Error analysis of undergraduate students in solving problems on ring theory

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Abstract. This study aimed to describe the types of error of undergraduate students in solving problems on ring theory. It involved 46 undergraduate students of Mathematics Education, Faculty of Teacher Training and Education, Sebelas Maret University academic year of 2018/2019. The research type used in this study was qualitative with a case study design. Data was gathered through tests and interviews. Validity tests of the data were a triangulation of methods and using reference materials. The subjects were chosen by purposive sampling technique. Consequently, three students selected to be interviewed. Errors found in this study included translation error, concept error, strategy error, and an encoding error. On translation errors, students could not understand the definition of the set and operations used in the questions. While concept and strategy errors were due to student's inability to understand and use mathematical definitions, such as the definitions of group, monoid, ring, and commutative ring. Students were unable to write mathematical definitions in the form of standard mathematical symbols, which indicated they did not understand them well. On encoding errors, students only focused on the final answers or conclusions without having proper reasons.

1. Introduction

Abstract algebra is one abstract branch of mathematics commonly harder to understand than other concrete mathematics branches. Abstract algebra barely talks about calculation and tends to focus on the abstract concept and interrelated facts and principles. Abstract algebra is essential in mathematics itself for being the central role in advanced mathematics and also in other fields, such as computer science, physics, chemistry, and data communications [1].

For undergraduate students, abstract mathematics is relatively harder than computational mathematics [2]. Further, Arikan, Ozkan, and Ozkan [3] concluded that in learning group theory, experiencing difficulties in dealing with abstract concepts is the most important one, whereas some researchers found out that undergraduate students often failed to construct proofs in abstract algebra [1,3,4]. Weber [4] stated that students failed to construct proof because they could not use syntactic knowledge that they had. They preferred to copy rather than think abstractly, which sounds believable that they could memorize the rules of theory without internalizing the descriptions [3]. Therefore, undergraduate students often make mistakes in solving abstract algebra problems.

Waluyo and Sari [2], in their research, stated that students in Indonesia also experience the low learning outcomes of students in an abstract algebra course. Based on the information from undergraduate students of Mathematics Education, Faculty of Teacher Training and Education of Sebelas Maret University, the materials on abstract algebra was hard to understand. From year to year, students have not been able to get satisfactory outcomes on this course. The percentage of students an academic year of 2017/2018 that did remedy on abstract algebra course is 56.76%, with a mean score of 59.27. This data shows that students have a low score, and it indicates that they made many mistakes in solving problems on abstract algebra.

Based on the analysis of the test, many students scored below 60 for ring problems, with a percentage of 70.73%, whereas ring and field are two of the most important mathematics structures on algebra [5]. Basic ideas about ring used to analyze integers and other familiar numbers systems. Other than that, ring plays an important role in the discussion of polynomial and Boolean algebra. The system of the polynomial in the matter of ring is very well-behaving [6]. Besides, Durbin [7] wrote, "In particular, Boolean algebras provide appropriate algebraic settings for formal logic and the theory of computer design."

Finding out the student's learning difficulties is an important first step on leaning, while an error analysis is one kind of diagnostic assessment to know precisely which steps the students having difficulty and misconception [1,2,3]. In order to help students to overcome their difficulties with ring theory, they should know their common errors in solving mathematics problems. To minimalize errors that are made by students during the test and seek alternative solutions so that the same errors do not arise again, it is necessary to analyze student's errors in solving problems on ring theory.

Student's difficulties can be traced by pinpointing their errors during the test in solving mathematical problems. One of the main methods to analyze student's errors is to classify them into categorization [8]. Many researchers proposed categorizations. Radatz, as cited in the research of Muzangwa and Chifamba [8], classified errors in terms of language difficulties, difficulties in processing iconic and visual representation of mathematical knowledge, deficiency in requisite skills, facts, and concepts, incorrect associations or rigidity, and application of irrelevant rules or strategies. Seah [9] classified errors and misconceptions that students may encounter into conceptual error as an inability to comprehend concepts and relationships in problems, procedural error as having a conceptual understanding but failing to perform manipulations or algorithms, and technical error as mathematical knowledge inadequacy and carelessness. The problems of abstract algebra are mostly about proving the statement. Weber [4], categorized mathematical proving errors into three types, failure to invoke syntactic knowledge, insufficient syntactic knowledge base, and logical error.

