \leq s \leq n; r = 3 \end{cases}$$

$$c(x_{s,1}) = \begin{cases} 3; & 1 \leq s \leq n; 2 \leq r \leq 3 \end{cases}$$

$$c(y_s) = \begin{cases} 3; & 1 \leq s \leq n; 2 \leq r \leq 3 \end{cases}$$

$$c(y_{s,1}) = \begin{cases} 1; & s \equiv 0 \ mod \ 2; \ r = 2 \\ 2; & s \equiv 1 \ mod \ 2; \ r = 2 \\ 4; & 1 \leq s \leq n; r = 3 \end{cases}$$

It is clear that $\chi_r(\mathcal{L}_n(2,1)) \leq r + 1$. We can conclude that $\chi_r(\mathcal{L}_n(2,1)) = r + 1$ for $2 \leq r \leq 3$.

Case 3: For $r \geq 4$
Based on Observation 1, since $\chi_r(G) \geq min\{r, \Delta\} + 1 = min\{r, 4\} + 1 = 4 + 1 = 5$, then the lower bound of $\mathcal{L}_n(2,1)$ is $\chi_r(G) \geq 5$. We need to prove the upper bound of r-dynamic vertex coloring of $\mathcal{L}_n(2,1)$ is $\chi_r(\mathcal{L}_n(2,1)) \leq 5$. Define the function $c : V(G) \to \{1, 2, ..., 5\}$ as follows:

$$c(a_s) = \begin{cases} 1; & s \equiv 2 \ mod \ 3 \\ 2; & s \equiv 1 \ mod \ 3 \\ 3; & s \equiv 0 \ mod \ 3 \end{cases}$$

$$c(x_s) = 5; \quad 1 \leq s \leq n$$

$$c(x_{s,1}) = 4; \quad 1 \leq s \leq n$$

$$c(y_s) = 4; \quad 1 \leq s \leq n$$

$$c(y_{s,1}) = \begin{cases} 1; & s \equiv 1 \ mod \ 3 \\ 2; & s \equiv 0 \ mod \ 3 \\ 3; & s \equiv 2 \ mod \ 3 \end{cases}$$

It is clear that $\chi_r(\mathcal{L}_n(2,1)) \leq 5$. We can conclude that $\chi_r(\mathcal{L}_n(2,1)) = 5$ for $r \geq 4$.

Theorem 2. *Let $L(\mathcal{L}_n(2,1))$ be a line graph of lobster graph for $n \geq 5$, the r-dynamic chromatic number of $L(\mathcal{L}_n(2,1))$ is*

$$\chi_r L(\mathcal{L}_n(2,1)) = \begin{cases} 2, & \text{r} = 1 \\ r+1, & 2 \leq r \leq 3 \\ 5, & \text{r} \geq 4 \end{cases}$$

Proof. The vertex set of $L(\mathcal{L}_n(2,1))$ is $VL(\mathcal{L}_n(2,1)) = \{a_s; 1 \leq s \leq n-1\} \cup \{x_s; 1 \leq s \leq n\} \cup \{x_{s,1}; 1 \leq s \leq n\} \cup \{y_s; 1 \leq s \leq n\} \cup \{y_{s,1}; 1 \leq s \leq n\}$ and the edge set of $L(\mathcal{L}_n(2,1))$ is $EL(\mathcal{L}_n(2,1)) = \{x_s x_{s,1}; 1 \leq s \leq n\} \cup \{x_s y_s; 1 \leq s \leq n\} \cup \{x_s a_s; 1 \leq s \leq n-1\} \cup \{x_s a_{s-1}; 2 \leq s \leq n\} \cup \{y_s a_s; 1 \leq s \leq n-1\} \cup \{y_s a_{s-1}; 2 \leq s \leq n\} \cup \{y_s y_{s,1}; 1 \leq s \leq n\}$. The vertex and edge cardinality of $L(\mathcal{L}_n(2,1))$ are $|VL(\mathcal{L}_n(2,1))| = 5n-1$, $|EL(\mathcal{L}_n(2,1))| = 7n-4$ respectively. We define three cases, namely for $r = 1$, $2 \leq r \leq 3$, and $r \geq 4$.

Figure 2 is the illustration of r-dynamic vertex coloring of $L(\mathcal{L}_n(2,1))$

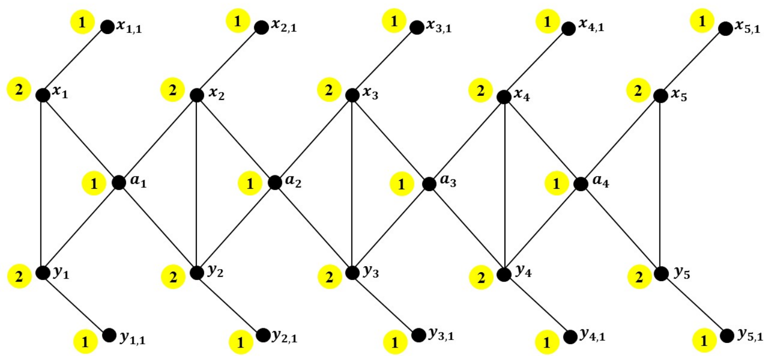

Figure 2. The r-dynamic vertex coloring of $L(\mathcal{L}_n(2,1))$ for $r = 1$

Case 1: For $r = 1$

Based on Observation 1, since $\chi_r(G) \geq min\{r, \Delta\} + 1 = min\{1, 4\} + 1 = 1 + 1 = 2$, then the lower bound of $L(\mathcal{L}_n(2,1))$ is $\chi_r(G) \geq 2$. We need to prove the upper bound of r-dynamic vertex coloring of $L(\mathcal{L}_n(2,1))$ is $\chi_r L(\mathcal{L}_n(2,1)) \leq 2$. Define the function $c : V(G) \to \{1, 2, ..., k\}$ as follows:

