IOP Publishing Journal of Physics G: Nuclear and Particle Physics

J. Phys. G: Nucl. Part. Phys. **47** (2020) 045003 (11pp) https://doi.org/10.1088/1361-6471/ab6af6

Investigating the effects of the phenomenological parameters changes in the final state interaction

Behnam Mohammadi[1] and Akbar Abdisaray

Department of Physics, Urmia University, Urmia, Iran

E-mail: be.mohammadi@urmia.ac.ir and ak.abdi@urmia.ac.ir

Received 25 June 2019, revised 9 January 2020
Accepted for publication 13 January 2020
Published 27 February 2020

Abstract

In this research, the decay of $B_c^+ \to B_s^0\pi^+$ is studied in two stages. In the first step, the QCD factorization (QCDF) approach is considered in the initial evaluation, the result of calculation is $\mathcal{B}(B_c^+ \to B_s^0\pi^+)_{\mathrm{QCDF}} = (16.52 \pm 2.21)\%$. While the available experimental result for this decay is $(\sigma(B_c^+)/\sigma(B_s^0)) \times \mathcal{B}(B_c^+ \to B_s^0\pi^+) = (2.37 \pm 0.31 \pm 0.11) \times 10^{-3}$, by applying the theoretical value of the $(\sigma(B_c^+)/\sigma(B_s^0))$ that span the range of [0.9, 8.0]%, the result for QCDF approach becomes $(\sigma(B_c^+)/\sigma(B_s^0))$ ** $\times\mathcal{B}(B_c^+ \to B_s^0\pi^+)_{\mathrm{QCDF}} = (1.49 \pm 0.20) \times 10^{-3} \sim (13.22 \pm 1.77) \times 10^{-3}$ and the branching ratio in the experimental observation is obtained in the range of 2.57%–29.76% which is a large range. Therefore, it is decided to calculate the theoretical branch ratio by applying the effects of the final state interaction (FSI) through only possible cross section channel. In this process, before the meson B_c^+ decays into two final mesons, $B_s^0\pi^+$, first, it decays into two intermediate mesons (as $B^+\bar{K}^0$), then these two intermediate mesons are converted into two final mesons by the exchange of another meson, (such as K^+ corresponding to $B^+\bar{K}^0$). The FSI effects are highly sensitive to the phenomenological parameters that appear in the form factor relationship, so that in most calculations, change the three units in the numeric value of this parameter changes the final result ten times. Therefore, the decision to use FSI is not unexpected. In this study, there are four intermediate states in the cross section channel in which the amplitude of each of them are calculated separately and included in the final amplitude. Considering $\sigma(B_c^+)/\sigma(B_s^0) = 0.90\%$, the numerical value of the $\sigma(B_c^+)/\sigma(B_s^0) \times \mathcal{B}(B_c^+ \to B_s^0\pi^+)$ is from $(1.54 \pm 0.20) \times 10^{-3}$ to $(2.89 \pm 0.41) \times 10^{-3}$ for which obtained by

[1] Author to whom any correspondence should be addressed.

entering the QCDF approach and FSI effects ($\eta = 0.5$–1.5). It should be noted that by choosing the value of the η according to the mass of the exchange meson, as $\eta = 1.5$ for exchange meson of B^* (or B) and $\eta = 0.5$ for exchange meson of K^* (or K) the obtained result is $\sigma(B_c^+)/\sigma(B_s^0) \times \mathcal{B}(B_c^+ \to B_s^0\pi^+) = (1.98 \pm 0.32) \times 10^{-3}$, that is in very good agreement with the experimental result.

Keywords: B meson decays, factorization, final state interaction

(Some figures may appear in colour only in the online journal)

1. Introduction

The weak decay channels of B_c^+ meson can occur in three classes: (1) c weak decay modes: $c \to (s, d)W^+$, in these transitions b-quark plays the role of the spectator, more than 70% of the B_c^+ width is due to c-quark decays, in which $c \to s$ transition has been observed with $B_c^+ \to B_s^0\pi^+$ decays [1]. (2) $\bar{b}$ weak decay modes: $\bar{b} \to (\bar{c}, \bar{u})W^+$, with c-quark as a spectator, around 20% of B_c^+ meson decay width is due to the b-quark decays [2]. (3) Pure weak annihilation channels, the $\bar{b}c \to W^+ \to \bar{q}q$ annihilation amplitudes account for only 10% of the B_c^+. In the study of the B_c^+ decay, all three classes are important.

The $B_c^+ \to B_s^0\pi^+$ decay has been observed by LHCb collaboration with a statistical significance of 5.1 standard deviations. They have been obtained a measurement of the branching fraction multiplied by the production rates for B_c^+ relative to B_s^0 mesons in the LHCb acceptance as [1]:

$$\frac{\sigma(B_c^+)}{\sigma(B_s^0)} \times \mathcal{B}(B_c^+ \to B_s^0\pi^+) = (2.37 \pm 0.31(\mathrm{stat}) \pm 0.11(\mathrm{syst})^{+0.17}_{-0.13}(\tau_{B_c^+})) \times 10^{-3}. \tag{1}$$