By examining the above categories and some more studies [1,10,11] about types of errors on different focuses of mathematics, in this study, types of error are categorized into four types, namely translation error, concept error, strategy error, an encoding error. These types of errors are developed and considered appropriate to classify errors that students may encounter while solving problems on ring theory.

Arti Sriarti in Jingga's research [10] defined translation error as an error in changing given information to mathematical sentences and symbols or in deriving the meaning of mathematical sentences and symbols. The translation is an ability to understand an idea and express it in another way from the original statement [13].

Agustyaningrum, Abadi, and Mahmudi [1] defined a conceptual error as the student's error due to misconception or a faulty understanding of the underlying principles and ideas connected to the mathematical problem. These principles and ideas are related to definitions, theorem, and properties in mathematics. Mathematical definition or theorem is said to be formally operable for a given individual if that individual is able to use it in creating or (meaningfully) reproducing a formal argument [14]. Therefore, concept error is an inability to comprehend mathematical definition, theorem, and properties related to the problem given. On the other hand, strategy error is an error in choosing algorithms and procedures, which are related to how to use mathematical definition or theorem to solve mathematical problems.

Encoding error, as stated by Newman, cited by Abdullah, Abidin, and Ali [11], is an error in making a conclusion or error in expressing the final answer. Selden and Selden, cited by Demir, Öztürk, and Güven [15], stated a mathematical proof not only shows whether a statement is valid or invalid but also why it is valid. Meanwhile, Almeida, cited by Demir, Öztürk, and Güven [15] emphasized reasoning is one of the most important components of the process of proof construction. The reasoning is a process of thinking in concluding [16]. Skill in constructing a conclusion requires an understanding of concepts and capabilities connecting the known arguments to learned concepts through systematic and deductive procedures [17]. Therefore, the encoding error is an error in determining the conclusion and also in reasoning the obtained conclusion.

2. Research Method

2.1. Research design
This study used a qualitative paradigm because it was conducted to find out the types of errors made by undergraduate students of Mathematics Education, Faculty of Teacher Training and Education of Sebelas Maret University. The research design used was a case study (research focused on one case only).

2.2. Participants
The subjects for this study comprised 46 undergraduate students academic year of 2018/2019 from abstract algebra course class of Mathematics Education, Faculty of Teacher Training and Education of Sebelas Maret University. The purposive sampling was employed, where the goal was to select samples, who are likely to be "information-rich". The criterion for choosing subjects were students make various mistakes and various types of error, students are minimally affected by researcher, students are able to communicate both verbally and in writing, and students are willing to be interviewed.

2.3. Data collection
In this study, the data was gathered from test and interview. The test was a written essay test, which consisted of 2 questions. Those questions have been validated by two expert judgement. The criterion aspect of the content validity that was carried out consisted of material, language, and construction aspects. The test is as follows.

1. $M_{2\times 2}(\mathbb{R})$ is a set of all 2×2 matrices with real number entries, and completed with the operations of matrix addition and matrix multiplication.

a. Prove that $M_{2\times 2}(\mathbb{R})$ has an identity element under matrix multiplication!

b. Prove that the combination of matrix addition and matrix multiplication in $M_{2\times 2}(\mathbb{R})$ is distributive!

c. Check whether all elements of $M_{2\times 2}(\mathbb{R})$ have inverse under matrix multiplication!

2. Given: Zn is a set of integers modulo n, with n is any positive integer and completed with binary modulo operations of $+_n$ and $\times_n$. If $(Zn,+_n)$ is a group and $(Zn,\times_n)$ is monoid, prove that $(Zn,+_n,\times_n)$ is a commutative ring!