$$c(a_s) = 1; \quad 1 \leq s \leq n-1$$

$$c(x_s) = 2; \quad 1 \leq s \leq n$$

$$c(x_{s,1}) = 1; \quad 1 \leq s \leq n$$

$$c(y_s) = 2; \quad 1 \leq s \leq n$$

$$c(y_{s,1}) = 1; \quad 1 \leq s \leq n$$

It is clear that $\chi_r L(\mathcal{L}_n(2,1)) \leq 2$. We can conclude that $\chi_r L(\mathcal{L}_n(2,1)) = 2$ for $r = 1$.

Case 2: For $2 \leq r \leq 3$
Based on Observation 1, since $\chi_r(G) \geq min\{r, \Delta\} + 1 = min\{r, 4\} + 1 = r + 1$, then the lower bound of $L(\mathcal{L}_n(2,1))$ is $\chi_r(G) \geq r + 1$. We need to prove the upper bound of r-dynamic vertex coloring of $L(\mathcal{L}_n(2,1))$ is $\chi_r L(\mathcal{L}_n(2,1)) \leq r + 1$. Define the function $c : V(G) \to \{1, 2, ..., k\}$ as follows:

$$c(a_s) = 2; \quad 1 \leq s \leq n-1; 2 \leq r \leq 3$$

$$c(x_s) = \begin{cases} 1; & 1 \leq s \leq n; r = 2 \\ & s \equiv 0 \ mod\ 2;\ r = 3 \\ 4; & s \equiv 1 \ mod\ 2;\ r = 3 \end{cases}$$

$$c(x_{s,1}) = \begin{cases} 1; & s \equiv 1 \ mod\ 2;\ r = 3 \\ 2; & 1 \leq s \leq n; r = 2 \\ 3; & s \equiv 0 \ mod\ 2;\ r = 3 \end{cases}$$

$$c(y_s) = \begin{cases} 3; & 1 \leq s \leq n; r = 2 \\ & s \equiv 1 \ mod\ 2;\ r = 3 \\ 4; & s \equiv 0 \ mod\ 2;\ r = 3 \end{cases}$$

$$c(y_{s,1}) = \begin{cases} 1; & s \equiv 1 \ mod\ 2;\ r = 3 \\ 2; & 1 \leq s \leq n; r = 2 \\ 3; & s \equiv 0 \ mod\ 2;\ r = 3 \end{cases}$$

It is clear that $\chi_r L(\mathcal{L}_n(2,1)) \leq r+1$. We can conclude that $\chi_r L(\mathcal{L}_n(2,1)) = r+1$ for $2 \leq r \leq 3$.

Case 3: For $r \geq 4$
Based on Observation 1, since $\chi_r(G) \geq min\{r, \Delta\} + 1 = min\{r, 4\} + 1 = 4 + 1 = 5$, then the lower bound of $L(\mathcal{L}_n(2,1))$ is $\chi_r(G) \geq 5$. We need to prove the upper bound of r-dynamic vertex coloring of $L(\mathcal{L}_n(2,1))$ is $\chi_r L(\mathcal{L}_n(2,1)) \leq 5$. Define the function $c : V(G) \to \{1, 2, ..., k\}$ as follows:

$$\mathrm{c}(\mathrm{a}_s) = \begin{cases} 1; & s \equiv 1 \ mod\ 5 \\ 2; & s \equiv 2 \ mod\ 5 \\ 3; & s \equiv 3 \ mod\ 5 \\ 4; & s \equiv 4 \ mod\ 5 \\ 5; & s \equiv 0 \ mod\ 5 \end{cases} \qquad \mathrm{c}(\mathrm{y}_s) = \begin{cases} 1; & s \equiv 3 \ mod\ 5 \\ 2; & s \equiv 4 \ mod\ 5 \\ 3; & s \equiv 0 \ mod\ 5 \\ 4; & s \equiv 1 \ mod\ 5 \\ 5; & s \equiv 2 \ mod\ 5 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_s) = \begin{cases} 1; & s \equiv 0 \ mod\ 5 \\ 2; & s \equiv 1 \ mod\ 5 \\ 3; & s \equiv 2 \ mod\ 5 \\ 4; & s \equiv 3 \ mod\ 5 \\ 5; & s \equiv 4 \ mod\ 5 \end{cases} \qquad \mathrm{c}(\mathrm{y}_{s,1}) = \begin{cases} 1; & s \equiv 4 \ mod\ 5 \\ 2; & s \equiv 0 \ mod\ 5 \\ 3; & s \equiv 1 \ mod\ 5 \\ 4; & s \equiv 2 \ mod\ 5 \\ 5; & s \equiv 3 \ mod\ 5 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_{s,1}) = \begin{cases} 1; & s \equiv 4 \ mod\ 5 \\ 2; & s \equiv 0 \ mod\ 5 \\ 3; & s \equiv 1 \ mod\ 5 \\ 4; & s \equiv 2 \ mod\ 5 \\ 5; & s \equiv 3 \ mod\ 5 \end{cases}$$

It is clear that $\chi_r L(\mathcal{L}_n(2,1)) \leq 5$. We can conclude that $\chi_r L(\mathcal{L}_n(2,1)) = 5$ for $r \geq 4$.