The ratio of the production cross-sections of the B_c^+ and B^+ mesons, $\sigma(B_c^+)/\sigma(B_s^0)$, can be get from the measurement involving another charmonium mode, $\sigma(B_c^+)/\sigma(B^+) \times \mathcal{B}(B_c^+ \to J/\psi\pi^+) = (7.0 \pm 0.3) \times 10^{-6}$ obtained from [3]. Although there is no experimental result for the $B_c^+ \to J/\psi\pi^+$ decay, but acceptable values have been calculated for it [4, 5]. So that the CMS [6] and LHCb [7, 8] collaborations have been used these theoretical values in their experimental estimates. Using the predictions listed in [4] for $\mathcal{B}(B_c^+ \to J/\psi\pi^+)$, which span the range $(0.34$–$2.91) \times 10^{-3}$, $\sigma(B_c^+)/\sigma(B^+) \sim [0.23, 2.15]\%$ is obtained [8]. Using the additional relation $\sigma(B_s^0)/\sigma(B^+) = 0.258 \pm 0.016$ quoted also in [8], it follows that the ratio of interest $\sigma(B_c^+)/\sigma(B_s^0)$ lies in the range [0.9, 8.0]%. So the branching fraction in the experimental observation becomes:

$$\mathcal{B}(B_c^+ \to B_s^0\pi^+) = (2.96 \pm 0.39)\% \sim (26.33 \pm 3.44)\%, \tag{2}$$

in this case its width will be from 2.57% to 29.76%. In the present work we calculate the branching fraction of the $B_c^+ \to B_s^0\pi^+$ decay by using the QCD factorization approach and effects of the FSI, the amount obtained from the first method is $(16.52 \pm 2.21)\%$ in which located in the middle of the above range. Entering the FSI effects the values are obtained in the range of $(16.92 \pm 2.26)\%$ to $(32.07 \pm 4.54)\%$, which are in good agreement with the range obtained in the equation (2). On the hand, for direct comparison with the experimental value, we multiply our predicted branching fraction by the estimates for $\sigma(B_c^+)/\sigma(B_s^0) \sim [0.9, 8.0]\%$ and get comparable result.

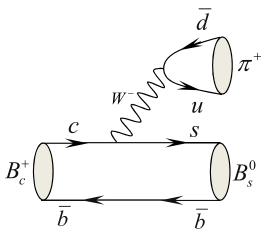

Figure 1. The Feynman diagram contributing to the $B_c^+ \to B_s^0 \pi^+$ decay.

2. Short-distance contributions

In the QCD factorization (QCDF) approach, the B_c^+ meson decays to $B_s^0\pi^+$ mesons via $c \to s$ transition, in this case there is a tree level a_1 Wilson coefficient that has the dominant contribution. Feynman diagram in figure 1 clearly shows this process. So the amplitude reads

$$A(B_c^+ \to B_s^0\pi^+)_{\text{QCDF}} = \mathrm{i}\frac{G_F}{\sqrt{2}} a_1 V_{cs} V_{ud}^* f_\pi (m_{B_c^+}^2 - m_{B_s^0}^2) F_0^{B_c^+ \to B_s^0}(m_{\pi^+}^2), \tag{3}$$

where $a_1 = c_1 + c_2/3$, f_π is the decay constant of the pion and $F_0^{B_c^+ \to B_s^0}(m_{\pi^+}^2)$ is the $B_c^+ \to B_s^0$ transition form factor which has been obtained in the $m_{\pi^+}^2$ region. The form factors for $c \to u$, d and s transitions are calculated via [9]

$$F(q^2) = \frac{F(0)}{1 - \left(\frac{q}{m_{fit}}\right)^2 + \delta\left(\frac{q}{m_{fit}}\right)^4}, \tag{4}$$

where $F(0) = 0.73 \pm 0.03$, $m_{\text{fit}} = 1.77 \pm 0.22$ and $\delta = 0.60 \pm 0.18$ for $F_0^{B_c^+ \to B_s^0}(m_{\pi^+}^2)$ form factor. The branching fraction of $B_c^+ \to B_s^0\pi^+$ in B_c^+ meson rest frame can be written as

$$\mathcal{B}(B_c^+ \to B_s^0\pi^+)_{\text{QCDF}} = \frac{\tau_{B_c^+}}{8\pi} \frac{|\vec{p}|}{m_{B_c^+}^2} |A(B_c^+ \to B_s^0\pi^+)_{\text{QCDF}}|^2, \tag{5}$$

in which $|\vec{p}|$ is the absolute value of the 3-momentum of the B_s^0 or π^+ mesons that can be calculated via: $\sqrt{(m_{B_c^+}^2 + m_{B_s^0}^2 - m_{\pi^+}^2)^2 - 4m_{B_c^+}^2 m_{B_s^0}^2} \Big/ (2m_{B_c^+})$ and $\tau_{B_c^+}$ is the lifetime of the B_c^+ meson. We obtain the following value for the branching ratio

$$\mathcal{B}(B_c^+ \to B_s^0\pi^+)_{QCDF} = (16.52 \pm 2.21)\%, \tag{6}$$

which is located in the middle of the branching fraction value in the experimental observation. It seems that, this decay mode requires a stronger model, a newer method or the effect of natural truth, which can compensate for this major difference. This is the effects of the FSI. Effect of FSI is a natural effect that has a significant contribution to some decays, especially in the $B_c^+ \to B_s^0\pi^+$ decay. This effect states that, before the final mesons are produced, other mesons may be produced along the way in the intermediate modes. In this case, two mesons produced in the middle state exchange another meson then they become the final two mesons. The Feynman graphs determine the type of intermediate and exchange mesons. These diagrams are presented in three different types for the quark model in FSI, (I) s-channel (II) t-channel (III) cross section channel. In B decays, in contrast to D decays, the resonant FSI will be expected to be suppressed relative to the rescattering effect arising from quark exchange owing to the lack of the existence of resonances at energies close to the B meson