In depth-interview was used to get data to confirm and compare to the written test data using triangulation of methods. Triangulation was done to determine the validity of the research data. If data obtained from the written test and interview tend to be the same or similar then the data can be said valid, whereas if the data from interview gives result that is pretty much different from the written test then the data is said to be invalid and later will be reduced.

2.4. Analysis of data
The data was analyzed referred to Miles and Huberman's opinions, namely data reduction, data display, and verification. Missing responses were excluded from the analysis because student's error can not be identified from blank response. Types of error used in this research were translation error,

concept error, strategy error, and encoding error. To facilitate the analysis, the following indicators of those types of error were formulated in table 1 based on the results of literature studies.

Table 1. Types of Error and Indicators.

Types of Error	Indicators
Translation error	a. Student makes error in giving meaning to a mathematical expression or symbol that is in the question. b. Student makes error in changing information that has been given to mathematical expression or symbol.
Concept error	a. Student makes error in determining the theorem, definition, or properties used to solve the problem. b. Student makes error in writing theorem, definition, or properties to solve the problem.
Strategy error	a. Student uses procedure or method that are not appropriate in solving problem. b. Student makes error in determining information to solve problem.
Encoding error	a. Student makes error in determining conclusion. b. Student makes error in determining the reason for the obtained conclusion.

3. Result

The sampling technique was purposive sampling with 3 students selected as research subjects. Furthermore, they will be mentioned as subjects S1, S2, and S3. Three selected subjects were interviewed to check to the written test data for validation using triangulation of methods. The result analysis of three subjects are shown in table 2.

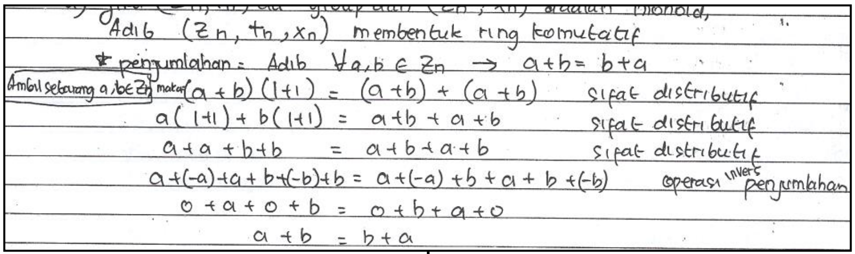
Table 2. The result analysis of students errors.

Types of Error	Indicator	Number			
		1(a)	1(b)	1(c)	2
Translation	(a)	-	-	-	S1 S2 S3
	(b)	S2	S1 S2	S2	S1 S2
Concept	(a)	-	-	-	S1 S2 S3
	(b)	S1 S2 S3	S1 S2 S3	S2	-
Strategy	(a)	-	S2 S3	S1	S1 S2 S3
	(b)	S2	S1 S2	S1 S2	S1 S2
Encoding	(a)	-	-	S1	-
	(b)	S2	S1 S2 S3	S1 S2	S1 S2 S3

Based on table 2, three selected subjects made 9 translation errors, 10 concept errors, 13 strategy errors, and 10 encoding errors. The table 2 shows that the most errors happened in the question number 2 and least errors happened in the question number 1(a).

3.1. Translation error

Translation error for indicator (a) was only made in the question number 2. Their error for this indicator is not being able to understand the definition of the set given in the question, the set *Zn.* This happened to subjects S1 and S3.



Poof:

$(Z_n, +_n, \times n)$ is a commutative ring

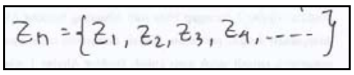
Addition: WTP (Want to prove) $\forall a, b \in Z_n \to a+b=b+a$

Given any $a, b \in Z_n$, then

$$(a+b)(1+1)=(a+b)+(a+b) \quad \textit{distributive property}$$
$$a(1+1)+b(1+1)=a+b+a+b \quad \textit{distributive property}$$
$$a+a+b+b=a+b+a+b \quad \textit{distributive property}$$
$$a+(-a)+a+b+(-b)+b=a+(-a)+b+a+b+(-b) \quad \textit{operation of invers of addition}$$
$$0+a+0+b=0+b+a+0$$
$$a+b=b+a$$

Figure 1. Translation error in the question number 2 of subject S1.

