Students' combinatorial generalization thinking skills in solving tessellation coloring pattern problems and its enhancement through problem-based learning

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Abstract. Combinatorial generalization thinking skills are important to support understanding of mathematical concepts, particularly in solving tessellation coloring pattern problems. This study aimed to analyze the students' combinatorial generalization thinking skills and improve them by using problem-based learning (PBL), then present them with phase portraits. This study applied explanatory sequential method that combined quantitative and qualitative sequentially. The subjects were 23 elementary school students in the experimental class and 23 elementary school students in the control class. The research findings show that the levels of students' combinatorial generalization thinking skills divided into weak, moderate, and strong. The result of independent sample t-test indicated that there was significant impact of PBL implementation with the significance of 0.03 ($p<0.05$). N-Gain shows that PBL is more effective than the conventional one. The enhancement on strong and moderate categories showed an increase of 43.47% and 8.69%, while the weak one showed a decrease of 52.18%. The descriptions of combinatorial generalization thinking skills are presented with phase portraits. The results reveal that the implementation of PBL increases the students' combinatorial generalization thinking skills in solving tessellation coloring pattern problems of elementary school students.

1. Introduction

Thinking is an activity or a process to draw a conclusion or make a new correct statement based on some statements that have been proven or assumed before. Thinking process will produce solution to solve the problem. Everyone will bring up the different concept of thought which is expressed in the form of a decision [22] [29]. The product of thinking is not only determined by the final decision, but also from the thought process, so the appreciation must also be given to thinking process.

Generalization thinking skills are the stage to understand the core of the subject matter that has been presented [1]. There are five generalizing indicators, namely counting object, checking whole object, difference, explicit, and guess and check [11]. The generalizing thinking activities are divided into relating, searching, and extending [5]. Meanwhile, combinatorial was the branch of mathematics about arrangement of objects without enumerate first [15]. Combinatorial thinking is built upon the ability to organize elements in a set into transparent tables, charts, schemes and lists [10]. Combinatorial thinking skills categories consist of identifying problems, re-understanding the problems given, systematic problem exposure, and changing problems into combinatorial one [16].

The combinatorial generalization thinking skills are the form of ability related to the thought process of making or drawing specific conclusions becomes something general in terms of the objects arrangement of group members obtained by repeated experiments. The aspect of the combinatorial generalization thinking skills consists of five categories, i.e.: investigating some cases, recognizing a pattern of all cases, generalizing all cases, proving mathematically, and conjecturing another combinatorial problem [24].

The combinatorial generalization thinking skills is suitable at the basic education level based on several theories. The elementary school students have started to think logically and also group objects into various ways or criteria or class. It also supports Piaget's theory [20], that the elementary school students were positioned at the pre-operational stage to concrete operations. They have begun to combine initial knowledge with new concepts. Based on these bases, the elementary school students begin to explore their thinking processes, including their combinatorial generalization thinking skills.

Table 1. The aspects and indicators of students' combinatorial generalization skills

Aspects	Indicators
Investigation some cases	a. Identifying the properties/ characteristics of the problem b. Applying some cases
Recognizing a pattern of all cases	a. Identifying the pattern of solution of the cases b. Expanding the pattern
Generalizing for all cases	a. Applying mathematical symbolization b. Counting the cardinalities c. Developing an algorithm

The tessellation color pattern is specific patterns consisting of geometric shapes arranged without separators or distances to cover a flat shape fully. Tessellation connects between geometric architectural designs with technologies [3]. Problem-based learning emphasizes the interaction process stages and active in providing appropriate solutions to the problems given [28]. It refers to the mental processes of individuals to find, analyze, and solve problems; it is also described as a learning environment that encourages processes such as interpretation, gathering information, and identifying solutions as possible [26] [6]. Furthermore, improving reasoning and mathematical communication skills can be done by guiding the students to engaging on learning activities. It will open the thinking opportunities in solving problems given [18]. It is able also to present the challenges for students [12].

This research has provided empirical and statistical evidence in the quantitative section. Then, it has presented students' thinking skills in solving problem given. Both of these are the difference between this research and the previous research. It will foster the ideas of readers and subsequent researchers to do more about the variable of this research.

The last presentation in the form of phase portrait is a novelty aspect in this research. The phase portrait will make it easier to understand about the pattern of students' thinking skills and can be used as a guide that provides an overview of research results more easily. This study aimed to analyze the students' combinatorial generalization thinking skills and improve them by using problem-based learning, then present the students' combinatorial generalization thinking skills with phase portraits.

Based on the explanations above, giving mathematical concepts deeply is very necessary. The combinatorial generalization thinking skills are in line with the achievement purpose of 21st century skills (communication, collaboration, critical thinking and problem solving, creativity and innovation).

2. Research Method

This study applied a combination method that combined qualitative and quantitative sequentially with explanatory sequential design. The explanatory sequential approach is a design that reveals the potential more deeply and extensively that has been known based on the session previously [4]. Qualitative methods are used to analyze students work product, while the qualitative ones are used to analyze data from the results of guided interviews more deeply [17].