Theorem 3. *Let $M(\mathcal{L}_n(2,1))$ be a middle graph of lobster graph for $n \geq 3$, the r-dynamic chromatic number of $M(\mathcal{L}_n(2,1))$ is*

$$\chi_r M(\mathcal{L}_n(2,1)) = \begin{cases} 3, & 1 \leq r \leq 2 \\ r+1, & 3 \leq r \leq 5 \\ 7, & \text{r} \geq 6 \end{cases}$$

Proof. The vertex set of $M(\mathcal{L}_n(2,1))$ is $VM(\mathcal{L}_n(2,1)) = \{x_{s,t}; 1 \leq s \leq n; 1 \leq t \leq m\} \cup \{y_s; 1 \leq s \leq n-1\}$ and the edge set of $M(\mathcal{L}_n(2,1))$ is $EM(\mathcal{L}_n(2,1)) = \{x_{s,t}x_{s,t+1}; 1 \leq s \leq n\ ; 1 \leq t \leq m-1\} \cup \{x_{s,t}x_{s,t+2}; 1 \leq s \leq n\ ; 2 \leq t \leq \left\lfloor \frac{m}{2} \right\rfloor - 1\} \cup \{x_s, \left\lceil \frac{m}{2} \right\rceil y_s; 1 \leq s \leq n-1\} \cup \{y_s x_{s+1}, \left\lceil \frac{m}{2} \right\rceil; 1 \leq s \leq n-1\} \cup \{x_s, \left\lceil \frac{m}{2} \right\rceil - 1 y_s; 1 \leq s \leq n-1\} \cup \{y_s x_{s+1}, \left\lceil \frac{m}{2} \right\rceil - 1; 1 \leq s \leq n-1\} \cup \{x_s, \left\lceil \frac{m}{2} \right\rceil + 1 y_s; 1 \leq s \leq n-1\} \cup \{y_s x_{1+1}, \left\lceil \frac{m}{2} \right\rceil + 1; 1 \leq s \leq n-1\}$. The vertex and edge cardinality of $M(\mathcal{L}_n(2,1))$ are $|VM(\mathcal{L}_n(2,1))| = mn+n-1$, $|EM(\mathcal{L}_n(2,1))| = mn+n\left\lfloor \frac{m}{2} \right\rfloor + 3n-6$ respectively. We define three cases, namely for $1 \leq r \leq 2$, $3 \leq r \leq 5$, and $r \geq 6$.

Figure 3 is the illustration of r-dynamic vertex coloring of $M(\mathcal{L}_n(2,1))$

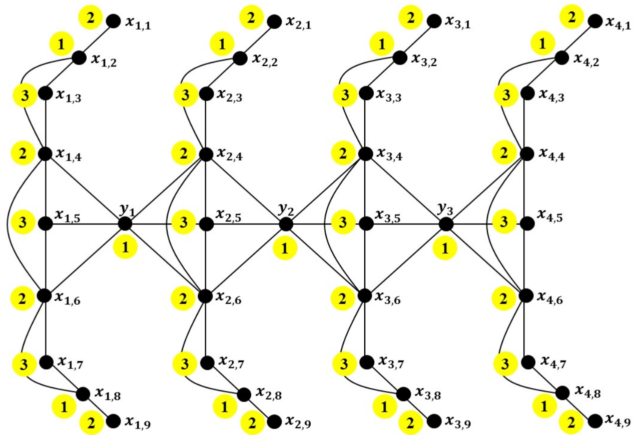

Figure 3. The r-dynamic vertex coloring of $M(\mathcal{L}_n(2,1))$ for $1 \leq r \leq 2$

Case 1: For $1 \leq r \leq 2$

Based on Observation 1, since $\chi_r(G) \geq min\{r, \Delta\} + 1 = min\{2, 6\} + 1 = 2 + 1 = 3$, then the lower bound of $M(\mathcal{L}_n(2,1))$ is $\chi_r(G) \geq 3$. We need to prove the upper bound of r-dynamic vertex coloring of $M(\mathcal{L}_n(2,1))$ is $\chi_r M(\mathcal{L}_n(2,1)) \leq 3$. Define the function $c : V(G) \rightarrow \{1, 2, ..., k\}$ as follows:

$$c(x_{s,1}) = 2; \quad 1 \leq s \leq n$$
$$c(x_{s,2}) = 1; \quad 1 \leq s \leq n$$
$$c(x_{s,3}) = 3; \quad 1 \leq s \leq n$$
$$c(x_{s,4}) = 2; \quad 1 \leq s \leq n$$
$$c(x_{s,5}) = 3; \quad 1 \leq s \leq n$$
$$c(x_{s,6}) = 2; \quad 1 \leq s \leq n$$
$$c(x_{s,7}) = 3; \quad 1 \leq s \leq n$$
$$c(x_{s,8}) = 1; \quad 1 \leq s \leq n$$

$$c(x_{s,9}) = 2; \quad 1 \leq s \leq n$$
$$c(y_s) = 1; \quad 1 \leq s \leq n-1$$

It is clear that $\chi_r M(\mathcal{L}_n(2,1)) \leq 3$. We can conclude that $\chi_r M(\mathcal{L}_n(2,1)) = 3$ for $1 \leq r \leq 2$.