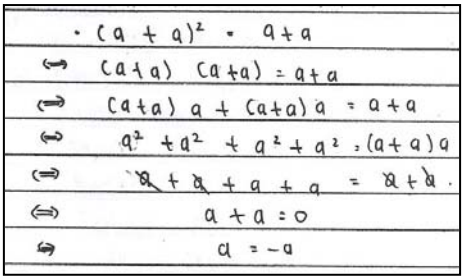
From the figure 1, it can be concluded that subject S1 did not understand what the set *Zn* is. Therefore, she wrote *a* and *b* to be the elements of *Zn,* without knowing what *a* and *b* actually are.



$$Z_n = \{Z_1, Z_2, Z_3, Z_4, ...\}$$

Figure 2. Error made by subject S1 in writing the set Zn.

In the interview, subject S1 could not define the set Zn well. When she was requested to write the elements of Zn, she wrote as shown in figure 2. Other than that, translation error for indicator (a) is not being able to understand the definition of the operation used in the question, which happened to subjects S1 and S2.



$$(a+a)^2 = a+a$$
$$\Leftrightarrow (a+a)(a+a) = a+a$$
$$\Leftrightarrow (a+a)a+(a+a)a = a+a$$
$$\Leftrightarrow a^2+a^2+a^2+a^2 = (a+a)a$$
$$\Leftrightarrow \cancel{a}+\cancel{a}+a+a = \cancel{a}+\cancel{a}$$
$$\Leftrightarrow a+a = 0$$
$$\Leftrightarrow a = -a$$

Figure 3. Translation error in the question number 2 of subject S2.

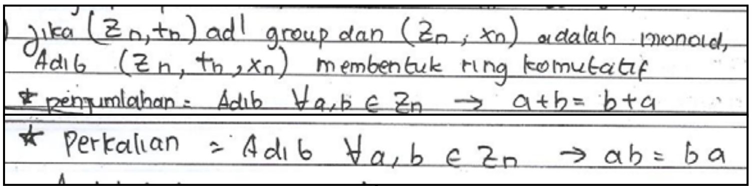
Figure 3 shows that subjects S2 did not understand the definition of the operation used in the question. Subject S2 wrote + sign instead of $+_n$, that was defined in the question. Based on the interview with subject S2, subject S2 could not explain the operations of $+_n$ and $\times_n$. Moreover, when subject S2 was requested to do "concrete" modulo operation, such as $+_4$ and $\times_4$, she did not understand the meaning completely how to operate them in the set Z_4.

Translation errors for indicator (b) were made in all of the questions. Their error for this indicator is ignoring the importance of operation sign, which was made by subjects S1 and S2. Subject S2 tent to often write symbols that were not up to standard. In other words, subject S2 did not realize the importance of writing mathematical symbols according to the standard. Both of them did not realize the importance of writing appropriate operation sign that was defined in the question.

It can be seen from table 2 that subject S2 always made translation error. This resulted errors in the next process. The inabilities of subject S2 for this aspect of translation are failed assimilation process and error in writing standard mathematical symbols, which shown in figure 2. Subject S2 could not perform the assimilation process properly, namely connecting new information with the existing knowledge. Based on the interview with subject S2, she could explain the set $M_{2\times2}(\mathbb{R})$ well, but she failed to process new information that was still related to the set $M_{2\times2}(\mathbb{R})$. This error caused subject S2 to make concept error, strategy error, and an encoding error.

3.2. Concept error

Concept errors for indicator (a) were made by all of the subjects in the question number 2. They did not understand the definition well and ignored the use of the definition to answer the problem. Moreover, they did not really consider its use.