The independent variable of this study is Problem-Based Learning model. The students' combinatorial generalization ability which was presented by learning outcomes becomes the independent variable. The research design used two classes were divided into control class and experimental class. The sample selection on quantitative was done by simple random sampling. Meanwhile, for the qualitative one used disproportionate stratified random sampling. It accommodates six students divided into 3 students each from the control class and the experimental class representing three levels of combinatorial generalization thinking skills.
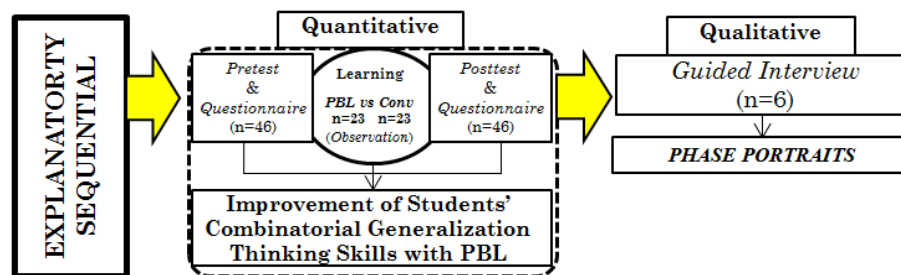

Figure 1. The flowchart of research method [4]

The quantitative research used the experimental research (*pre-test post-test control group design*) with problem-based learning for the experimental class. Meanwhile, the control class used the conventional learning to make difference in learning between both of the classes.

Table 2. The pretest posttest control group design [23]

Group	Pretest	Treatment	Posttest
A (n=23)	O_1	X	O_2
B (n=23)	O_3	-	O_4

2.1 Population and sample
The populations of this study were 4th grade students of elementary school. The samples were 23 students as the control class and 23 students as the experimental class. The guided interview conducted on six students representing 3 levels of combinatorial generalization thinking skills from both classes.
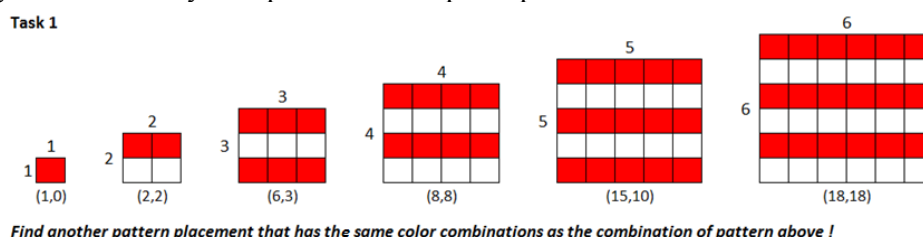
2.2 Instrument
The instruments of this study included tests, questionnaires, interviews, and learning process observations. The researcher distributed the observation of learning activities to all samples in the experimental class. The observation used a scale encompassing *Very Active, Active, Hesitate, Inactive, Very Inactive*. The instruments were validated by expert and statistical feasibility test.
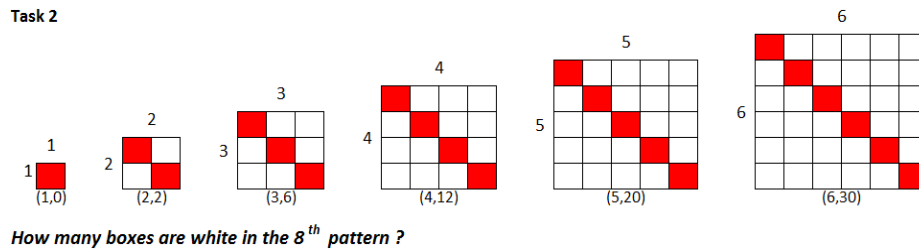
2.3 Data analysis
The data analysis method of this study used analysis of test results, questionnaire, and observation of learning process. The statistical tests used the t-independent sample test. N-Gain relative effectiveness and resume table becomes additional proof of the problem-based learning implementation. Meanwhile, for the qualitative session, the guided interviews results were presented in phase portraits.

2.4 Task
The following figure was the tasks to explore the students' combinatorial generalization thinking skills more deeply and extensively then presented with phase portraits.

Find another pattern placement that has the same color combinations as the combination of pattern above !

Figure 2. The task for the phase portraits

Some important things to solve those tasks are generalization pattern, geometry transformation, and numbers pattern. Generalization of the color patterns placement (vertical, horizontal, and diagonal) can be applied to different positions but still have similarities to the color pattern of the tasks. These tasks above were almost same or identic to the task of trigonometry in advanced level [17].
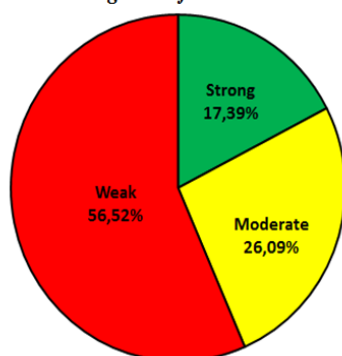
3. Results and Discussion
Preliminary findings from this study are related to the distribution of students' combinatorial generalization thinking skills in solving tessellation coloring problems.
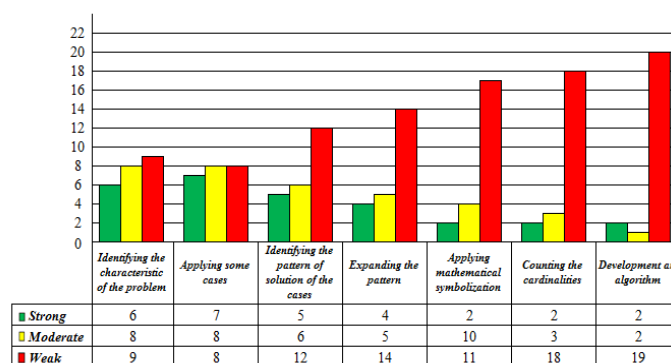
3.1 The distribution of students' combinatorial generalization thinking skills
The abilities distribution in the control class on the pre-test session shows 56.52% for the weak ability, while 26.09% and 17.38% for the moderate and strong one.
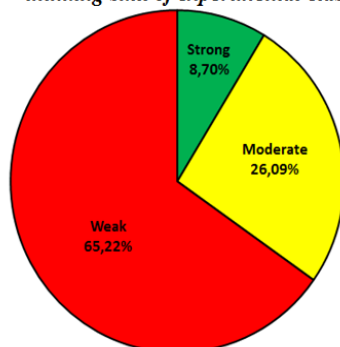