Case 2: For $3 \leq r \leq 5$
Based on Observation 1, since $\chi_r(G) \geq min\{r, \Delta\} + 1 = min\{r, 6\} + 1 = r + 1$, then the lower bound of $M(\mathcal{L}_n(2,1))$ is $\chi_r(G) \geq r+1$. We need to prove the upper bound of r-dynamic vertex coloring of $M(\mathcal{L}_n(2,1))$ is $\chi_r M(\mathcal{L}_n(2,1)) \leq r+1$. Define the function $c : V(G) \rightarrow \{1, 2, ..., k\}$ as follows:

$$\mathrm{c}(\mathrm{x}_{s,1}) = \begin{cases} 1; & 1 \leq s \leq n; 4 \leq r \leq 5 \\ 4; & 1 \leq s \leq n; r = 3 \end{cases} \qquad \mathrm{c}(\mathrm{x}_{s,6}) = \begin{cases} 1; & 1 \leq s \leq n; r = 3 \\ 4; & 1 \leq s \leq n; 4 \leq r \leq 5 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_{s,2}) = \begin{cases} 2; & 1 \leq s \leq n; r = 3 \\ 5; & 1 \leq s \leq n; 4 \leq r \leq 5 \end{cases} \qquad \mathrm{c}(\mathrm{x}_{s,7}) = \begin{cases} 3; & 1 \leq s \leq n; r = 3 \\ 5; & 1 \leq s \leq n; r = 4 \\ 6; & 1 \leq s \leq n; r = 5 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_{s,3}) = \begin{cases} 1; & 1 \leq s \leq n; r = 3 \\ 2; & s \equiv 0 \; mod \; 2; \; r = 5 \\ 4; & 1 \leq s \leq n; r = 4 \\ 6; & s \equiv 1 \; mod \; 2; \; r = 5 \end{cases} \qquad \mathrm{c}(\mathrm{x}_{s,8}) = \begin{cases} 2; & 1 \leq s \leq n; 3 \leq r \leq 4 \\ & s \equiv 0 \; mod \; 2; \; r = 5 \\ 5; & s \equiv 1 \; mod \; 2; \; r = 5 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_{s,4}) = \begin{cases} 2; & 1 \leq s \leq n; r = 4 \\ & s \equiv 1 \; mod \; 2; \; r = 5 \\ 3; & 1 \leq s \leq n; r = 3 \\ 6; & s \equiv 0 \; mod \; 2; \; r = 5 \end{cases} \qquad \mathrm{c}(\mathrm{x}_{s,9}) = \begin{cases} 1; & 1 \leq s \leq n; 4 \leq r \leq 5 \\ 4; & 1 \leq s \leq n; r = 3 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_{s,5}) = \begin{cases} 1; & s \equiv 0 \; mod \; 3; \; 4 \leq r \leq 5 \\ 2; & 1 \leq s \leq n; r = 3 \\ 3; & s \equiv 1 \; mod \; 3; \; 4 \leq r \leq 5 \\ 5; & s \equiv 2 \; mod \; 3; \; 4 \leq r \leq 5 \end{cases} \qquad \mathrm{c}(\mathrm{y}_s) = \begin{cases} 1; & s \equiv 1 \; mod \; 3; \; 4 \leq r \leq 5 \\ 3; & s \equiv 2 \; mod \; 3; \; 4 \leq r \leq 5 \\ 4; & 1 \leq s \leq n; r = 3 \\ 5; & s \equiv 0 \; mod \; 3; \; 4 \leq r \leq 5 \end{cases}$$

It is clear that $\chi_r M(\mathcal{L}_n(2,1)) \leq r+1$. We can conclude that $\chi_r M(\mathcal{L}_n(2,1)) = r+1$ for $3 \leq r \leq 5$.

Case 3: For $r \geq 6$
Based on Observation 1, since $\chi_r(G) \geq min\{r, \Delta\} + 1 = min\{r, 6\} + 1 = 6 + 1 = 7$, then the lower bound of $M(\mathcal{L}_n(2,1))$ is $\chi_r(G) \geq 7$. We need to prove the upper bound of r-dynamic vertex coloring of $M(\mathcal{L}_n(2,1))$ is $\chi_r M(\mathcal{L}_n(2,1)) \leq 7$. Define the function $c : V(G) \rightarrow \{1, 2, ..., k\}$ as follows:

$$\mathrm{c}(\mathrm{x}_{s,1}) = 1; \quad 1 \leq s \leq n \qquad \mathrm{c}(\mathrm{x}_{s,6}) = \begin{cases} 4; & s \equiv 1 \; mod \; 2 \\ 7; & s \equiv 0 \; mod \; 2 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_{s,2}) = \begin{cases} 4; & s \equiv 0 \; mod \; 2 \\ 7; & s \equiv 1 \; mod \; 2 \end{cases} \qquad \mathrm{c}(\mathrm{x}_{s,7}) = \begin{cases} 2; & s \equiv 0 \; mod \; 2 \\ 7; & s \equiv 1 \; mod \; 2 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_{s,3}) = \begin{cases} 2; & s \equiv 0 \; mod \; 2 \\ 6; & s \equiv 1 \; mod \; 2 \end{cases} \qquad \mathrm{c}(\mathrm{x}_{s,8}) = \begin{cases} 4; & s \equiv 0 \; mod \; 2 \\ 6; & s \equiv 1 \; mod \; 2 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_{s,4}) = \begin{cases} 2; & s \equiv 1 \; mod \; 2 \\ 6; & s \equiv 0 \; mod \; 2 \end{cases} \qquad \mathrm{c}(\mathrm{x}_{s,9}) = 1; \quad 1 \leq s \leq n$$

$$\mathrm{c}(\mathrm{x}_{s,5}) = \begin{cases} 1; & s \equiv 0 \; mod \; 3 \\ 3; & s \equiv 1 \; mod \; 3 \\ 5; & s \equiv 2 \; mod \; 3 \end{cases} \qquad \mathrm{c}(\mathrm{y}_s) = \begin{cases} 1; & s \equiv 1 \; mod \; 3 \\ 3; & s \equiv 2 \; mod \; 3 \\ 5; & s \equiv 0 \; mod \; 3 \end{cases}$$

It is clear that $\chi_r M(\mathcal{L}_n(2,1)) \leq 7$. We can conclude that $\chi_r M(\mathcal{L}_n(2,1)) = 7$ for $r \geq 6$.

Theorem 4. *Let $T(\mathcal{L}_n(2,1))$ be a total graph of lobster graph for $n \geq 6$, the r-dynamic chromatic number of $T(\mathcal{L}_n(2,1))$ is*

$$\chi_r T(\mathcal{L}_n(2,1)) = \begin{cases} 3, & 1 \leq r \leq 2 \\ r+1, & 3 \leq r \leq 7 \\ 9, & \text{r} \geq 8 \end{cases}$$

Proof. The vertex set of $T(\mathcal{L}_n(2,1))$ is $VT(\mathcal{L}_n(2,1)) = \{x_{s,t}; 1 \leq s \leq n; 1 \leq t \leq m\} \cup \{y_s; 1 \leq s \leq n-1\}$ and the edge set of $T(\mathcal{L}_n(2,1))$ is $ET(\mathcal{L}_n(2,1)) = \{x_{s,t}x_{s,t+1}; 1 \leq s \leq n\ ; 1 \leq t \leq m-1\} \cup \{x_{s,t}x_{s,t+2}; 1 \leq s \leq n\ ; 1 \leq t \leq m-2\} \cup \{x_s, \left\lceil \frac{m}{2} \right\rceil -1 y_s; 1 \leq s \leq n-1\} \cup \{x_s, \left\lceil \frac{m}{2} \right\rceil +1 y_s; 1 \leq s \leq n-1\} \cup \{x_{s+1}, \left\lceil \frac{m}{2} \right\rceil -1 y_s; 1 \leq i \leq n-1\} \cup \{x_{s+1}, \left\lceil \frac{m}{2} \right\rceil +1 y_s; 1 \leq s \leq n-1\} \cup \{x_s, \left\lceil \frac{m}{2} \right\rceil y_s; 1 \leq s \leq n-1\} \cup \{x_{s+1}, \left\lceil \frac{m}{2} \right\rceil y_s; 1 \leq s \leq n-1\} \cup \{x_s, \left\lceil \frac{m}{2} \right\rceil x_{s+1} \left\lceil \frac{m}{2} \right\rceil; 1 \leq s \leq n-1\}$. The vertex and edge cardinality of $T(\mathcal{L}_n(2,1))$ are $|VT(\mathcal{L}_n(2,1))| = nm + n - 1$, $|ET(\mathcal{L}_n(2,1))| = 2nm + 4n - 7$ respectively. We define three cases, namely for $1 \leq r \leq 2$, $3 \leq r \leq 7$, and $r \geq 8$.

Figure 4 is the illustration of r-dynamic vertex coloring of $T(\mathcal{L}_n(2,1))$

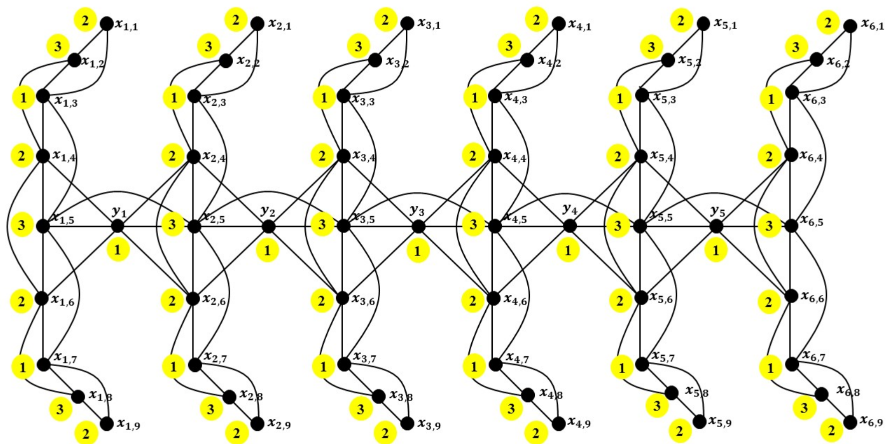

Figure 4. The r-dynamic vertex coloring of $T(\mathcal{L}_n(2,1))$ for $1 \leq r \leq 2$

