

Quantum cosmology in four dimensions

Teresa Bautista^{1,2} , André Benevides^{3,4}, Atish Dabholkar^{1,2,3}
and Akash Goel⁵

¹ Sorbonne Universités, UPMC Univ Paris 06. UMR 7589, LP THE, Paris, F-75005, France

² CNRS, UMR 7589, LP THE, Paris, F-75005, France

³ International Centre for Theoretical Physics. ICTP-UNESCO, Strada Costiera 11, Trieste 34151, Italy

⁴ SISSA, Via Bonomea 265, Trieste 34136, Italy

⁵ Indian Institute of Technology Kanpur, Kanpur 208016, India

Received 16 August 2019, revised 3 December 2019

Accepted for publication 16 December 2019

Published 27 January 2020



Abstract

We analyze the cosmological solutions to the recently proposed nonlocal quantum effective action for gravity with a cosmological term. We show that the vacuum energy decays with a slow-roll parameter proportional to the anomalous gravitational dressings

Keywords: cosmology, inflation, Weyl anomaly

1. Introduction

It was pointed out recently [1, 2] that Weyl anomalies in the renormalized quantum effective action of gravity can have important consequences for the gravitational dynamics on cosmological scales. The relevant Weyl anomalies can be computed explicitly in a two-dimensional model of gravity and are summarized by a nonlocal quantum effective action [3]. The resulting quantum momentum tensor is nonlocal in general but simplifies for an isotropic and homogeneous universe and the cosmological equations can be solved analytically. The scale factor exhibits power law expansion driven entirely by the slowly decaying vacuum energy [1, 3].

A four-dimensional phenomenological action (1) was proposed in [1, 2] motivated by these two-dimensional results and from considerations of the local renormalization group [4–7]. This nonlocal action parametrizes possible Weyl anomalies arising from the renormalization of composite operators. The resulting integro-differential equations describe the effective *classical* dynamics of the spacetime metric at long distances.

The nonlocality may be surprising at first but one must bear in mind that the action (1) should be regarded as the 1PI effective action and not the Wilsonian effective action. In general, the 1PI effective action is expected to be nonlocal. A class of non-local generalizations

of the Einstein–Hilbert term have been considered earlier to analyze cosmological evolution [8–14] and can lead to interesting cosmology [15, 16]. The nonlocality that we consider is of a very specific kind and is constrained by the requirement that the action should be a solution of the local renormalization group equation. The main idea is that anomalous dimensions of composite operators modify the trace of the quantum momentum tensor as encapsulated by the local renormalization group equation. Therefore the momentum tensor itself must be modified, which in turn can modify the gravitational dynamics. This reasoning suggests a specific nonlocal generalization of Einstein Gravity.

In this paper we analyze the cosmological consequences of this action in a Robertson–Walker spacetime. Somewhat surprisingly, the effective classical dynamics can be solved analytically even in four dimensions using the Weyl-invariant formulation discussed in [1, 2]. Our main conclusion is that nonzero anomalous gravitational dressings lead to a slow quantum decay of vacuum energy just as in two dimensions.

The paper is organized as follows. In section 2 we review the nonlocal four-dimensional action proposed in [1, 2] and derive the cosmological evolution equations for an isotropic and homogeneous universe. In section 3 we present the solutions to these equations which describe an expanding universe with a decaying vacuum energy density. We give estimates of the anomalous gravitational dressings and conclude with a discussion of the theoretical and cosmological implications.

2. Quantum gravity at long distances

We are interested in the quantum effective action for the metric obtained by integrating out the quantum fluctuations of various fields valid at distances large compared to the Planck distance. The essential lesson that emerges from the study of the two-dimensional model [1, 3] is that the physical coupling constants are the couplings of the gravitationally dressed operators. The anomalous dimensions of the dressed operators are in principle different from the anomalous dimensions of the undressed operators. This applies in particular to the square-root of the determinant of the metric corresponding to the cosmological term as well as to the Einstein–Hilbert term. The quantum effective action (1) should take into account these anomalous gravitational dressings and can be obtained either by using the background field method or as a solution of the local renormalization group equation [2].