	Identifying the characteristic of the problem	Applying some cases	Identifying the pattern of solution of the cases	Expanding the pattern	Applying mathematical symbolization	Counting the cardinalities	Development an algorithm
Strong	6	7	5	4	2	2	2
Moderate	8	8	6	5	10	3	2
Weak	9	8	12	14	11	18	19

Figure 3. The combinatorial generalization thinking skills in the control class on pretest

	Identifying the characteristic of the problem	Applying some cases	Identifying the pattern of solution of the cases	Expanding the pattern	Applying mathematical symbolization	Counting the cardinalities	Development an algorithm
Strong	4	2	3	2	2	1	0
Moderate	9	10	8	7	2	3	3
Weak	10	11	12	14	19	19	20

Figure 4. The combinatorial generalization thinking skills in the experimental class on pretest
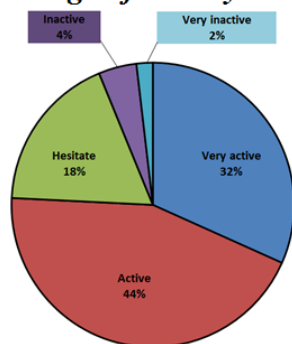
Meanwhile, the results for the experimental class shows 65.22% for the weak category, while the moderate and the strong category respectively 26.09% and 8.70%. Based on those data, it can reveal that the distribution of students' combinatorial generalization thinking skills in solving tessellation coloring problems spread over strong, moderate, and weak.

These results provide reinforcement of [22]; [2]; [29], that everyone will display each different thinking product. Understanding the core of the problem as stated by [1], influences the students' combinatorial generalizations in solving tessellation coloring problems is also seen in this study. Each student presents their understanding [5], the generalization activities greatly influenced of thinking skills displayed, more and more indicators that can be exceeded will produce better grades.

3.2 The enhancement of students' combinatorial generalization thinking skills
The observation results of learning activity in the experimental class with problem-based learning show that 32% were very active, 44% were active, 18% were hesitant, and for inactive and very inactive were only 4% and 2%. These results provide additional confidence of the effectiveness of Problem-Based Learning has been applied in the experimental class.

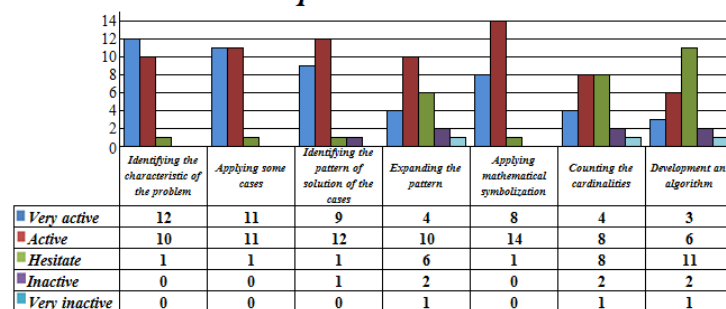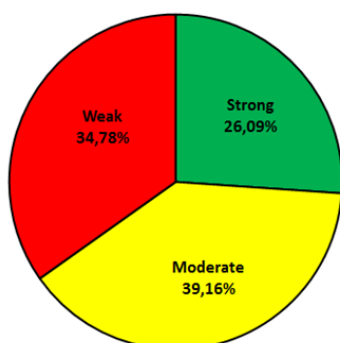|	Identifying the characteristic of the problem	Applying some cases	Identifying the pattern of solution of the cases	Expanding the pattern	Applying mathematical symbolization	Counting the cardinalities	Development an algorithm
Very active	12	11	9	4	8	4	3
Active	10	11	12	10	14	8	6
Hesitate	1	1	1	6	1	8	11
Inactive	0	0	1	2	0	2	2
Very inactive	0	0	0	1	0	1	1

Figure 5. The observation result of learning process in the experimental class

Based on the figure above, 76% of students were in a very active and active position. It indicates that the implementation of problem-based learning has been proven to improve the students' combinatorial generalization thinking skills in solving tessellation coloring problems. It reinforces [8] and [19] that the maximum impact of problem-based learning can be achieved by engaging and improving of students' readiness and motivation aspects; students actively engaged while solving the problem can develop their thinking skills. It also supports [22], that the variety of students' thinking abilities has an impact on their achievement results.