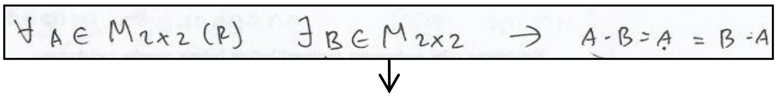

Jika $(Z_n, +_n)$ ad! group dan $(Z_n ; \times_n)$ adalah monoid,
Adib $(Z_n, +_n, \times_n)$ membentuk ring komutatif
* penjumlahan : Adib $\forall a,b \in Z_n \rightarrow a+b = b+a$
* Perkalian : Adib $\forall a,b \in Z_n \rightarrow ab = ba$

If $(Z_n, +_n)$ is a group and $(Z_n, \times_n)$ is a monoid,
WTP (Want to prove) $(Z_n, +_n, \times_n)$ is a commutative ring.
Addition: WTP $\forall a,b \in Z_n \rightarrow a+b = b+a$
Multiplication: WTP $\forall a,b \in Z_n \rightarrow ab = ba$

Figure 4. Concept error in the question number 2 of subject S1.

Figure 4 shows that subject S1 intended to prove commutative property under addition and multiplication, without considering the distributive property needed to prove a commutative ring in the question number 2. Based on the interview, all subjects could not mention or determine the definitions of group, monoid, or commutative ring correctly.

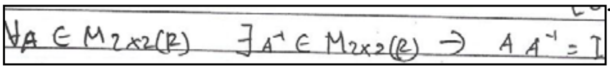
Concept errors for indicator (b) were made by the subjects in the questions number 1(a) and 1(b). In question 1(a), all three subjects made errors in using quantification, while in question number 1(b) the error made was writing an incomplete definition.


$\forall A \in M_{2\times2}(R)$ $\exists B \in M_{2\times2} \rightarrow A \cdot B = A = B \cdot A$

$\forall A \in M_{2\times2}(\mathbb{R}) . \exists B \in M_{2\times2} \rightarrow A.B = A = B.A$

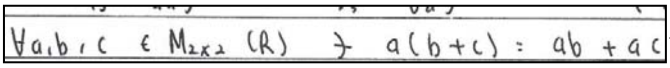
Figure 5. Concept error in the question number 1(a) of subject S1.

Based on the figure 5, subject used the wrong universal and existential quantifier which were reversed. Subject S1 knew what definition used to solve the problem in the question number 1(a) but could not write it correctly in mathematical symbols.



$\forall A \in M_{2\times 2}(\mathbb{R}).\ \exists A^{-1} \in M_{2\times 2}(\mathbb{R}) \rightarrow AA^{-1} = I$

Figure 6. Concept error in the question number 1(c) of subject S1.



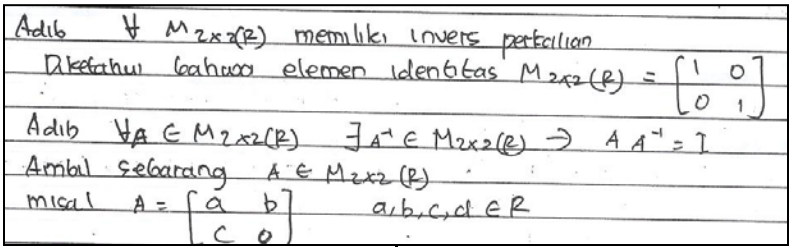
$\forall a,b,c \in M_{2\times 2}(\mathbb{R}) \rightarrow a(b+c) = ab + ac$

Figure 7. Concept error in the question number 1(b) of subject S2 as well as subject S3.

Figure 6 and figure 7 show three subjects wrote incomplete definitions. Other than that, subject S2 had a misconception by assuming that the proof was sufficient to prove distributive property in a ring.