2.1. A nonlocal action for gravity

The nonlocal action in the physical gauge is given by:

$$I_G[g] = \frac{M_p^2}{2} \int d^4x \sqrt{-g} \left(R[g] e^{-\Gamma_K(\Sigma_g)} - 2\Lambda e^{-\Gamma_\Lambda(\Sigma_g)} \right) \quad (1)$$

where M_p is the reduced Planck mass and the $\Gamma_i(\Sigma_g)$, $i = K, \Lambda$ are the integrated anomalous gravitational dressings. The field Σ_g is a nonlocal functional of the metric $g_{\mu\nu}$ defined by [17–19]

$$\Sigma_g(x) = \frac{1}{4} \int d^4y \sqrt{-g} G_4(x, y) F_4[g](y), \quad (2)$$

where

$$F_4[g] = E_4[g] - \frac{2}{3} \nabla^2 R[g], \quad E_4[g] = (R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2)[g]; \quad (3)$$

and $G_4(x, y)$ is the Green function of the Weyl covariant quartic differential operator

$$\Delta_4[g] = (\nabla^2)^2 + 2R^{\mu\nu} \nabla_\mu \nabla_\nu + \frac{1}{3} (\nabla^\nu R) \nabla_\nu - \frac{2}{3} R \nabla^2 \quad (4)$$

on the $g_{\mu\nu}$ background satisfying

$$\Delta_4^x[g] G_4(x, y) = \delta^{(4)}(x, y) := \frac{\delta^{(4)}(x - y)}{\sqrt{-g}}. \quad (5)$$

For metrics related by a Weyl rescaling

$$g_{\mu\nu} = e^{2\Sigma_g(x)} \bar{\eta}_{\mu\nu}, \quad (6)$$

the scalars F_4 are related by

$$F_4[g] = e^{-4\Sigma_g(x)} (F_4[\bar{\eta}] + 4 \Delta_4[\bar{\eta}] \Sigma_g), \quad (7)$$

and the operators Δ_4 are related by

$$\Delta_4[g] = e^{-4\Sigma_g} \Delta_4[\bar{\eta}]. \quad (8)$$

One can choose a gauge in which $\Sigma_g(x)$ becomes the conformal factor of the metric with respect to a reference metric $\bar{\eta}_{\mu\nu}$ which satisfies the F-flatness condition $F_4[\bar{\eta}] = 0$. The expression (2) is obtained in this F-flat gauge by inverting (7). Note that the action (1) should be regarded as the in-in effective action and hence one must impose retarded boundary conditions. This ensures that the propagation is causal.

We emphasize that the action (1) is the result of having performed a path integral and is not to be quantized further but is to be used for studying the effective classical dynamics. In principle, the dressing functions can be computed *ab initio* in a given microscopic theory [2]. For now, we view these functions as a phenomenological parametrization of possible Weyl anomalies and analyze the cosmological solutions of (1) in terms of these functions.

2.2. Weyl-invariant nonlocal action

The variation of (1) with respect to $g_{\mu\nu}$ is very cumbersome because both Δ_4 and F_4 have a complicated dependence on the metric. The Weyl-invariant formulation discussed in [1, 2] leads to considerable simplification by exploiting the fact that the spatially flat Robertson–Walker metric is Weyl equivalent to the flat Minkowski metric.

In the Weyl-invariant formulation, one introduces a Weyl compensator field $\Omega(x)$ and a fiducial metric $h_{\mu\nu}$ to write the physical metric as

$$g_{\mu\nu} = e^{2\Omega(x)} h_{\mu\nu}. \quad (9)$$

With this arbitrary split, the physical metric is invariant under a ‘fiducial Weyl transformation’

$$h_{\mu\nu} \rightarrow e^{2\xi(x)} h_{\mu\nu}, \quad \Omega(x) \rightarrow \Omega(x) - \xi(x). \quad (10)$$

The fiducial metric can be further parametrized in terms of an F-flat reference metric $\bar{\eta}_{\mu\nu}$ as

$$h_{\mu\nu} = e^{2\Sigma_h(x)} \bar{\eta}_{\mu\nu}. \quad (11)$$

Then, the Weyl factor $\Sigma_h(x)$ is given by (2) evaluated on $h_{\mu\nu}$. Equation (9) implies

$$\Sigma_g = \Omega + \Sigma_h \quad (12)$$

and (10) implies that $\Sigma_g(x)$ is invariant under the fiducial Weyl symmetry. Let M_0 be the UV cutoff scale below the Planck scale. One can then define the gravitational coupling κ^2 by

$$M_p^2 = \frac{M_0^2}{\kappa^2}. \quad (13)$$

Henceforth we choose units so that $M_0 = 1$. Substituting (9) and (12) into the action (1), one obtains the Weyl-invariant quantum action

$$I_G[h, \Omega] = \frac{1}{2\kappa^2} \int d^4x \sqrt{-h} e^{4\Omega} \left(R[h e^{2\Omega}] e^{-\Gamma_K(\Omega + \Sigma_h)} - 2\Lambda e^{-\Gamma_\Lambda(\Omega + \Sigma_h)} \right). \quad (14)$$

Using the Weyl transformation of the Ricci scalar and integrating by parts, we obtain

$$I_G[h, \Omega] = \frac{1}{2\kappa^2} \int d^4x \left[\left(R[h] + 6(1 - \Gamma_K^{(1)}) |\nabla\Omega|^2 - 6\Gamma_K^{(1)} \nabla\Omega \cdot \nabla\Sigma_h \right) e^{2\Omega - \Gamma_K} - 2\Lambda e^{4\Omega - \Gamma_\Lambda} \right] \quad (15)$$

where $d^4x \equiv d^4x \sqrt{-h}$ and $\Gamma_i^{(n)}(\Omega + \Sigma_h)$ are the n th derivatives of the dressing functions.