Case 1: For $1 \leq r \leq 2$
Based on Observation 1, since $\chi_r(G) \geq min\{r, \Delta\} + 1 = min\{2, 8\} + 1 = 2 + 1 = 3$, then the lower bound of $T(\mathcal{L}_n(2,1))$ is $\chi_r(G) \geq 3$. We need to prove the upper bound of r-dynamic vertex coloring of $T(\mathcal{L}_n(2,1))$ is $\chi_r T(\mathcal{L}_n(2,1)) \leq 3$. Define the function $c : V(G) \to \{1, 2, ..., k\}$ as follows:

$$c(x_{s,1}) = 2; \quad 1 \leq s \leq n$$

$$c(x_{s,2}) = 3; \quad 1 \leq s \leq n$$

$$c(x_{s,3}) = 1; \quad 1 \leq s \leq n$$

$$c(x_{s,4}) = 2; \quad 1 \leq s \leq n$$

$$c(x_{s,5}) = 3; \quad 1 \leq s \leq n$$

$$c(x_{s,6}) = 2;\quad 1 \le s \le n$$
$$c(x_{s,7}) = 1;\quad 1 \le s \le n$$
$$c(x_{s,8}) = 3;\quad 1 \le s \le n$$
$$c(x_{s,9}) = 2;\quad 1 \le s \le n$$
$$c(y_s) = 1;\quad 1 \le s \le n-1$$

It is clear that $\chi_r T(\mathcal{L}_n(2,1)) \le 3$. We can conclude that $\chi_r T(\mathcal{L}_n(2,1)) = 3$ for $1 \le r \le 2$.

Case 2: For $3 \le r \le 7$
Based on Observation 1, since $\chi_r(G) \ge min\{r, \Delta\} + 1 = min\{r, 8\} + 1 = r + 1$, then the lower bound of $T(\mathcal{L}_n(2,1))$ is $\chi_r(G) \ge r + 1$. We need to prove the upper bound of r-dynamic vertex coloring of $T(\mathcal{L}_n(2,1))$ is $\chi_r T(\mathcal{L}_n(2,1)) \le r + 1$. Define the function $c : V(G) \to \{1, 2, ..., k\}$ as follows:

$$\mathrm{c}(\mathrm{x}_{s,1}) = \begin{cases} 1; & s \equiv 1\ mod\ 2;\ r = 4 \\ & s \equiv 0\ mod\ 2;\ r = 5 \\ 2; & s \equiv 0\ mod\ 2;\ r = 4 \\ & s = 1; 6 \le r \le 7 \\ 3; & s \equiv 1\ mod\ 2;\ r = 5 \\ 4; & 1 \le s \le n; r = 3 \\ & s \equiv 2\ mod\ 3;\ 6 \le r \le 7 \\ 5; & s \equiv 0\ mod\ 3;\ 6 \le r \le 7 \\ 6; & s \equiv 1\ mod\ 3;\ 6 \le r \le 7; s \ne 1 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_{s,6}) = \begin{cases} 1; & s \equiv 1\ mod\ 2;\ s \ne 1; r = 7 \\ 2; & s \equiv 0\ mod\ 2;\ 4 \le r \le 5 \\ 3; & 1 \le s \le n, r = 3 \\ & s \equiv 1\ mod\ 2;\ 4 \le r \le 5 \\ 4; & s \equiv 3\ mod\ 6;\ r = 6 \\ 5; & s \equiv 1\ mod\ 6;\ r = 6 \\ & s = 1; r = 7 \\ 6; & s \equiv 5\ mod\ 6;\ r = 6 \\ 7; & s \equiv 0\ mod\ 2;\ 6 \le r \le 7 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_{s,2}) = \begin{cases} 2; & s \equiv 0\ mod\ 3;\ 6 \le r \le 7 \\ 3; & 1 \le s \le n; r = 3 \\ & s \equiv 1\ mod\ 2;\ r = 4 \\ & s \equiv 0\ mod\ 2;\ 6 \le r \le 7 \\ 4; & s \equiv 0\ mod\ 2;\ 4 \le r \le 5 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_{s,7}) = \begin{cases} 1; & 1 \le s \le n; r = 4 \\ & s \equiv 0\ mod\ 2;\ r = 7 \\ 2; & 1 \le s \le n; r = 3 \\ & s \equiv 1\ mod\ 2;\ s \ne 1; r = 6 \\ 3; & s \equiv 0\ mod\ 2;\ r = 6 \\ 4; & s \equiv 0\ mod\ 2;\ r = 5 \\ 5; & s \equiv 1\ mod\ 2;\ r = 5 \\ 6; & s = 1; r = 6 \\ 7; & s \equiv 1\ mod\ 2;\ r = 7 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_{s,3}) = \begin{cases} 1; & 1 \le s \le n; r = 3 \\ & s \equiv 0\ mod\ 2;\ r = 4, 6 \\ 3; & s \equiv 0\ mod\ 2;\ r = 5 \\ 5; & s \equiv 1\ mod\ 2;\ r = 4 \\ 6; & s \equiv 1\ mod\ 2;\ r = 5 \\ 7; & s \equiv 1\ mod\ 2;\ r = 6 \\ 8; & 1 \le s \le n; r = 7 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_{s,8}) = \begin{cases} 1; & 1 \le s \le n; r = 3 \\ & s \equiv 0\ mod\ 2;\ r = 6 \\ 2; & s \equiv 1\ mod\ 2;\ s \ne 1; r = 7 \\ 3; & s \equiv 0\ mod\ 2;\ r = 5, 7 \\ 4; & s \equiv 0\ mod\ 2;\ r = 4 \\ 5; & s \equiv 1\ mod\ 2;\ r = 4 \\ 6; & s \equiv 1\ mod\ 2;\ r = 5 \\ & s = 1; r = 7 \\ 7; & s \equiv 1\ mod\ 2;\ r = 6 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_{s,4}) = \begin{cases} 1; & s \equiv 3 \ mod \ 4;\ r = 6 \\ 2; & 1 \le s \le n; r = 3 \\ & s \equiv 1 \ mod \ 2;\ 4 \le r \le 5 \\ 3; & s \equiv 0 \ mod \ 2;\ r = 4 \\ & s \equiv 1 \ mod \ 4;\ r = 6 \\ & s = 1, r = 7 \\ 4; & s \equiv 0 \ mod \ 6;\ r = 6 \\ & s \equiv 0 \ mod \ 3;\ r = 7 \\ 5; & s \equiv 4 \ mod \ 6;\ r = 6 \\ & s \equiv 1 \ mod \ 3;\ s \ne 1; r = 7 \\ 6; & s \equiv 0 \ mod \ 2;\ r = 5 \\ & s \equiv 2 \ mod \ 6;\ r = 6 \\ & s \equiv 2 \ mod \ 3;\ r = 7 \end{cases} \qquad \mathrm{c}(\mathrm{x}_{s,9}) = \begin{cases} 1; & s \equiv 0 \ mod \ 2;\ r = 5 \\ 2; & s \equiv 1 \ mod \ 2;\ 4 \le r \le 5 \\ & s = 1; 6 \le r \le 7 \\ 3; & s \equiv 0 \ mod \ 2;\ r = 4 \\ 4; & 1 \le s \le n; r = 3 \\ & s \equiv 2 \ mod \ 3;\ 6 \le r \le 7 \\ 5; & s \equiv 0 \ mod \ 3;\ 6 \le r \le 7 \\ 6; & s \equiv 1 \ mod \ 3;\ 6 \le r \le 7; 1 \ne 1 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_{s,5}) = \begin{cases} 1; & s \equiv 1 \ mod \ 4;\ 6 \le r \le 7 \\ 2; & s \equiv 0 \ mod \ 2;\ 6 \le r \le 7 \\ 3; & s \equiv 3 \ mod \ 4;\ 6 \le r \le 7 \\ 4; & 1 \le s \le n; r = 3 \\ & s \equiv 1 \ mod \ 2;\ 4 \le r \le 5 \\ 5; & s \equiv 0 \ mod \ 2;\ 4 \le r \le 5 \end{cases} \qquad \mathrm{c}(\mathrm{y}_s) = \begin{cases} 1; & 1 \le s \le n; 3 \le r \le 5 \\ 4; & s \equiv 1 \ mod \ 3;\ 6 \le r \le 7 \\ 5; & s \equiv 2 \ mod \ 3;\ 6 \le r \le 7 \\ 6; & s \equiv 0 \ mod \ 3;\ 6 \le r \le 7 \end{cases}$$

It is clear that $\chi_r T(\mathcal{L}_n(2,1)) \le r+1$. We can conclude that $\chi_r T(\mathcal{L}_n(2,1)) = r+1$ for $3 \le r \le 7$.

Case 3: For $r \ge 8$
Based on Observation 1, since $\chi_r(G) \ge min\{r, \Delta\} + 1 = min\{r, 8\} + 1 = 8 + 1 = 9$, then the lower bound of $T(\mathcal{L}_n(2,1))$ is $\chi_r(G) \ge 9$. We need to prove the upper bound of r-dynamic vertex coloring of $T(\mathcal{L}_n(2,1))$ is $\chi_r T(\mathcal{L}_n(2,1)) \le 9$. Define the function $c : V(G) \to \{1, 2, ..., k\}$ as follows:

$$\mathrm{c}(\mathrm{x}_{s,1}) = \begin{cases} 3; & s \equiv 2 \ mod \ 3 \\ 5; & s \equiv 0 \ mod \ 3 \\ 6; & s \equiv 1 \ mod \ 3;\ s \ne 1 \\ 7; & s = 1 \end{cases} \qquad \mathrm{c}(\mathrm{x}_{s,6}) = \begin{cases} 5; & s = 1 \\ 7; & 1 \le s \le n, s \ne 1 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_{s,2}) = \begin{cases} 1; & s \equiv 2 \ mod \ 3 \\ 2; & s \equiv 0 \ mod \ 3 \\ 3; & s \equiv 1 \ mod \ 3;\ s \ne 1 \\ 6; & s = 1 \end{cases} \qquad \mathrm{c}(\mathrm{x}_{s,7}) = \{\ 9; \quad 1 \le s \le n$$