3.3. Strategy error

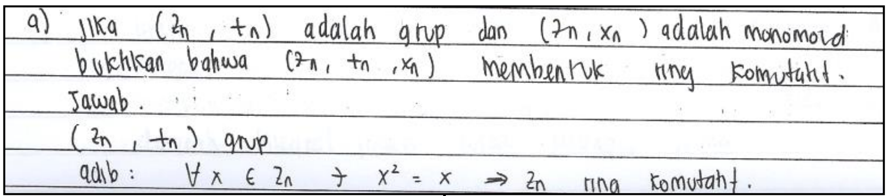
Strategy error for indicator (a) was not arisen from number 1(a). It means that the subjects knew how to solve that problem. On the other hand, the errors made were errors following previous errors, improper use of procedures, beginning with the wrong assumption, proofing unrelated statements, which was made by subjects S1 and S2, and gap between definition and the proof provided, which was done by subject S3.



WTP (Want to prove) $\forall M_{2\times 2}(\mathbb{R})$ has invers of multiplication. Given that identity element $M_{2\times 2}(\mathbb{R}) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. WTP $\forall M_{2\times 2}(\mathbb{R}).\ \exists A^{-1} \in M_{2\times 2}(\mathbb{R}) \rightarrow AA^{-1} = I$.

Given any $A \in M_{2\times 2}(\mathbb{R})$. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $a,b,c,d \in \mathbb{R}$

Figure 8. Strategy error in the question number 1(c) of subject S1.

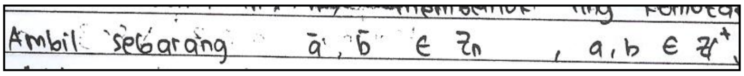


a) If $(Z_n, +_n)$ is a group and $(Z_n, \times_n)$ is a monoid, prove that $(Z_n, +_n, \times_n)$ is a commutative ring.
Answer: $(Z_n, +_n)$ group. WTP (Want to prove): $\forall x \in Z_n \rightarrow x^2 = x \Rightarrow Z_n$ is commutativ e ring

Figure 9. Strategy error in the question number 2 of subject S2.

In figure 8, subject S1 assumed that all elements of that set have an inverse under multiplication and in figure 9, subject S2 tried to proof using improper procedure, which was an unrelated statement.

Strategy errors for indicator (b) were made in all of the questions. The error were choosing the wrong information, not paying attention to the information carefully, and using any random information, which occurred to three of the subjects.


Ambil sebarang $\bar{a}, \bar{b} \in Z_n$, $a, b \in Z^+$

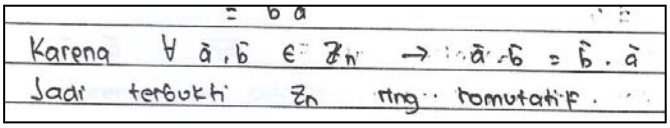
→ Given any $\bar{a},\bar{b} \in Z_n,\ a,b \in Z^+$

Figure 10. Strategy error in the question number 2 of subject S3.

In figure 10, subject S3 did not pay attention to the given information in the question. She wrote $a,b \in Z^+$, when in fact $a,b \in Z$ and $n \in Z^+$.

3.4. Encoding error

Encoding error for indicator (a) was only made by subject S1 in the question number 1(c). The error was arisen because subject S1 had begun with the wrong assumption. Therefore, she made a wrong conclusion. Encoding errors for indicator (b) were occurred due to improper procedures, experienced by subject S1 and S2 (figure 9), gap between the proof process and the final answer (figure 11), and due to the use of incorrect definitions.


= b a
Karena $\forall \bar{a}, \bar{b} \in Z_n \rightarrow \bar{a} \cdot \bar{b} = \bar{b} \cdot \bar{a}$
Jadi terbukti Z_n ring komutatif.

→ Because of $\forall \overline{a},\overline{b} \in Z_n \rightarrow \overline{a}.\overline{b} = \overline{b}.\overline{a}$
So, it was proven that Z_n is commutative ring

Figures 11. Encoding error in the question number 2 of subject S3.

In figure 11, subject S3 concluded that the mathematical system was a commutative ring by proofing commutative property under multiplication without considering commutative property under addition and distributive property that were needed to solve the problem.