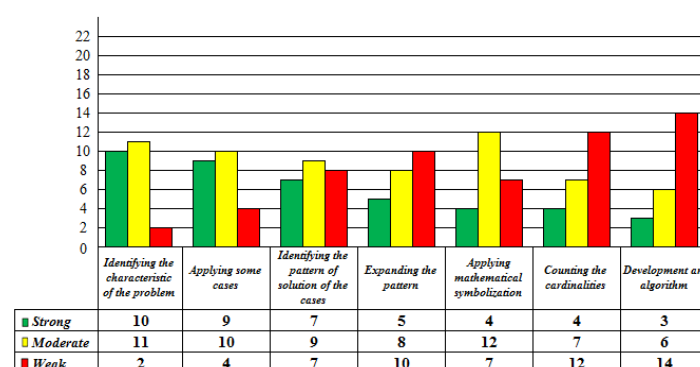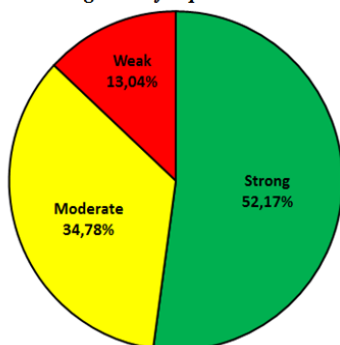|	Identifying the characteristic of the problem	Applying some cases	Identifying the pattern of solution of the cases	Expanding the pattern	Applying mathematical symbolization	Counting the cardinalities	Development an algorithm
Strong	10	9	7	5	4	4	3
Moderate	11	10	9	8	12	7	6
Weak	2	4	7	10	7	12	14

Figure 6. The combinatorial generalization thinking skills in the control class on posttest
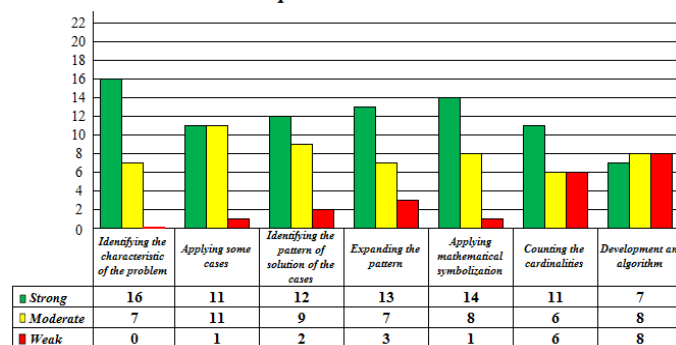
Figure 7. The combinatorial generalization thinking skills in the experimental class on posttest

The figures above show that 34.78% for the weak, 39.13% for the moderate and 26.09% for the strong category for the control class. The distribution of students' combinatorial generalization thinking skills in the experimental class on post-test show that 13.04% for weak, 34.78% and 52.17% for moderate and strong category. The experimental class showed better results than the control class.

The next finding is t-test independent sample, N-Gain score result, and the presentation table resume. The average of control class is 65.22 while 79.35 for the experimental. The homogeneity and normality previously tested gave homogeneous and normal distributed results for both of them.

Table 3. The result of t-test independent sample

		Independent Samples Test							
		Levene's Test for Equality of Var		t-test for Equality of Means					
		F	Sig.	t	df	Sig. (2-tailed)	Mean Diff	Std. Error Diff	95% Confidence Interval of the Diff	
									Lower	Upper
Post test	Equal variances assumed	,36	,55	-2,24	44	,030	-14,13	6,32	-26,86	-1,40
	Equal variances not assumed			-2,24	43,09	,030	-14,13	6,32	-26,86	-1,40

Table 3 above shows the result of independent sample t-test. The sig. (2-tailed) of independent sample t-test is 0.03 (p=<0.05), thus it is significant. It proves that the two classes are different in terms of student achievement test after the implementation of problem-based learning. This finding reinforces [6], that problem-based learning is more suitable for improving problem solving skills.

Table 4. The N-Gain relative effectiveness result

Problem-Based Learning	$= \dfrac{1825 - 1150}{2300 - 1150}$	$= \dfrac{675}{1150}$	$= 0,58696$	$= \begin{array}{l} 58,70\% \\ \text{(medium)} \end{array}$
Conventional Learning	$= \dfrac{1500 - 1250}{2300 - 1250}$	$= \dfrac{250}{1050}$	$= 0,23810$	$= \begin{array}{l} 23,81\% \\ \text{(low)} \end{array}$
N-Gain	$= \dfrac{0,58696}{0,23810}$	$= $	**2,465 (n>1)** *[Problem-Based Learning is more effective]*	

The table above shows the result of N-Gain relative effectiveness. Problem-based learning contributes 58.70% effectively, while the conventional gives 23.81%. The final calculation shows 2.465 (n-gain>1) for problem-based learning. It supports [14] and [9], that the effectiveness of BL and PBL are able to improve students' algebra learning outcomes. N-Gain for problem-based learning in each indicator of mathematical critical thinking ability shows higher value than conventional one.

Table 5. The enhancement of students' combinatorial generalization thinking skills

Combinatorial Generalization thinking skills potential	Pretest		Posttest		$\triangle$
	n	Percentage	n	Percentage	
Control class with Conventional Learning					
Strong	4	17,39%	6	26,09%	+ 8,70%
Moderate	6	26,09%	9	39,13%	+ 13,04%
Weak	13	56,52%	8	34,78%	- 21,74%
Experimental class with Problem-Based Learning					
Strong	2	8,70%	12	52,17%	+ 43,47%
Moderate	6	26,09%	8	34,78%	+ 8,69%
Weak	15	65,22%	3	13,04%	- 52,18%

The strong category in the control class showed an increase of 8.70%, the moderate category showed an increase of 13.04%, and the weak category showed a decrease of 21.74%. While the strong category in the experimental class showed an increase of 43.47%, the moderate category showed an increase of 8.69%, and the weak category showed a decrease of 52.18%. It gives further evidence of an increase in the students' combinatorial generalization thinking skills in solving tessellation coloring problems. Based on these evidences, we conclude that the problem-based learning in the experimental class is more effective than the conventional one in the control class.

