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Conical dispersions induced interface states in two-dimensional photonic crystals

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Abstract – It is known that interface states are not guaranteed at the interface between two two-dimensional photonic crystals. To create an interface state, it is usual to decorate the structure near the interface. In this work, we construct a photonic crystal in an oblique lattice through tilting from a square lattice. Conical dispersion, existing in the oblique lattice, induces band inversion and results in two band gaps, separated by the projected band structure of it, possessing opposite signs of surface impedance. Therefore, interface states are guaranteed to exist at the interface between the square lattice and its "tilted" partner.

An interface state is characterized by the field localized at the interface between two materials exponentially decaying in the direction perpendicular to the interface. For the field enhancement and subwavelength properties, interface states can be used to guide electromagnetic waves for many interesting applications [1,2]. In homogeneous materials, there exists an analytical condition for the existence of interface states [2]. If the effective medium description is valid, this condition can be easily extended to photonic crystals (PCs) [3]. However, generally, the existence of interface states in PCs is usually verified by full-wave simulations based on a trial-and-error method [1,4–6]. It is well known that at a surface formed by a semi-infinite PC and air, in order to generate surface states, it is needed to cut the cylinders at the boundary [1]. Replacing air with another PC to form a new interface, there are no simple ways to predict the existence of interface states. To create interface states, it is usual to decorate the interface through shifting two PCs laterally along the interface direction or modifying separation between them [4]. It has been shown that interface states can be found in two-dimensional (2D) PCs, which possess Dirac-like cone dispersions in the Brillouin zone center [7]. Interface states are also found at the boundary between a 2D PC and its "inverted" partner [8]. Recently, Hang's group realized interface states in 2D PCs in two different ways: one is using a PC and its partner with translating each unit cell by half a lattice constant [9], the other is using two PCs with different radii and relative permittivities [10]. All of these interface states are resulted from different geometric phases of the bulk band structures of the two PCs at the interface. Inspired by the topological edge states in electronic systems [11,12], topological surface/interface states in photonic systems have attracted much attention [13–29]. In this work, we report that interface states can be found in a simple way in 2D PCs at the boundary formed by a PC in a square lattice and its "tilted" partner. These interface states are different from those reported in refs. [7–10], in which different materials or different geometries of scatterers are applied. Here, only the same material and geometry of scatterers are needed, which helps the sample fabrication and experimental measurement. These interface states are determined by the conical dispersion of the oblique lattice, which leads to the band inversion for varying the wave vector across the Dirac point. This induces the π Zak phase jump. As a consequence, the band gaps separated by the Dirac cone should have opposite signs of imaginary parts of the surface impedance.

In a 2D PC with a square lattice, only considering the monopole and dipole modes, as the C_{4v} symmetry, the bulk band structure possesses a quadratic dispersion at the M point [30]. Through tilting the square lattice to an oblique lattice for maintaining the side length of the unit cell, the first Brillouin zone is changed to a hexagon as shown in fig. 1(b). The tilted angle is θ and the lattice constant is a as displayed in fig. 1(a). As a result, the

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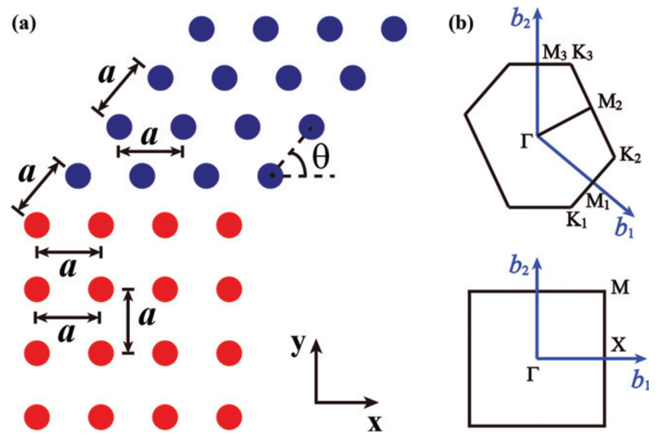

Fig. 1: (a) The schematic of an interface formed by two 2D photonic crystals in a square lattice and an oblique lattice. a is the lattice constant of both the two lattices. θ is the tilted angle of the oblique lattice. (b) The first Brillouin zones of a square lattice (bottom) and an oblique lattice (top).