4. Discussion

Based on the table 2, all subjects made all types of errors in the question number 2. It has happened because they could not understand the definition of the set and the operations that were given in the question. Related to the set *Zn*, it has happened because students are not familiar with the set. Although in class, the lecturer had explained about that set and used examples of "concrete" sets such as $Z, Z_2, Z_3, \ldots, Z_n$, students seemed not to understand. This is what Titova [18] found in her research that students preferred to dwell on concrete objects and often experienced difficulties if the problem was expressed in a general form. Therefore, they made translation error.

Besides the set used in the question, students found difficulty in the written mathematical symbol. Yetkin [12] revealed that standard written symbols play an essential role in student learning of mathematics, but students may experience difficulties in constructing mathematical meanings of them. Most of the difficulty in understanding symbols came from the fact that in their standard form, written symbols might take on a different meaning in different settings. Therefore, students led to the misuse of mathematical terms and misuse of mathematical symbols [19], which are two of nine commonly committed mathematical writing errors.

In the translation error indicator (b), there is a student with a failed assimilation process. According to Netti, Nusantara, Subanji, Abadyo, and Anwar [20], who concerned the failure to construct proof based on assimilation and accommodation framework from Piaget, it was revealed that students only had a single scheme of a mathematical object, they did not have other schemes of the elements associated with the objects. This caused them not to be able to create or manipulate the object into another form or associate the object with other objects.

The most problematic concept error is that all subjects did not understand the meaning of definitions. In this case, Demir, Öztürk, and Güven [15] referred it as deficiencies of using mathematical language (not aware of mathematical definitions or mathematical theorems and symbols during proof). The tent not to be able to understand the definitions of group, monoid, and commutative ring, which are needed to solve question number 2. Moreover, when they knew the definitions, they could not apply and use them to solve mathematical problems.

Besides not being able to understand the definitions well, they found difficulties in using quantifiers in expressing the definition in the form of mathematical symbols. The inability of students to understand and use quantifiers is in line with the finding of Titova [18], which stated that the concept of abstract algebra was a complex structure so that students made repeated errors of the elements used during problem-solving and one of the elements was quantification. This is considered the reason why students could not apply and use mathematical definitions to solve problems.

Strategy errors came from using incomplete definitions, beginning with the wrong assumption, and using the wrong information. According to Edwards and Ward, in the research of Stavrou [21], students did not understand the role of formal definitions in mathematics. They observed that students did not understand that definition was set and depended on context, whereas students who could provide definitions correctly did not apply them correctly. As stated by Arikan, Ozkan, and Ozkan [3] and Yetkin [12], students are more comfortable to copy rather than think abstractly; therefore, they focus on rote memorization. In this case, students memorized and copied what the lecturer taught in class then applied them in solving “similar” problems. However, this result is not in line with what Stavrou [21] found that standard error in the proof process was using specific examples that were not found in all subjects.

Based on the result of encoding errors, it shows that students only focused on conclusions, without considering whether the answers were correct. As stated by Guce [19], “Students in mathematics class tend not to be mindful of how they would explain, in writing, their solutions to given problems as they think that the teacher focuses only on the correctness of their answers.” Even according to Weber, in the research of Demir, Öztürk, and Güven [15], students are not able to realize whether the proof provided is valid or not.

5. Conclusion

The subjects made four types of errors in each of the questions. They made translation errors due to the inability to understand the definition of the set and operations given in the problems. In addition, they also often wrote mathematical symbols that were not up to the standard. Students need to know what the symbols represent. Besides, they need to learn multiple meanings of the symbols depending on the given problem context. Therefore, they should be provided with a variety of appropriate materials that represent the written mathematical symbols, and they should also be aware of the meaning of mathematical symbols in different problem contexts.

Concept and strategy errors began with not being able to understand and to use mathematical definitions needed to solve problems. Teachers should provide a context to help students bring about their intuitive mathematical concepts and procedures, encourage them to argue whether they are reasonable, and guide them to make connections between their intuitive and formal mathematical concepts and procedures. To help the students learning ring theory subject, they should be a self-study module. Besides, the lecturer is expected to implement remedy using a certain approach, such as diagnostic conflict teaching.