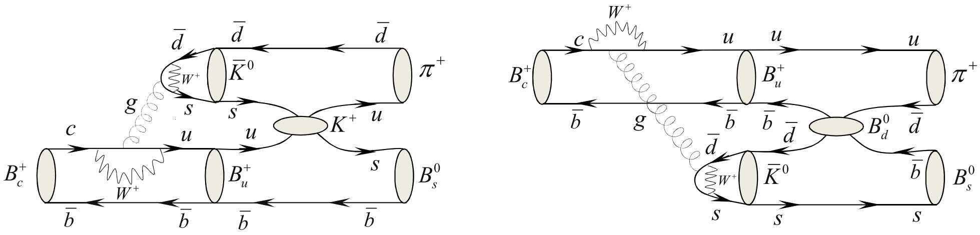

Figure 2. Contributions to $B_c^+ \to B_s^0\pi^+$ from the color-allowed weak decay $B_u^+\bar{K}^0$ followed by quark exchange.

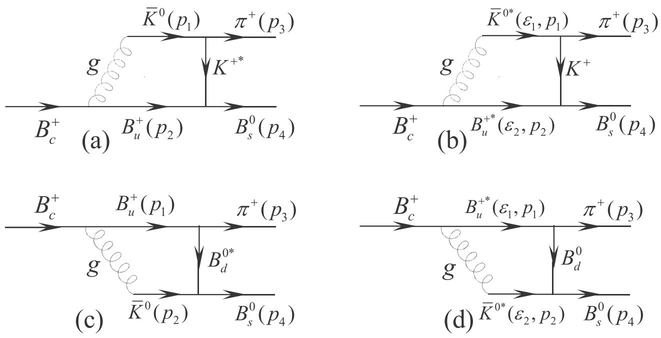

Figure 3. The long-distance cross section channel contributions to the color-allowed penguin amplitude in $B_c^+ \to B_s^0\pi^+$.

mass, this means that one can neglect the s-channel contribution. In the t-channel, two final mesons, $B_s^0(s\bar{b})$ and $\pi^+(u\bar{d})$ should share their similar quark that have the same flavor, but they do not have the same quark. In fact this process is only done with the cross section channel. In the cross section channel, each of the two final mesons exchanges a quark (these quarks do not have the same flavors) crosswise with the intermediate mesons. For the $B_c^+ \to B_s^0\pi^+$ decay, final state mesons of B_s^0 and π^+ exchange the s and u quarks with the $\bar{K}^0$ and B^+ mesons in the intermediate states, respectively. In this case the K^+ meson can participate as a meson of exchange. In the another process the final state mesons of B_s^0 and π^+ can be exchanged the $\bar{b}$ and $\bar{d}$ anti-quarks, respectively. Wherever the intermediate state mesons are the same as before ($\bar{K}^0$ and B^+) and the exchange meson become B^0. These processes are shown in figure 2. In general, the amplitude of the $B_c^+ \to B_s^0\pi^+$ decay, using the FSI effects in cross section channel, is obtained from calculating the following four decay amplitudes:

$$
\begin{aligned}
&B_c^+ \to B^+\bar{K}^0 \to B_s^0\pi^+ \text{ (exchange mesons are: } K^{+*} \text{ and } B^{0*}\text{)},\\
&B_c^+ \to B^{+*}\bar{K}^{0*} \to B_s^0\pi^+ \text{ (exchange mesons are: } K^+ \text{ and } B^0\text{)}.
\end{aligned}
\tag{7}
$$

3. Long-distance contributions

The diagrams which determine the long-distance FSI contributions on the rate of $B_c^+ \to B_s^0\pi^+$ decay are depicted in figure 3. The amplitude of the mode

$B_c^+ \to \bar{K}^0(p_1)B_u^+(p_2) \to \pi^+(p_3)B_s^0(p_4)$ with the exchange of the K^{+*}, it follows that the absorptive part of figure 3(a) is given by

$$
\begin{aligned}
Abs(3a) &= \frac{1}{2}\int \frac{\mathrm{d}^3\vec{p}_1}{2E_1(2\pi)^3}\frac{\mathrm{d}^3\vec{p}_2}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_{B_c^+} - p_1 - p_2)A(B_c^+ \to B_u^+\bar{K}^0)\\
&\quad \times (-\mathrm{i}g_{K^*K\pi})(\epsilon_{K^{+*}}.p_1)(-\mathrm{i}g_{B_sBK^*})(\epsilon_{K^{+*}}.p_4)\frac{F^2(q^2, m_{K^{+*}}^2)}{T_a}\\
&= -\frac{(g_{B_sBK^*})(g_{K^*K\pi})A(B_c^+ \to B_u^+\bar{K}^0)}{16\pi m_{B_c^+}}\int_{-1}^{1}|\vec{p}_1|\mathrm{d}(\cos\theta)\frac{F^2(q^2, m_{K^{+*}}^2)}{T_a}H_a,
\end{aligned} \tag{8}
$$

where

$$
A(B_c^+ \to B_u^+\bar{K}^0) = \mathrm{i}\frac{G_F}{\sqrt{2}}a_3V_{cb}V_{ub}^*f_K(m_{B_c^+}^2 - m_{B_u^+}^2)F_0^{B_c^+\to B_u^+}(m_K^2), \tag{9}
$$