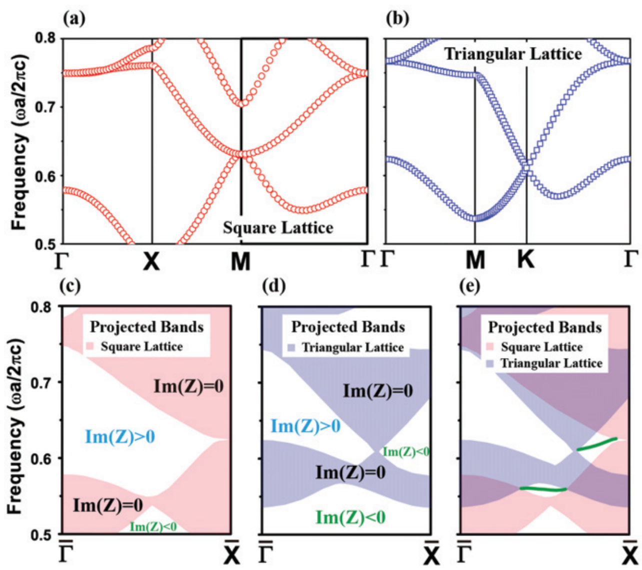

Fig. 2: The bulk band structures in the transverse electric polarization for the PCs with a square lattice (a) and an oblique lattice with $\theta = 60°$ (b). (c)–(d) The projected band structures of these two PCs. (e) The interface wave dispersions at the interface. The relative permittivity and permeability of the PCs are $\varepsilon = 10, \mu = 1$. The radius of the cylinder is $R = 0.15a$. The green line represents the interface wave dispersions.

quadratic dispersion will evolve to a pair of conical dispersions along the (ΓK_1)- or (K_3M_2)-direction. To specify this point, we first calculate the bulk band structure of a triangular lattice with $\theta = 60°$ in transverse electric polarization (electric field perpendicular to the plane) as shown in fig. 2(b). Meanwhile, the bulk band structure for a square lattice is plotted in fig. 2(a). The relative permittivity, permeability and radii of the cylinders are $\varepsilon = 10, \mu = 1, R = 0.15a$. Conical dispersion is at the K-point in the triangular lattice, and the quadratic dispersion is at the M point in the square lattice. The interface along the x-direction constructed by these two PCs is schematically demonstrated in fig. 1(a).

Then, we will study the existence of interface states at this interface. The condition for the existence of interface states is described by $Z_U(\omega, k_{||}) + Z_L(\omega, k_{||}) = 0$, where $Z_{U(L)}(\omega, k_{||})$ is the surface impedance of the PC on the upper (lower) side for certain ω and $k_{||}$ [31,32]. In lossless materials, $Z(\omega, k_{||})$ is real or pure imaginary, $Z(\omega, k_{||})$ is real, *i.e.*, $\mathrm{Im}(Z(\omega, k_{||})) = 0$, for the pass band, and $Z(\omega, k_{||})$ is pure imaginary inside a gap. To study the existence of interface states, the necessary condition is the existence of a common band gap, then the projected bands along the interface ($\overline{\Gamma}\,\overline{X}$)-direction are calculated in figs. 2(c) and (d). The purple and blue regions represent the pass band with $\mathrm{Im}(Z(\omega, k_{||})) = 0$. The white ones are for band gaps with $\mathrm{Im}(Z(\omega, k_{||})) > 0$ or $\mathrm{Im}(Z(\omega, k_{||})) < 0$. There are two band gaps in the PC with a square lattice, the lower gap with $\mathrm{Im}(Z(\omega, k_{||})) < 0$, and the upper one with $\mathrm{Im}(Z(\omega, k_{||})) > 0$. Whereas there are three gaps in the PC with a triangular lattice. The lower gap is also with $\mathrm{Im}(Z(\omega, k_{||})) < 0$, while in the upper frequency region, there are two gaps separated by the projected bands of a Dirac cone. The key point is that $\mathrm{Im}(Z(\omega, k_{||}))$ carries opposite signs in these two gaps. It should be noted that for a given $k_{||}$, $\mathrm{Im}(Z(\omega, k_{||}))$ decreases monotonically from 0 to $-\infty$ in the gap with $\mathrm{Im}(Z(\omega, k_{||})) < 0$ accompanying with frequency increasing, whereas $\mathrm{Im}(Z(\omega, k_{||}))$ decreases monotonically from $+\infty$ to 0 in the gap with $\mathrm{Im}(Z(\omega, k_{||})) > 0$ [7,8]. Therefore, as long as there is a common gap with opposite signs of $\mathrm{Im}(Z(\omega, k_{||}))$, the interface wave condition should be satisfied. Considering the frequency region near $0.65c/a$, there are two common gaps between these two PCs. $\mathrm{Im}(Z(\omega, k_{||})) > 0$ is for the square lattice, while both $\mathrm{Im}(Z(\omega, k_{||})) > 0$ and $\mathrm{Im}(Z(\omega, k_{||})) < 0$ are for the oblique lattice. As a consequence, the interface wave condition is satisfied for the band gap on the right side with larger $k_{||}$. In addition, the interface wave condition is also satisfied in the lower common gap near $0.525c/a$. Two interface wave dispersions are shown in fig. 2(d) denoted by the green lines.