The action (15) now has an enlarged gauge symmetry that includes local Weyl symmetry in addition to diffeomorphisms. We have introduced an additional scalar degree of freedom in the process, but the number of physical degrees of freedom remains unchanged. Using the fiducial Weyl invariance one can choose a ‘physical gauge’ $\Omega = 0$ so that the physical metric is identified with the fiducial metric and one recovers the action (1). Alternatively, one can keep Ω arbitrary and impose a scalar gauge condition on the fiducial metric such as $F_4[h] = 0$. In this F-flat gauge $\Sigma_h = 0$ and $\Sigma_g = \Omega$.

2.3. Evolution equations for cosmology

A homogeneous and isotropic universe is described by the Robertson–Walker metric. For simplicity we consider the case when the spatial section is flat. In this case, one can choose a gauge in which the fiducial metric $h_{\mu\nu}$ equals the flat Minkowski metric $\eta_{\mu\nu}$. Since this is determined purely by the symmetry of the problem, the entire dynamics now resides in the Weyl compensator. Moreover, the Minkowski metric is not only F-flat but Riemann-flat. Consequently the variation of the nonlocal terms in the action (15) simplifies considerably.

Since $F_4[\eta] = 0$, the variation of the Green function does not contribute, and we obtain

$$\delta\Sigma_h(x) = \frac{1}{4} \int dy G_4(x, y) \delta F_4[h](y). \quad (16)$$

Furthermore, the quadratic terms involving the Riemann curvature tensors do not contribute to the variation of $F_4[h]$ when evaluated around $\eta_{\mu\nu}$. The only nonzero variation comes from the variations of the term linear in the curvatures:

$$\delta F_4[h](y) = -\frac{2}{3} \nabla^2 \delta R[h]. \quad (17)$$

The total variation of Σ_h after an integration by parts is then given by

$$\delta\Sigma_h(x) = \frac{1}{6} \int dy \delta h^{\mu\nu}(y) (\nabla_\mu \nabla_\nu - h_{\mu\nu} \nabla^2) \nabla^2 G_4(x, y). \quad (18)$$

⁶Henceforth, all covariant derivatives and contractions are with respect to the fiducial metric $h_{\mu\nu}$.

After performing the variation in the fiducial frame, it is convenient to rewrite the equations of motion in terms of the gauge-invariant physical metric using (9). Under a Weyl transformation, the Einstein tensor transforms as [2]

$$\begin{aligned} E_{\mu\nu}[g] &= E_{\mu\nu}[h] + D_{\mu\nu}[h, \Omega], \\ D_{\mu\nu}[h, \Omega] &:= -2 (\nabla_\mu \nabla_\nu - h_{\mu\nu} \nabla^2) \Omega + 2 \left(\nabla_\mu \Omega \nabla_\nu \Omega + \frac{1}{2} h_{\mu\nu} |\nabla \Omega|^2 \right). \end{aligned} \quad (19)$$

Substituting the above in the variation of (15) yields the equations of motion for the physical metric

$$E_{\mu\nu}[g] = \kappa^2 (T_{\mu\nu}^K + T_{\mu\nu}^\Lambda) \quad (20)$$

where $T_{\mu\nu}^K$ is the momentum tensor of the ‘gravifluid’ [2], of purely geometric origin

$$\begin{aligned} \kappa^2 T_{\mu\nu}^K(x) &= 2 \Gamma_K^{(1)} \left(\nabla_\mu \Omega \nabla_\nu \Omega + \frac{1}{2} \eta_{\mu\nu} |\nabla \Omega|^2 \right) \\ &+ \left((\Gamma_K^{(1)})^2 - \Gamma_K^{(2)} \right) (\nabla_\mu \Omega \nabla_\nu \Omega - \eta_{\mu\nu} |\nabla \Omega|^2) - \Gamma_K^{(1)} (\nabla_\mu \nabla_\nu - \eta_{\mu\nu} \nabla^2) \Omega \\ &- e^{-2\Omega + \Gamma_K} \int dy \Gamma_K^{(1)} e^{2\Omega - \Gamma_K} (\nabla^2 \Omega + |\nabla \Omega|^2) (\nabla_\mu \nabla_\nu - \eta_{\mu\nu} \nabla^2) \nabla^2 G_4(x, y); \end{aligned} \quad (21)$$