Acknowledgement

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References

[1] Agustyaningrum N, Abadi A M, Sari R N and Mahmudi A 2018 Analysis of Students’ Error in

Solving Abstract Algebra Tasks *J. Phys.: Conf. Ser.* **1097** 012118
[2] Waluyo M and Sari C K 2017 Error Reasoning in Proving Problems in Algebra Structure Aljabar *Scientific Journal of Mathematics Education* **2** 116-21
[3] Arikan E E, Ozkan A and Ozkan E M 2015 An Examination in Turkey: Error analysis f Mathematics students on group theory *Educ. Res. Rev.* **10** 2352-61
[4] Weber K 2001 Student difficulty in constructing proofs: The need for strategic knowledge *Educational Studies in Mathematics* **48** 101-19
[5] Thompson R C and Yaqub A 1970 *Introduction to Abstract Algebra* (Illinois: Scott, Foresman and Company)
[6] Herstein I N 1996 *Abstract Algebra (Third Edition)* (United States: A Viacom Company)
[7] Durbin J R 1979 *Modern Algebra: An Introduction* (Canada: John Wiley & Sons, Inc)
[8] Muzangwa J and Chifamba P 2012 Analysis of Errors and Misconceptions in the Learning of Calculus by Undergraduate Students *Acta Didacta Napocensia* **5** 1-10
[9] Seah E K 2005 Analysis of Students' Difficulties in Solving Integration Problems *The Mathematics Educator* **9** 39-59
[10] Jingga A A, Mardiyana and Setiawan R 2017 Analysis of Student Errors on Solving Trigonometry Identity Problems at Class X Semester 2 Students of SMA Negeri 1 Kartasura in the 2015/2016 School year. *Journal of Mathematics and Mathematics Education* **1** 48-62
[11] Abdullah A H, Abidin N Z L and Ali M 2015 Analysis of Students' Errors in Solving Higher Order Thinking Skills (HOTS) Problems for the Topic of Fraction *Canadian Center of Science Education* **11** 133-42
[12] Yetkin E 2003 Student difficulties in learning elementary mathematics *ERIC* ED482727
[13] Ramdhani M R, Usodo B and Subandi S 2017 Student's mathematical understanding ability based on self-efficacy *J. Phys.: Conf. Ser.* **909** 012065
[14] Bills L and Tall D 1998 Operable Definitions in Advanced Mathematics: The Case of the Least Upper Bound *Proc. PME* vol 22 (Stellenbosch) pp 104-11
[15] Demir E, Öztürk T and Güven B 2018 Examining pre-service mathematics teachers' reasoning errors, deficiencies and gaps in the proof process *European Journal of Science and Mathematics Education* **6** 44-61
[16] Tambunan H 2018 Impact of Heuristic Strategy on Students' Mathematics Ability in High Order Thinking *International Electronic Journal of Mathematics Education* **13** 321-28
[17] Feriyanto 2018 The Ability of Students' Mathematical Proof in Determining the Validity of Argument Reviewed from Gender Differences *J. Phys.: Conf. Ser.* **947** 012042
[18] Titova A S 2007 *Understanding Abstract Algebra Concepts* (Doctoral Dissertations: 362) (Durham: University of New Hampshire)
[19] Guce I K 2017 Mathematical Writing Errors in Expository Writings of College Mathematics Students *International Journal of Evaluations and Research in Education* **6** 233-42
[20] Netti S, Nusantara T, Subanji, Abadyo and Anwar L 2016 The Failure to Construct Proof Based on Assimilation and Accommodation Framework from Piaget *International Education Studies* **9** 12-22
[21] Stavrou S G 2014 Common errors and misconceptions in mathematical proving by education undergraduate *Issues in the Undergraduate Mathematics Preparation of School Teachers: The Journal* **1** ISSN 2165-7874