The interface wave dispersions are obtained through the full-wave simulations. Actually, the existence of interface waves can be predicted by the properties of the bulk bands. It is shown that the ratio of the signs of surface impedance $\mathrm{Im}(Z(\omega, k_{||}))$ in two adjacent gaps is related to the Zak phase φ_{i-1} of the band [33] in between in the following equation [7,8,34]:

$$\frac{\mathrm{Sgn}[\mathrm{Im}[Z_i(\omega, k_{||})]]}{\mathrm{Sgn}[\mathrm{Im}[Z_{i-1}(\omega, k_{||})]]} = e^{i(\varphi_{i-1}+\pi)}.$$

The Zak phase of the i-th band is illustrated in fig. 3 for the oblique lattice with different k_x. As the Zak phase is dependent on the choice of the origin, the origin for the calculation is demonstrated in the inset of

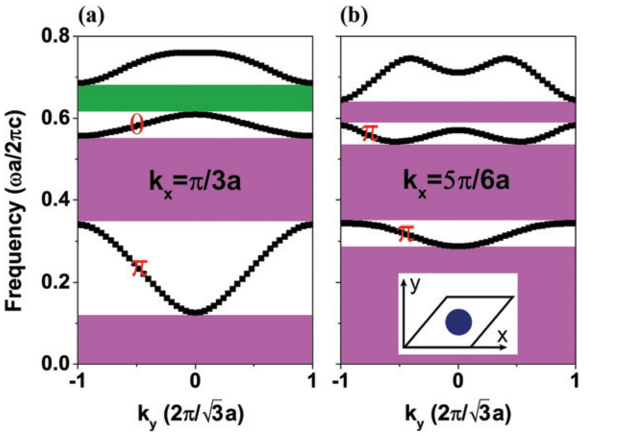

Fig. 3: The bulk band structures with a fixed $k_x = \pi/3a$ (a) and $k_x = 5\pi/6a$ (b) for the PC in a triangular lattice. The Zak phase of the bulk band is either 0 or π labeled with red color. The purple and green regions denote different signs of $\mathrm{Im}(Z)$, purple color for $\mathrm{Im}(Z) < 0$, green one for $\mathrm{Im}(Z) > 0$. The inset denotes the coordinate for calculating the Zak phase. The parameters of the PC is the same as those in fig. 2.