the form factor of the $F_0^{B_c^+\to B_u^+}(m_K^2)$ is calculated via equation (4) by applying the values of the $F(0) = 0.63 \pm 0.04$, $m_{\text{fit}} = 1.52 \pm 0.20$ and $\delta = 0.52 \pm 0.13$. In equation (8) θ is the angle between $\vec{p}_1$ and $\vec{p}_3$, q and m_i are the momentum and mass of the exchange K^{+*} meson, respectively, and

$$
\begin{aligned}
H_a &= -p_1 \cdot p_4 + \frac{(m_1^2 - p_1 \cdot p_3)(m_4^2 - p_2 \cdot p_4)}{m_{K^{+*}}^2},\\
T_a &= q^2 - m_i^2 = m_{\bar{K}^0}^2 + m_{\pi^+}^2 - m_{K^{+*}}^2 - 2E_{\bar{K}^0}E_{\pi^+} + 2|\vec{p}_{\bar{K}^0}||\vec{p}_{\pi^+}|\cos\theta,
\end{aligned} \tag{10}
$$

and $F(q^2, m_i^2)$ is the form factor defined to take care of the off-shell character of the exchange particles, defined as [10, 11]

$$
F(q^2, m_i^2) = \left(\frac{\Lambda^2 - m_i^2}{\Lambda^2 - q^2}\right)^n. \tag{11}
$$

The form factor (i.e. $n = 1$) normalized to unity at $q^2 = m_i^2$. m_i and q are the physical parameters of the exchange particle and Λ is a phenomenological parameter. It is obvious that for $q^2 \to 0$, $F(q^2, m_i^2)$ becomes a number. If $\Lambda \gg m_i$ then $F(q^2, m_i^2)$ turns to be unity, whereas, as $q^2 \to \infty$ the form factor approaches to zero and the distance becomes small and the hadron interaction is no longer valid. Since Λ should not be far from the m_i and q, we choose

$$
\Lambda = m_i + \eta\Lambda_{\text{QCD}}, \tag{12}
$$

where η is the phenomenological parameter that its value in the form factor is expected to be of the order of unity and can be determined from the measured rates. According to the exchanged mesons, variable used values from 0.5 to 5 can be found for it. In [12] the exchanged mesons are D and D^*, so the authors have chosen $\eta = 0.5$–3.0. However, the authors of [13] with the same exchanged mesons have fixed $\eta \sim 5$. In this regard, in the [14], the value of the 4 has been fixed for this phenomenological parameter. In this work since the exchanged mesons in figure 2 are B_d^0 and K^+ (heavy and light mesons), to select the values of η from 0.5 until 1.5, we follow the [15] in which their exchanged mesons are both heavy and light. On the other hand, in the [10] the value of the η is selected corresponding to the mass of the meson exchanged: $\eta = 2.2$ for the exchanged particle D^* (or D) and $\eta = 1.1$ for ρ (or π). This type of choice is also applied in this work, as $\eta = 1.5$ is selected for exchange meson B (or B^*) and $\eta = 0.5$ for K (or K^*). The result of this selection is presented separately.

Likewise, for diagram 3(b), the amplitude of the $B_c^+ \to \bar{K}^{0*}(\epsilon_1, p_1)B_u^{+*}(\epsilon_2, p_2) \to \pi^+(p_3)B_s^0(p_4)$ (where K^+ is exchanged particle) is given by

$$\begin{aligned}
Abs(3b) = & -\mathrm{i}\frac{G_F}{4} f_{\bar{K}^{0*}} m_{\bar{K}^{0*}} a_3 V_{cb} V_{ub}^* \int \frac{\mathrm{d}^3\vec{p}_1}{2E_1(2\pi)^3} \frac{\mathrm{d}^3\vec{p}_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_{B_c^+} - p_1 - p_2) \\
& \times (-\mathrm{i}g_{K^*K\pi})(2\epsilon_1.p_3)(-\mathrm{i}g_{B^*B_sK})(\epsilon_2.p_4) \frac{F^2(q^2, m_{K^+}^2)}{T_b} \\
& \times \left\{ (\epsilon_1^*.\epsilon_2^*)(m_{B_c^+} + m_{B_u^{+*}}) A_1^{B_c^+B_u^{+*}}(m_{\bar{K}^{0*}}^2) - (\epsilon_1.p_{B_c^+})(\epsilon_2.p_{B_c^+}) \frac{2A_2^{B_c^+B_u^{+*}}(m_{\bar{K}^{0*}}^2)}{m_{B_c^+} + m_{B_u^{+*}}} \right\} \\
= & \, \mathrm{i}\frac{G_F}{32\pi m_{B_c^+}} g_{B^*B_sK} g_{K^*K\pi} f_{\bar{K}^{0*}} m_{\bar{K}^{0*}} a_3 V_{cb} V_{ub}^* \int_{-1}^{1} |\vec{p}_1| \mathrm{d}(\cos\theta) \frac{F^2(q^2, m_{K^+}^2)}{T_b} \\
& \times \left\{ (m_{B_s^0} + m_{B_u^{+*}}) A_1^{B_c^+B_u^{+*}}(m_{\bar{K}^{0*}}^2) H_b - \frac{2A_2^{B_c^+B_u^{+*}}(m_{\bar{K}^{0*}}^2)}{m_{B_c^+} + m_{B_u^{+*}}} H'_b \right\},
\end{aligned} \tag{13}$$