3.3 The phase portraits of students' combinatorial generalization thinking skills
The symbolization that needed to be considered in this session is that the subscript 1 shows the selected student from the control class and subscript 2 for the selected student from the experimental class. This phase portraits were obtained from the derivatives of the guided interview transcript (1st December 2018). According to [7], that attempt to find the way out of a difficulty to achieve a goal that is not so easily reached referred to solving mathematical problem. The concept of geometry transformation is also an alternative for solving the tasks. The correlation between lines, angles, triangles and squares, pythagorean theorem, circles, cubes, prisms, pyramids, nets, congruence, tubes, cones, balls into problem solving were interpreted as the geometry materials [27]. Geometry transformation is the position shift or change [13]. The concept of number patterns can be used to find the cardinalities formula. The pattern of mathematical numbers has a value of attractiveness on the learning activity application, both in terms of material and substantial of learning [12].

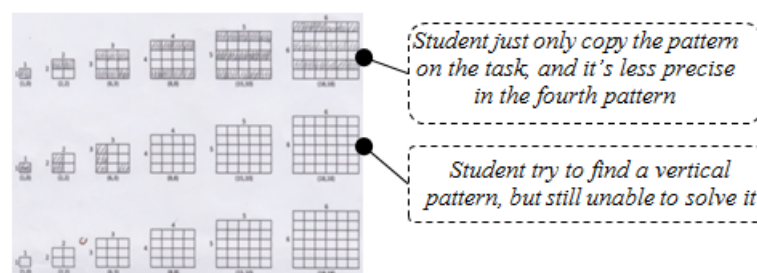*3.3.1 The weak combinatorial generalization thinking skills*
a. Selected student from the control class (S_1)
R : What is the task about?
S_1 : *(Student still remains silent, but after being triggered)* Color
R : How about this task, easy, medium, or difficult? How to solve it?
S_1 : *(Student smiles, then answers hesitantly)* It's easy Sir. It needs memory to solve it

Figure 8. The results of student completion

R : Ok, have you used your memory when you finishing it?
S$_1$: Yes Sir
R : Good, did you really pay attention to the pattern reference?
S$_1$: Yes Sir *(Students' completion results are still wrong)*
R : Have you found similar pattern based on the reference?
S$_1$: *(Student still remains silent)*
R : Well, do you understand the numbers below for each box?
S$_1$: *(Student still remains silent)*
R : How many red squares are on the 8th?
S$_1$: *(Student shakes her head)* I don't know Sir.
R : Ok, explain how did you finish task #1?
S$_1$: I give the same color as the above one Sir
R : For task #2, can you find how many white squares are on the 8th?
S$_1$: *(Student observes the task, then shakes her head)*
R : What do you think if the image continued until pattern 8, can you find the number of white squares? *(The researcher tries to trigger)*
S$_1$: *(Student doesn't give respond)*

Source: transcript of guided interview (1st December, 2018)

b. Selected student from the experimental class (S$_2$)
R : What is the task about?
S$_2$: Color and pattern
R : How about the task, easy, medium, or difficult? How to solve it?
S$_2$: It is at medium level Sir. I use formula to solve it
R : Well, have you used the formula while finishing it?
S$_2$: *(Student smile, then answer hesitantly)* Yes Sir
R : Ok, did you really pay attention to the pattern reference?
S$_2$: Yes Sir *(Students' completion results are not complete)*

Student finds other pattern, but it's wrong at the 4th and the 6th patterns

Student tries to find a vertical pattern, but it's not right at the 6th pattern

Student tries to find a vertical pattern, but it's not right at the 3rd pattern
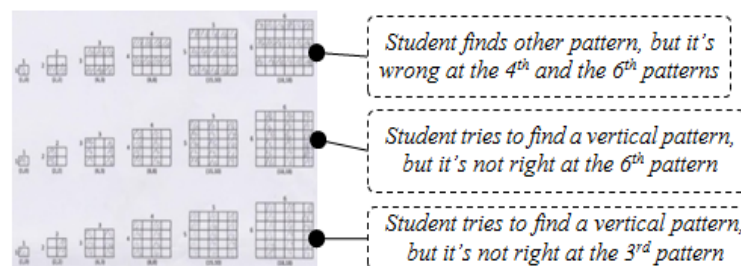
Figure 9. The results of student completion

R : Have you tried to find it according to the reference pattern?
S$_2$: Yes Sir
R : Well, do you understand the numbers below for each box?
S$_2$: The number of red boxes
R : How many red squares are on the 8th?
S$_2$: *(Student still remains silent)*
R : Ok, explain how do you finish task #1?
S$_2$: I give the same color as the above one Sir
R : Can you find how many white squares are on the 8th pattern for task #2?
S$_2$: *(Student observes the task)*. Yes Sir. Continued drawing up till the 8th
R : Ok, can you find without drawing it?
S$_2$: *(Student tries to think, then shakes his head)*

Source: transcript of guided interview (1st December, 2018)