fig. 3(b), which is consistent with the boundary of the interface as shown in fig. 1. As the Dirac point locates at $(k_x, k_y) = (2\pi/3a, 2\pi/\sqrt{3}a)$, for $k_x = 2\pi/3a$, the band gap between the second and third bands as shown in fig. 3 will close. Thus, the linear dispersion of the Dirac point leads to the band inversion of the second and third bands for $k_x < 2\pi/3a$ and $k_x > 2\pi/3a$. This gives rise to the π Zak phase jump of the second bands for $k_x = \pi/3a$ and $k_x = 5\pi/6a$ as shown in fig. 3. The sign of $\mathrm{Im}(Z(\omega, k_{||}))$ in the lowest gap is always negative as in the long wavelength limit the effective medium is valid to determine the sign of $\mathrm{Im}(Z(\omega, k_{||}))$. Based on the equation for the relation between $\mathrm{Im}(Z(\omega, k_{||}))$ and the Zak phase, we can obtain $\mathrm{Im}(Z(\omega, k_{||})) < 0$ and $\mathrm{Im}(Z(\omega, k_{||})) > 0$ for the second and third gaps with $k_x = \pi/3a$, respectively (fig. 3(a)), while all the three gaps are with $\mathrm{Im}(Z(\omega, k_{||})) < 0$ for $k_x = 5\pi/6a$ as shown in fig. 3(b). Obviously, the third gaps have opposite signs of $\mathrm{Im}(Z(\omega, k_{||}))$ for two different k_x on the two sides of the Dirac point. The Zak phases of the lowest two bands for the PC with a square lattice are the same as those in fig. 3(a). Therefore, at the interface formed by the PCs with square and oblique lattices, in the common gap, the interface states should appear in the right side gap possessing opposite signs of $\mathrm{Im}(Z(\omega, k_{||}))$. This explanation is consistent with the full-wave simulation in fig. 2.

In the previous discussion, we choose a PC in a triangular lattice, which is a well-known structure possessing conical dispersion at the high symmetry point [35]. In fact, for the fixed size of the cylinders, the tilted angle θ can be varied from 20° to 90°. Our results discussed above can be fulfilled to other tilted angles in this large tilted angle range. For instance, we calculate the bulk band structure of an oblique lattice with $\theta = 70°$ as shown in fig. 4(a).

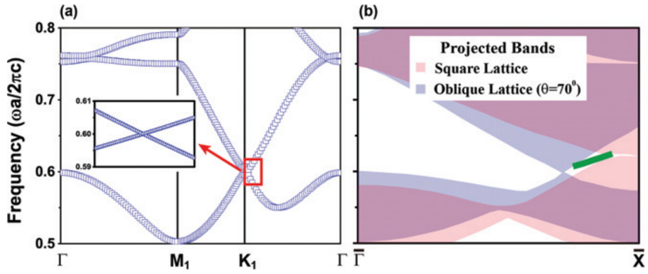

Fig. 4: (a) Bulk band structure of a PC in an oblique lattice with $\theta = 70°$. (b) Projected band structures and interface wave dispersion of an interface formed by two PCs in a square lattice and an oblique lattice. The green line represents the interface wave dispersion. The other parameters of the two PCs are the same as in fig. 2.

There is also conical dispersion in the ΓK_1-direction. The inset demonstrates the enlarged region near the K_1-point. To find the interface states, the projected band structures for two PCs in the square and oblique lattices are plotted in fig. 4(b). The green line denotes the dispersion of interface states. $\mathrm{Im}(Z(\omega, k_{||}))$ in the gaps have the same distributions as that shown in fig. 2 for the triangular lattice. The left gap has $\mathrm{Im}(Z(\omega, k_{||})) > 0$, while $\mathrm{Im}(Z(\omega, k_{||})) < 0$ in the right gap. Therefore, in the right gap, the sign of $\mathrm{Im}(Z(\omega, k_{||}))$ in the oblique lattice is opposite to that of the square lattice. This guarantees the existence of interface states.

In summary, we demonstrate a simple way to create interface states at the interface formed by two PCs with one PC in a square lattice and another one being its "tilted" partner. Based on the current fabrication and measurement technologies [8–10], our idea can be experimentally realized at microwave and even near-infrared frequencies. This way can be used to create interface waves for propagating electromagnetic waves for communication. In addition, this method considered here can be extended to other classical wave systems.

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