where the form factor of the $A_1^{B_c^+ \to B_u^{+*}}(m_{K^{0*}}^2)$ is calculated via equation (4) by applying the values of the $F(0) = 0.43 \pm 0.01$, $m_{\rm fit} = 1.16 \pm 0.07$ and $\delta = 0.27 \pm 0.03$. The form factors of $A_1^{B_c^+ \to B_u^{+*}}(m_{K^{0*}}^2)$ and $A_2^{B_c^+ \to B_u^{+*}}(m_{K^{0*}}^2)$ are related to each other via:

$$A_2^{B_c^+ \to B_u^{+*}}(m_{K^{0*}}^2) = \frac{m_{B_c^+} + m_{K^{0*}}}{m_{B_c^+} - m_{K^{0*}}} A_1^{B_c^+ \to B_u^{+*}}(m_{K^{0*}}^2) - \frac{2m_{K^{*0}}}{m_{B_c^+} - m_{K^{0*}}} A_0^{B_c^+ \to B_u^{+*}}(m_{K^{0*}}^2), \tag{14}$$

where the values of the $F(0) = 0.47 \pm 0.01$, $m_{\rm fit} = 0.99 \pm 0.04$ and $\delta = 0.31 \pm 0.03$ are used for $A_0^{B_c^+ \to B_u^{+*}}(m_{K^{0*}}^2)$ form factor, and

$$\begin{aligned}
H_b &= p_3 \cdot p_4 - \frac{(p_1 \cdot p_3)(p_1 \cdot p_4)}{m_1^2} - \frac{(p_2 \cdot p_3)(p_2 \cdot p_4)}{m_2^2} - \frac{(p_1 \cdot p_2)(p_1 \cdot p_3)(p_2 \cdot p_4)}{m_1^2 m_2^2}, \\
H'_b &= (-p_3 \cdot p_{B_c^+} + \frac{(p_1 \cdot p_{B_c^+})(p_1 \cdot p_3)}{m_1^2})(-p_4 \cdot p_{B_c^+} + \frac{(p_2 \cdot p_{B_c^+})(p_2 \cdot p_4)}{m_2^2}), \\
T_b &= m_{\bar{K}^{0*}}^2 + m_{\pi^+}^2 - m_{K^+}^2 - 2E_{\bar{K}^{0*}} E_{\pi^+} + 2|\vec{p}_{\bar{K}^{0*}}||\vec{p}_{\pi^+}| \cos\theta.
\end{aligned} \tag{15}$$

The amplitude of the mode $B_c^+ \to B_u^+(p_1)\bar{K}^0(p_2) \to \pi^+(p_3)B_s^0(p_4)$ with the exchange of the B_d^{0*} is given by

$$\begin{aligned}
Abs(3c) = & \frac{1}{2} \int \frac{\mathrm{d}^3\vec{p}_1}{2E_1(2\pi)^3} \frac{\mathrm{d}^3\vec{p}_2}{2E_2(2\pi)^3} (2\pi)^4 \delta^4(p_{B_c^+} - p_1 - p_2) A(B_c^+ \to B_u^+ \bar{K}^0) \\
& \times (-\mathrm{i}g_{B^*B\pi})(\epsilon_{B_d^{0*}}.p_1)(-\mathrm{i}g_{B^*B_sK})(\epsilon_{B_d^{0*}}.p_4) \frac{F^2(q^2, m_{B_d^{0*}}^2)}{T_c} \\
= & -\frac{(g_{B^*B\pi})(g_{B^*B_sK}) A(B_c^+ \to B_u^+ \bar{K}^0)}{16\pi m_{B_c^+}} \int_{-1}^{1} |\vec{p}_1| \mathrm{d}(\cos\theta) \frac{F^2(q^2, m_{B_d^{0*}}^2)}{T_c} H_c,
\end{aligned} \tag{16}$$

where

$$H_c = -p_1 \cdot p_4 + \frac{(m_1^2 - p_1 \cdot p_3)(m_4^2 - p_2 \cdot p_4)}{m_{B_d^{0*}}^2},$$
$$T_c = q^2 - m_i^2 = m_{B_u^+}^2 + m_{\pi^+}^2 - m_{B_d^{0*}}^2 - 2E_{B_u^+}E_{\pi^+} + 2|\vec{p}_{B_u^+}||\vec{p}_{\pi^+}|\cos\theta. \tag{17}$$

The amplitude of the $B_c^+ \to B_u^{+*}(\epsilon_1, p_1)\bar{K}^{0*}(\epsilon_2, p_2) \to \pi^+(p_3)B_s^0(p_4)$ (where B_d^0 is exchanged particle) is given by