3.3.2 The moderate combinatorial generalization thinking skills
a. Selected student from the control class (S_1)

    R : What is the task about?
    S_1 : *(Student demonstrates shading)* Shading the shape
    R : How about the task, easy, medium, or difficult? How to solve it?
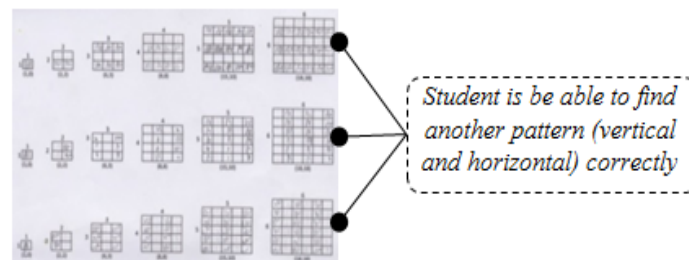 S_1 : *(Student smiles).* It's easy Sir. I need formula to solve it

Figure 10. The results of student completion

    R : Well, have you used the formula while you finishing it?
    S_1 : Yes Sir
    R : Good, did you really pay attention to the pattern reference?
    S_1 : Yes Sir *(Students' completion results were right and complete)*
    R : Have you tried to find it according to the reference pattern?
    S_1 : *(Student smiles)* Anyway, I just walk and adapt the pattern Sir
    R : Ok, Well, do you understand the numbers below for each box?
    S_1 : Red and white Sir
    R : How many red squares are on the 8th?
    S_1 : *(Student tries to think, then give answer but it's still not right)*
    R : Ok, explain how did you finish task #1 according to the reference?
    S_1 : I give the same color as the above one Sir
    R : For task #2, can you find how many white squares are on the 8th?
    S_1 : *(Student observes the task)* Continued drawing up till the 8th
    R : Ok, can you find without drawing it?
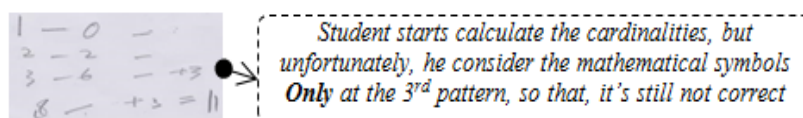 S_1 : *(Student starts to think, then tries to count in the worksheet)*

Figure 11. The results of the cardinalities

    R : Explain your calculation process
    S_1 : Three for red, six for white (added by 3). If eight for red, so eleven for white
    R : Ok, then, can you find the number of red squares on the 8th (task #1)?
    S_1 : *(Student tries to think).* I don't know Sir

Source: transcript of guided interview (1st December, 2018)

b. Selected student from the experimental class (S_2)

    R : What is the task about?
    S_2 : Color and shading the shape
    R : How about the task, easy, medium, or difficult? How to solve it?
    S_2 : It is medium Sir. I need memory to solve it
    R : Well, have you used your memory when you finishing it?
    S_2 : Yes Sir
    R : Good, did you really pay attention to the pattern reference?

S_2 : Yes Sir *(Students' completion results were right and complete)*
R : Have you tried to find it according to the reference pattern?
S_2 : Yes Sir, I have been done
R : Ok, Well, do you understand the numbers below for each box?
S_2 : Red and white number Sir
R : How many red squares are on the 8th?
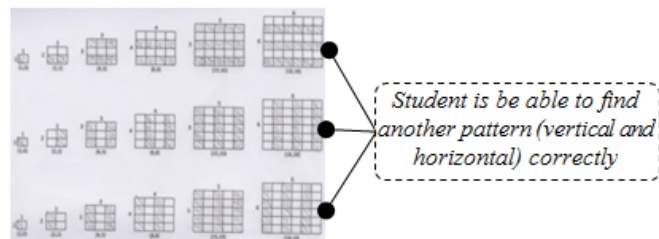S_2 : *(Student tries to think, then gives answer but it's still not right)*

Figure 12. The results of student completion

R : Ok, explain how did you finish task #1?
S_2 : The number of color's box must be the same but the placement can be different
R : For task #2, can you find how many white squares on the 8th?
S_2 : *(Student observes the task)* Continued drawing up till the 8th
R : Ok, can you find without draw it?
S_2 : *(Student starts to think, then tries to count in the worksheet)*
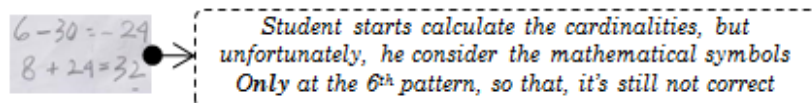

Figure 13. The results of the cardinalities

R : Explain your calculation process
S_2 : 6 for red and 30 for white (added by 24). If 8 for red, so (8+24=32) for white
R : Ok, then, can you find the number of red squares on the 8th (task #1)?
S_2 : *(Student tries to think, then gives answer but it's still not right)*

Source: transcript of guided interview (1st December, 2018)

3.3.3 The strong combinatorial generalization thinking skills
a. Selected student from the control class (S_1)
R : What is the task about?
S_1 : Squares, color, and pattern
R : How about the task, easy, medium, or difficult? How to solve it?
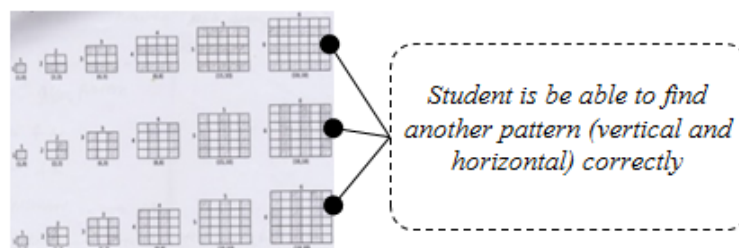S_1 : It is easy Sir. I need formula and see to the pattern reference