$$\mathrm{c}(\mathrm{x}_{s,3}) = 8; \quad 1 \le s \le n \qquad \mathrm{c}(\mathrm{x}_{s,8}) = \begin{cases} 1; & s \equiv 2 \ mod \ 3 \\ 2; & s \equiv 0 \ mod \ 3 \\ 3; & s \equiv 1 \ mod \ 3;\ s \ne 1 \\ 6; & s = 1 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_{s,4}) = \begin{cases} 3; & s = 1 \\ 4; & s \equiv 0 \ mod \ 3 \\ 5; & s \equiv 1 \ mod \ 3;\ s \ne 1 \\ 6; & s \equiv 2 \ mod \ 3 \end{cases} \qquad \mathrm{c}(\mathrm{x}_{s,9}) = \begin{cases} 3; & s \equiv 2 \ mod \ 3 \\ 5; & s \equiv 0 \ mod \ 3 \\ 6; & s \equiv 1 \ mod \ 3;\ s \ne 1 \end{cases}$$

$$\mathrm{c}(\mathrm{x}_{s,5}) = \begin{cases} 1; & s \equiv 1 \ mod \ 3 \\ 2; & s \equiv 2 \ mod \ 3 \\ 3; & s \equiv 0 \ mod \ 3 \end{cases} \qquad \mathrm{c}(\mathrm{y}_s) = \begin{cases} 4; & s \equiv 1 \ mod \ 3 \\ 5; & s \equiv 2 \ mod \ 3 \\ 6; & s \equiv 0 \ mod \ 3 \end{cases}$$

It is clear that $\chi_r T(\mathcal{L}_n(2,1)) \le 9$. We can conclude that $\chi_r T(\mathcal{L}_n(2,1)) = 9$ for $r \ge 8$.

3. Conclusion

In this study, we obtained of the r-dynamic vertex coloring of line, middle, total of lobster graph $\mathcal{L}_n(2,1)$.

Open Problem 1. *Find the r-dynamic chromatic number of $\mathcal{L}_n(l,m)$ for $l \geq 3$ and $m \geq 2$.*

Acknowledgement

We gratefully the support from CGANT Univesity of Jember 2019.

References

[1] Akbari S, Ghanbari M, and Jahanbekam S 2014 On the Dynamic Coloring of Cartesian Product Graphs *Ars Combinatoria* **114** 161-168

[2] Alishahi M 2011 On the Dynamic Coloring of Graphs *Discrete Applied Mathematics* **159(2-3)** 152-156

[3] Alishahi M 2012 Dynamic Chromatic Number of Regular Graphs *Discrete Applied Mathematics* **160(15)** 2098-2103

[4] Capobianco M and Molluzzo J C 1978 Examples and Counterexamples in Graph Theory *Journal of Graph Theory* **2(3)** 274-274

[5] Dafik, Meganingtyas D E W, Purnomo K D, Tarmidzi M D, and Agustin I H 2017 Several Classes of Graphs and their r-Dynamic Chromatic Numbers *Journal of Physics: Conference Series* **855(1)** 012011

[6] Gross J L and Yellen J 2005 Graph Theory and its Applications *CRC Press*

[7] Jahanbekam S, Kim J, Suil O, and West D B 2016 On r-Dynamic Coloring of Graphs *Discrete Applied Mathematics* **206** 65-72

[8] Kang R, Mller T, and West D B 2015 On r-Dynamic Coloring of Grids *Discrete Applied Mathematics* **186** 286-290

[9] Kristiana A I, Dafik, Utoyo M I, and Agustin I H 2017 On r-Dynamic Chromatic Number of the Corronation of Path and Several Graphs *International Journal of Advanced Engineering Research and Science* **4(4)**

[10] Kristiana A I and Utoyo M I 2018 On the r-Dynamic Chromatic Number of the Corronation by Complete Graph *In Journal of Physics: Conference Series* **1008(1)** 012033

[11] Kristiana A I, Utoyo M I, and Dafik 2018 The Lower Bound of the r-Dynamic Chromatic Number of Corona Product by Wheel Graphs *In AIP Conference Proceedings* **2014(1)** 020054

[12] Merly E E R and Gnanadhas N 2012 Linear Star Decomposition of Lobster *International Journal Contemporer Mathematics Science* **7(6)** 251-261

[13] Montgomery B 2001 Dynamic Coloring of Graphs (Ph.D Dissertation) West Virginia University

[14] Septory B J, Kristiana A I, Agustin I H, and Wardani D A R 2019 On r-Dynamic Chromatic Number of Coronation of Order Two of any Graphs with Path Graph *In IOP Conference Series: Earth and Environmental Science* **243(1)** 012113

[15] Taherkhani A 2016 On r-Dynamic Chromatic Number of Graphs *Discrete Applied Mathematics* **201** 222-227

[16] Thenmozhi B and Prabha R 2017 Power Domination of Middle Graph of Path, Cycle and Star *International Journal of Pure and Applied Mathematics* **114(5)** 13-19