$$\begin{aligned}
Abs(3\mathrm{d}) &= -\mathrm{i}\frac{G_F}{4} f_{\bar{K}^{0*}} m_{\bar{K}^{0*}} a_3 V_{cb} V_{ub}^* \int \frac{\mathrm{d}^3\vec{p}_1}{2E_1(2\pi)^3}\frac{\mathrm{d}^3\vec{p}_2}{2E_2(2\pi)^3}(2\pi)^4\delta^4(p_{B_c^+} - p_1 - p_2)\\
&\quad \times (-\mathrm{i}g_{B^*B\pi})(2\epsilon_1.p_3)(-\mathrm{i}g_{BB_sK^*})(\epsilon_2.p_4)\frac{F^2(q^2, m_{B_\mathrm{d}^0}^2)}{T_\mathrm{d}}\\
&\quad \times \left\{(\epsilon_1^*.\epsilon_2^*)(m_{B_c^+} + m_{B_u^{+*}})A_1^{B_c^+B_u^{+*}}(m_{\bar{K}^{0*}}^2) - (\epsilon_1.p_{B_c^+})(\epsilon_2.p_{B_c^+})\frac{2A_2^{B_c^+B_u^{+*}}.(m_{\bar{K}^{0*}}^2)}{m_{B_c^+} + m_{B_u^{+*}}}\right\}\\
&= \mathrm{i}\frac{G_F}{32\pi m_{B_c^+}} g_{B^*B\pi} g_{BB_sK^*} f_{\bar{K}^{0*}} m_{\bar{K}^{0*}} a_3 V_{cb} V_{ub}^* \int_{-1}^{1} |\vec{p}_1| \mathrm{d}(\cos\theta)\frac{F^2(q^2, m_{B_\mathrm{d}^0}^2)}{T_\mathrm{d}}\\
&\quad \times \left\{(m_{B_s^0} + m_{B_u^{+*}})A_1^{B_c^+B_u^{+*}}(m_{\bar{K}^{0*}}^2)H_b - \frac{2A_2^{B_c^+B_u^{+*}}(m_{\bar{K}^{0*}}^2)}{m_{B_c^+} + m_{B_u^{+*}}}H'_b\right\},
\end{aligned} \tag{18}$$

where

$$T_d = m_{\bar{K}^{0*}}^2 + m_{\pi^+}^2 - m_{K^+}^2 - 2E_{\bar{K}^{0*}}E_{\pi^+} + 2|\vec{p}_{\bar{K}^{0*}}||\vec{p}_{\pi^+}|\cos\theta. \tag{19}$$

The dispersive part of the rescattering amplitude can be obtained from the absorptive parts via the dispersion relation [10, 16]

$$Dis3(m_{B_c^+}^2) = \frac{1}{\pi}\int_s^\infty \frac{Abs_{3a}(s') + Abs_{3b}(s') + Abs_{3c}(s') + Abs_{3d}(s')}{s' - m_{B_c^+}^2}\mathrm{d}s'. \tag{20}$$

where s is the threshold of intermediate states, in this case $s \sim m_{B_c^+}^2$. The decay amplitude of $B_c^+ \to B_s^0\pi^+$ via the FSI corrections is

$$A(B_c^+ \to B_s^0\pi^+)_{\mathrm{FSI}} = \mathrm{i}Abs(3a) + \mathrm{i}Abs(3b) + \mathrm{i}Abs(3c) + \mathrm{i}Abs(3d) + Dis3. \tag{21}$$

Then the decay amplitude for the $B_c^+ \to B_s^0\pi^+$ decay by using the QCDF approach considering the effects of FSI turn to

$$A(B_c^+ \to B_s^0\pi^+) = A(B_c^+ \to B_s^0\pi^+)_{\mathrm{QCDF}} + A(B_c^+ \to B_s^0\pi^+)_{\mathrm{FSI}}. \tag{22}$$

The numerical values of the absorptive and dispersive parts used in the calculation are given separately in table 1. Finally, using the above amplitude in the equation (5) and the input parameters of table 2, we are able to calculate the branching ratio of $B_c^+ \to B_s^0\pi^+$ decay with different values of η. The results are shown in table 3 and figure 4. In figure 4, we present the dependence of the branching ratio of $B_c^+ \to B_s^0\pi^+$ decay on η. In this work the absorptive and dispersive parts of the four diagrams in figure 3 are calculated using the same value of η (for example $\eta = 1$ for all four diagrams) and contributed to the FSI amplitude. Then the amplitude resulting from the FSI effects with amplitude of the QCDF method are considered

Table 1. Absorptive and dispersive parts of the FSI amplitude (in units of 10^{-6}).

η	Absorptive	Dispersive
0.50	1.05 ± 0.07	0.23 ± 0.02
0.75	2.18 ± 0.15	0.49 ± 0.03
1.00	3.26 ± 0.23	0.74 ± 0.05
1.25	4.46 ± 0.31	1.01 ± 0.07
1.5	6.42 ± 0.46	1.51 ± 0.11

Table 2. Default values of the input parameters.

$m_{B_c^+} = 6274.9 \pm 0.8$, $m_{B_s^0} = 5366.89 \pm 0.19$, $m_{B^*} = 5324.65 \pm 0.25$, $m_{B_u^+} = 5279.32 \pm 0.14$, $m_{K^{+*}} = m_{\bar{K}^{0*}} = 891.76 \pm 0.25$, $m_{\bar{K}^0} = 497.611 \pm 0.013$, $m_{K^+} = 493.677 \pm 0.016$, $m_{\pi^+} = 139.57061 \pm 0.00024$ (MeV) [17]
$f_K = 159.80 \pm 1.84$, $f_{K^*} = 217 \pm 5$, $f_\pi = 130.70 \pm 0.46$ (MeV) [17]
$V_{ub} = 0.003\,94 \pm 0.000\,36$, $V_{cb} = 0.0422 \pm 0.0008$, $V_{cs} = 0.997 \pm 0.017$, $V_{ud} = 0.974\,20 \pm 0.000\,21$ [17]
$\Lambda_{\mathrm{QCD}} = 0.225$ GeV, $\tau_{B_c^+} = 513.4 \pm 11.0 \pm 5.7$ fs [18, 19]
$F_0^{B_c^+ B_s^0}(m_\pi^2) = 0.73 \pm 0.03$, $F_0^{B_c^+ B_u^+}(m_K^2) = 0.70 \pm 0.03$, $A_1^{B_c^+ B_u^{+*}}(m_{K^*}^2) = 0.85 \pm 0.15$, $A_2^{B_c^+ B_u^{+*}}(m_{K^*}^2) = 2.95 \pm 0.75$ [9]
$c_1 = 1.081$, $c_2 = -0.190$, $c_3 = 0.014$, $c_4 = -0.036$ [20]
$g_{B^* B_s K} = 23.43 \pm 2.13$, $g_{B B_s K^*} = 14.71 \pm 0.90$, $g_{B^* B \pi} = 32.00 \pm 5.00$ [21, 22], $g_{K^* K \pi} = 4.60$ [11]