Figure 14. The results of student completion

R : Well, have you used your memory while you finishing it?
S_1 : Yes, I have

R : Good, did you really pay attention to the pattern reference?
S_1 : Yes I did *(Students' completion results were right and complete)*
R : Have you tried to find it according to the reference pattern?
S_1 : Yes Sir, I have been done
R : Ok, Well, did you understand the numbers below for each box?
S_1 : The number of red squares and white squares
R : Can you find the numbers of red squares are on the 8th?
S_1 : *(Student tries to think, then gives answer but it's still not right)*
R : Ok, explain how did you finish task #1?
S_1 : The number of color's box must be the same with different placement
R : For task #2, can you find how many white squares are on the 8th?
S_1 : *(Student observes the task)* Continued drawing up till the 8th
R : Ok, can you find without drawing it?
S_1 : *(Student starts to think, then tries to count in the worksheet)*
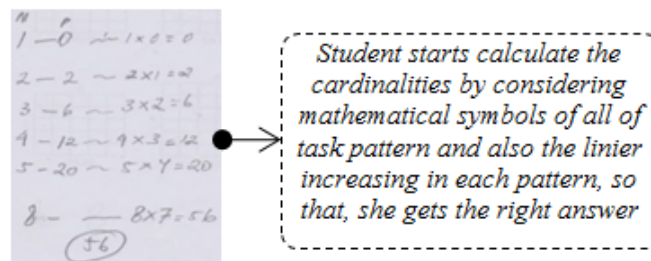

Figure 15. The results of cardinalities #1

R : Explain your calculation process
S_1 : Red sequentially (1, 2, 3…) white box is searched by multiplying with sequence starting at 0. If 8 for red, so it must multiplied by 7 = 56 for white
R : Ok, then how many red squares are on the 8th (task #1)? Pay attention to even and odd pattern
S_1 : *(Student starts to think, then tries to count in the worksheet)*
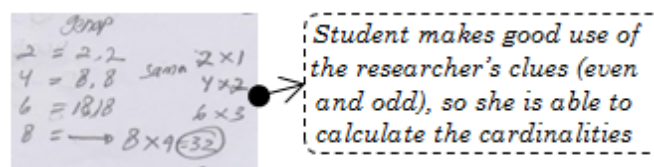

Figure 16. The results of cardinalities #2

R : Good, please explain the calculation process
S_1 : The number of red squares and white squares are always in the even pattern. Pattern 8 is in the even pattern. Pattern 8 is in the 4th of even pattern, so 8x4=32 for the red.
R : Good, thank for your explanation

Source: transcript of guided interview (1st December, 2018)

b. Selected student from the experimental class (S_2)
R : What is the task about?
S_2 : Color, the number, squares, and shading pattern
R : How about the task, easy, medium, or difficult? How to solve it?
S_2 : It is easy Sir. I need memory and see to the pattern reference
R : Well, have you used your memory while you finishing it?
S_2 : Yes, I have
R : Good, did you really pay attention to the pattern reference?
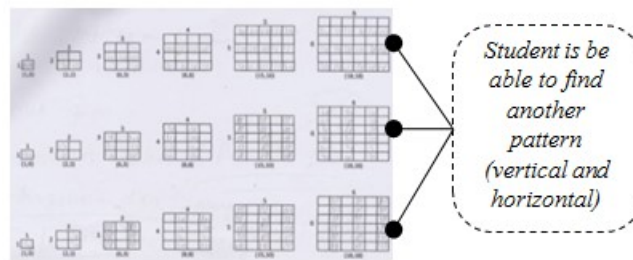S_2 : Yes I did *(Students' completion results were right and complete)*

Figure 17. The results of student completion

R : Have you tried to find it according to the reference pattern?

S_2 : Yes Sir, I have been done

R : Ok, Well, did you understand the numbers below for each box?

S_2 : The number of red and white squares

R : Can you find the numbers of red squares are on the 8th?

S_2 : *(Student tries to think, then gives answer but it's still not right)*

R : Ok, explain how did you finish task #1?

S_2 : Find the placement of pattern and the number according to the reference

R : For task #2, can you find how many white squares are on the 8th?

S_2 : *(Student observes the task)* Continued drawing up till the 8th

R : Ok, can you find without drawing it?

S_2 : *(Student starts to think, then tries to count in the worksheet)*
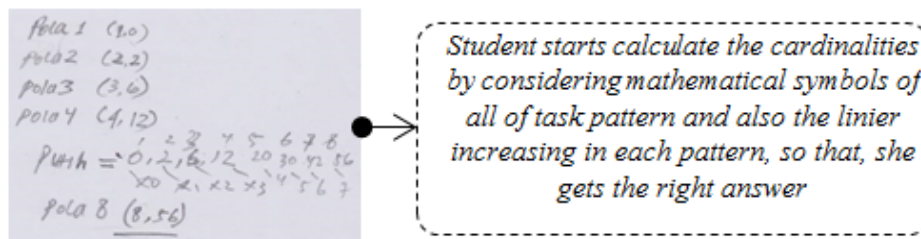

Figure 18. The results of cardinalities #1

R : Explain your calculation process

S_2 : p1 (1, 0); p2 (2, 2); p3 (3, 6); p4 (4, 12). Red pattern sequenced starting by 1. White pattern sequenced starting by 0. The 8th pattern = 8x7=56 for white

R : Ok, then how many red squares are on the 8th (task #1)?