Table 3. The branching ratio of $B_c^+ \to B_s^0 \pi^+$ decay with $\sigma(B_c^+)/\sigma(B_s^0) = [0.9, 8.0]\%$, $\eta = 0.5$–1.5 and experimental data (BR in EXP is the branch ratio present in the experimental observation).

Contributions	η	$\frac{\sigma(B_c^+)}{\sigma(B_s^0)} \times \mathcal{B}(B_c^+ \to B_s^0 \pi^+)(\times 10^{-3})$	$\mathcal{B}(B_c^+ \to B_s^0 \pi^+)$ $(\times 10^{-2})$
QCDF	—	$(1.49 \pm 0.20) \sim (13.22 \pm 1.77)$	16.52 ± 2.21
QCDF+FSI	0.50	$(1.54 \pm 0.20) \sim (13.53 \pm 1.78)$	16.92 ± 2.26
	0.75	$(1.64 \pm 0.22) \sim (14.62 \pm 1.94)$	18.28 ± 2.48
	1.00	$(1.84 \pm 0.25) \sim (16.38 \pm 2.23)$	20.47 ± 2.79
	1.25	$(2.16 \pm 0.31) \sim (19.45 \pm 2.94)$	23.93 ± 3.33
	1.5	$(2.89 \pm 0.41) \sim (25.66 \pm 3.63)$	32.07 ± 4.54
EXP [1]	—	$2.37 \pm 0.31 \pm 0.11$	—
BR in EXP	—	—	$(2.96 \pm 0.39) \sim (26.33 \pm 3.44)$

as the total amplitude. Note that, in addition to the conventional selection of η, we choose its value according to the mass of the exchange meson, as $\eta = 1.5$ for exchange meson of B^* (or B) and $\eta = 0.5$ for exchange meson of K^* (or K). The result is as follow:

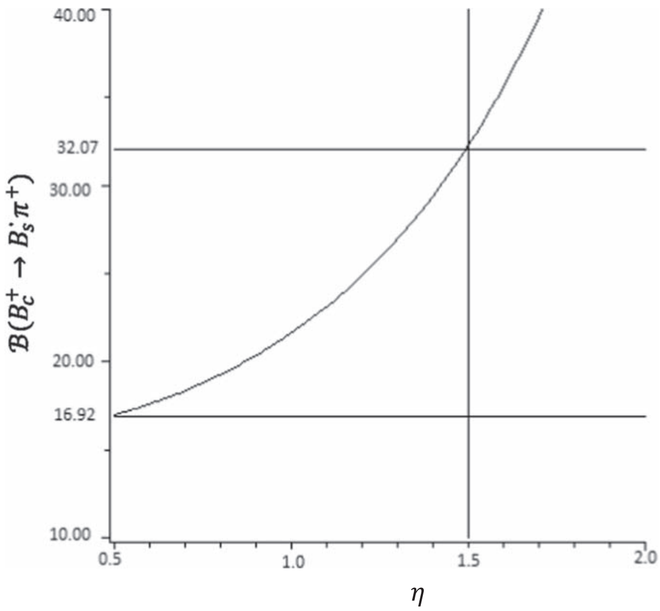

Figure 4. The dependence of the branching ratio of $B_c^+ \to B_s^0 \pi^+$ on η.

$$\frac{\sigma(B_c^+)}{\sigma(B_s^0)} \times \mathcal{B}(B_c^+ \to B_s^0 \pi^+)_{\text{QCDF+FSI}} = (1.98 \pm 0.32) \times 10^{-3} \sim (17.57 \pm 2.81) \times 10^{-3}. \tag{23}$$

In this work we find that the best choice for η is proportional to the mass of the intermediate mesons and besides choosing $\sigma(B_c^+)/\sigma(B_s^0) = 0.90\%$ guide us to the closet answer to the experience. In addition to the changes of η from 0.5 to 1.5, in many cases of input parameters, there are uncertainties arising from the variation of CKM parameters, meson masses, form factors and decay constants. The uncertainties in table 2 are due to these uncertainties.