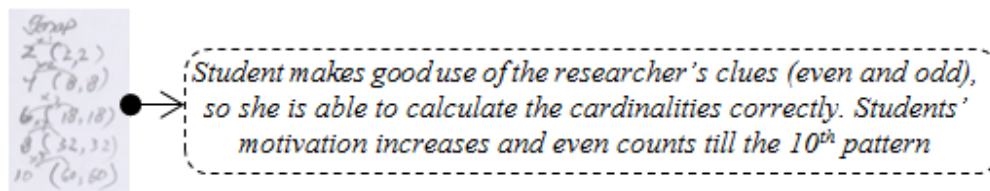S_2 : *(Student starts to think, then tries to count on the worksheet)*

Figure 19. The results of cardinalities #2

R : Good, please explain the calculation process

S_2 : The number of red squares and white squares are always the same in the even patter n. Pattern 8 is in the 4th of even pattern, so 8x4=32 for the red.

R : Good, thank for the explaining. Why did you calculate it till to the 10th pattern?

S_2 : I am too excited to prove it, Sir

Source: transcript of guided interview (1st December, 2018)

The conversation above was carried out to find out more deeply about the students' thinking pattern that will be presented into the phase portraits. Circles 1a, 1b, 2a, 2b, 3a, 3b, and 3c are seven indicator points of the students' combinatorial generalization skills. S_1 shows the student from the control class, while the S_2 for the student from the experimental class. "S_{12}" coded with three different colors, the red is for weak, the purple is for moderate, and the green is for strong one.

Indicators 1a till 3a can be exceeded by all selected students. Moderate and strong categories continue up to 3b. Strong students from the control and experimental classes and the student with moderate ability from the experimental class continue until the last indicator. The last, strong students from the control and experimental classes are able to complete the pattern until the 3c with correctly and perfectly. Backward activities of 2a on the 2b were done by all students. Calling activities 2a and 3a on the 3b carried out by students with moderate and strong abilities from the control and experimental classes. While calling 2a and 3a on the 3c was done by the moderate student from the experimental class and two strong students from the control and experimental classes.
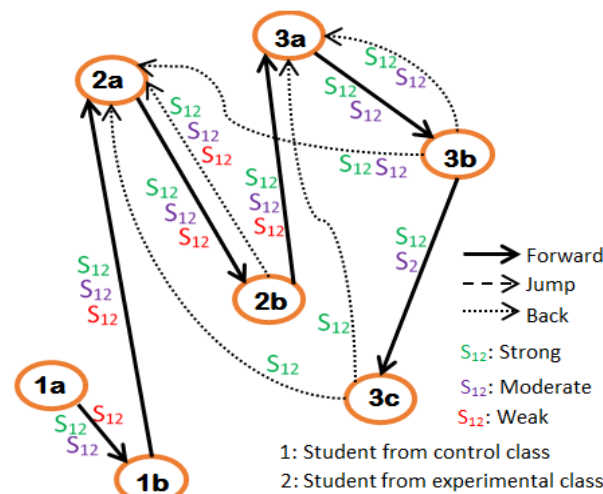

Figure 20. The combination phase portraits of all selected students

The moderate student from the control class gives negative response when the researcher gives trigger directions about the cardinalities, but positive response were given by student from the experimental class. The strong students from the experimental class tried to count until the 10th pattern independently. Based on the both concerns, there is a positive correlation between the implementation of problem-based learning and student motivation in getting the core of completion problem. Problem-based learning implementation in the experimental class influenced student motivation in getting the core of completion problem. The improvement of students' competencies and that the students can be personally responsible are the significant effects of self-assessment [21], thus all tasks that are responsibility of students can be completed properly. Problem-solving included in high level of thinking ability. Therefore, the accompanying impact makes the students were able to adapt and participate actively in 21th century learning better and comprehensive [25].

4. Conclusion

Based on the research findings, it can be revealed that the distribution of the students' combinatorial generalization thinking skills in solving tessellation coloring problems spread over strong, moderate, and weak. The weak students are only able to reach the second level and its indicators. The moderate ones are be able to reach the third level of the second indicators, while the strong students are be able to reach all levels and indicators. So, the implementation of problem-based learning can increase the students' combinatorial generalization thinking skills in solving tessellation coloring pattern problems. These things were proven by the results of the statistical evidences. The evidence of this study is also strengthened by the results of in-depth qualitative analysis.

The phase portraits display illustrates that the students' combinatorial generalization thinking skills in solving tessellation coloring problems forming a logical and linear pattern. The strong students from the both of classes are able to surpass all indicators correctly and perfectly. The moderate student from the control class is only be able to surpass at the indicator counting the cardinalities indicator but still not perfect, while students from the experimental class is able to complete all indicators even though he is not perfect at the last indicator. And the weak students from the control and experimental classes are only able to surpass at the mathematical symbols application indicator.

The suggestions are that the types of students' skills can be improved. The application of approaches, models, strategies, and learning techniques can be done to improve them. Then, the whole support of the learning aspects will influence on the result of the problem-based learning implementation. Lastly, based on the phase portraits discussion, it was recommended to give more attention on indicators that haven't been exceeded by students.

Acknowledgment

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