4. Conclusion

The decay of $B_c^+ \to B_s^0 \pi^+$ has been observed by the LHCb collaboration with the measurement of the branching fraction multiplied by the production rates for B_c^+ relative to B_s^0 mesons as $(\sigma(B_c^+)/\sigma(B_s^0))\mathcal{B}(B_c^+ \to B_s^0 \pi^+)_{\text{EXP}} = (2.37 \pm 0.31 \pm 0.11) \times 10^{-3}$. In this paper we have calculated the branching ratio of the $B_c^+ \to B_s^0 \pi^+$ decay using the QCDF theorem. The numerical value of this calculation is $\mathcal{B}(B_c^+ \to B_s^0 \pi^+)_{\text{QCDF}} = (16.52 \pm 2.21)\%$. For direct comparison with the experimental result we have multiplied this value by $\sigma(B_c^+)/\sigma(B_s^0) = [0.9, 8.0]\%$ which is theoretically obtained, and got $(\sigma(B_c^+)/\sigma(B_s^0))\mathcal{B}(B_c^+ \to B_s^0 \pi^+)_{\text{QCDF}} = (1.49 \pm 0.20) \times 10^{-3} \sim (13.22 \pm 1.77) \times 10^{-3}$. On the other hand, by applying theoretical number of production rates $(\sigma(B_c^+)/\sigma(B_s^0))$, we have obtained the branch ratio value that exists within the experimental observation and got $(2.96 \pm 0.39) \sim (26.33 \pm 3.44)\%$ in which our predicted value using the QCDF approach is located in the middle of the branching fraction value in the experimental observation. We investigated then the FSI effects, in which such decays are highly dependent on the phenomenology parameter which appear in the form factors of the long distance distributions. By applying these effects and considering that the decay in this work is done only through the cross section processes, four intermediate states have been created. The contribution of all these intermediate decays have been included in the final amplitude. Finally, we have calculated the branching ratio by using the QCDF approach and FSI effects together for various values of the phenomenological parameter, $\eta = 0.5$–1.5. The obtained results have covered the branch ratio that exists inside the experimental view. In another selection of η, we have fixed $\eta = 0.5$ and $\eta = 1.5$ for light and heavy intermediate mesons, respectively, with this selection and fixing $\sigma(B_c^+)/\sigma(B_s^0)$ where a more acceptable result of 0.9% has been obtained.

ORCID iDs

Behnam Mohammadi https://orcid.org/0000-0002-7622-935X

References

[1] Aaij R *et al* LHCb Collaboration 2013 Observation of the decay $B_c^+ \to B_s^0\pi^+$ *Phys. Rev. Lett.* **111** 181801
[2] Gouz I P, Kiselev V V, Likhoded A K, Romanovsky V I and Yushchenko O P 2004 Prospects for the B_c studies at LHCb *Phys. Atom. Nucl.* **67** 1559
[3] Aaij R *et al* LHCb Collaboration 2015 Measurement of B_c^+ production in proton-proton collisions at $\sqrt{s} = 8$ TeV *Phys. Rev. Lett.* **114** 132001
[4] Rui Z and Zhou Z T 2014 S-wave ground state charmonium decays of B_c mesons in the perturbative QCD *Phys. Rev.* D **90** 114030
[5] Qiao C F, Sun P, Yang D and Zhu R L 2014 B_c exclusive decay to charmonium and a light meson at next to leading order accuracy *Phys. Rev.* D **89** 034008
[6] CMS Collaboration 2018 *Measurement of production cross section times branching fraction of $B_c^+ \to J/\psi\pi^+$ and $B^+ \to J/\psi K^+$ in pp collision at $\sqrt{s} = 7$TeV at CMS (2018).* CMS Physics Analysis Summary CMS-PAS-BPH-13-002 CERN
[7] Aaij R *et al* LHCb Collaboration 2017 Observation of $B_c^+ \to D^0K^+$ decays *Phys. Rev. Lett.* **118** 111803
[8] Aaij R *et al* LHCb Collaboration 2016 Study of B_c^+ decays to the $K^+K^-\pi^+$ final state and evidence for the decay $B_c^+ \to \chi_{c0}\pi^+$ *Phys. Rev.* D **94** 091102
[9] Wang W, Shen Y L and Lu C D 2009 *Phys. Rev.* D **79** 054012
[10] Cheng H Y, Chua C K and Soni A 2005 Final state interactions in hadronic B decays *Phys. Rev.* D **71** 014030
[11] Lu C D, Shen Y L and Wang W 2006 Final state interaction in $B \to KK$ decays *Phys. Rev.* D **73** 034005
[12] Liu X, Zhang B and Zhu S L 2007 The hidden charm decay of X(3872), Y(3940) and final state interaction effects *Phys. Lett.* B **645** 185
[13] Colangelo P, Fazio F D and Pham T N 2002 $B^- \to K^-\chi_{c0}$ decay from charmed meson rescattering *Phys. Lett.* B **542** 71
[14] Meng C and Chao K T 2007 Decays of the X(3872) and $\chi_{c1}(2P)$ charmonium *Phys. Rev.* D **75** 114002
[15] Liu X, Wei Z T and Li X Q 2009 Contribution of final state interaction to the branching ratio of $B \to J/\psi D$ *Eur. Phys. J.* C **59** 683
[16] Zhang B, Lu X and Zhu S L 2007 The dispersive contribution of ρ(1450, 1700) decays and X(1576) *Chin. Phys. Lett.* **24** 2537
[17] Tanabashi M and Particle Data Group 2018 Review of particle physics *Phys. Rev.* D **98** 030001
[18] Kiselev V V 2003 Decays of the B_c meson arXiv:hep-ph/0308214
[19] Aaij R *et al* LHCb Collaboration 2015 Measurement of the lifetime of the B_c^+ meson using the $B_c^+ \to J/\psi\pi^+$ decay mode *Phys. Lett.* B **742** 29
[20] Beneke M, Buchalla G, Neubert M and Sachrajda C T 2001 QCD factorization in decays $B \to \pi K, \pi\pi$ and extraction of Wolfenstein parameters *Nucl. Phys.* B **606** 245

[21] Cheng H Y and Yang K C 2002 Nonresonant three-body decays of D and B mesons *Phys. Rev.* D **66** 054015

[22] Melikhov D and Stech B 2000 Weak form factors for heavy meson decays: an update *Phys. Rev.* D **62** 014006