and $T_{\mu\nu}^\Lambda$ is the momentum tensor of the ‘vacuum fluid’

$$\begin{aligned} \kappa^2 T_{\mu\nu}^\Lambda(x) &= -\Lambda \eta_{\mu\nu} e^{2\Omega + \Gamma_K - \Gamma_\Lambda} \\ &- \frac{\Lambda}{3} e^{-2\Omega + \Gamma_K} \int dy \Gamma_\Lambda^{(1)} e^{4\Omega - \Gamma_\Lambda} (\nabla_\mu \nabla_\nu - \eta_{\mu\nu} \nabla^2) \nabla^2 G_4(x, y). \end{aligned} \quad (22)$$

We emphasize that the contribution from the ‘gravifluid’ is purely geometric in origin and in principle belongs to the left hand side of the equation (20) on the same footing as the Einstein tensor. Since the fiducial metric is flat in this context, the equation (20) reduces to

$$D_{\mu\nu}[\eta, \Omega] = \kappa^2 (T_{\mu\nu}^K + T_{\mu\nu}^\Lambda). \quad (23)$$

2.4. Cosmological equations in an alternative gauge

As described in [2], it is possible to choose an alternative gauge in which the conformal factor $\tilde{\Sigma}_g(x)$ is defined with respect to an R-flat reference metric $\tilde{\eta}_{\mu\nu}$ [20, 21]. In the R-flat gauge, the expression for the conformal factor follows from the Weyl transformation of the Ricci scalar

$$R[g] = e^{-2\tilde{\Sigma}_g} \left(R[\tilde{\eta}] - 6\nabla^2 \tilde{\Sigma}_g - 6|\nabla \tilde{\Sigma}_g|^2 \right). \quad (24)$$

Imposing $R[\tilde{\eta}] = 0$ and inverting the above equation gives

$$\tilde{\Sigma}_g(x) = -\ln \left(1 - \int d^d y \sqrt{-g} \tilde{G}(x, y) R[g](y) \right), \quad (25)$$

where $\tilde{G}(x, y)$ is the Green function of the differential operator

$$(-6\nabla^2 + R)^x [g] \tilde{G}(x, y) = \delta^{(4)}(x, y). \quad (26)$$

To use this gauge in the quantum action, we express the fiducial metric as

$$h_{\mu\nu} = e^{2\tilde{\Sigma}_h} \tilde{\eta}_{\mu\nu}, \quad \tilde{\Sigma}_g = \Omega + \tilde{\Sigma}_h. \quad (27)$$

We introduce the Weyl split (9) into the analog of the action (1) in the R-flat gauge with gravitational dressing functions⁷ $\tilde{\Gamma}_i(\tilde{\Sigma}_g)$. The Weyl-invariant action then becomes

$$I_G[h, \Omega] = \frac{1}{2\kappa^2} \int dx \left[(R[h] + 6(1 - \tilde{\Gamma}_K^{(1)}) |\nabla\Omega|^2 - 6\tilde{\Gamma}_K^{(1)} \nabla\Omega \cdot \nabla\tilde{\Sigma}_h) e^{2\Omega - \tilde{\Gamma}_K} - 2\Lambda e^{4\Omega - \tilde{\Gamma}_\Lambda} \right] \quad (28)$$

where now $\tilde{\Gamma}_i = \tilde{\Gamma}_i(\Omega + \tilde{\Sigma}_h)$.

The equations of motion for a spatially-flat Robertson–Walker spacetime then follow from the variation of this action around $h_{\mu\nu} = \eta_{\mu\nu}$. On a flat background, the variation of $\tilde{\Sigma}_h$ (25) receives no contribution from the variation of the Green function and is given by

$$\delta\tilde{\Sigma}_h = \int d^4y \tilde{G}(x, y) \delta(\sqrt{-h} R_h(y)) = \int dy \delta h^{\mu\nu} (-\nabla_\mu \nabla_\nu + h_{\mu\nu} \nabla^2) \tilde{G}(x, y). \quad (29)$$

Furthermore, since $R_\eta = 0$, the equation (26) becomes the Green equation for the flat Laplacian. Comparing with the Green equation (5) of Δ_4 on a flat background, we find that the two Green functions are related through

$$\tilde{G}(x, y) = -\frac{1}{6} \nabla^2 G_4(x, y). \quad (30)$$

Introducing this in (29) we recover the same variation of $\Sigma_h(x)$ in a flat background (18):

$$\delta\tilde{\Sigma}_h = \frac{1}{6} \int dy \delta h^{\mu\nu} (\nabla_\mu \nabla_\nu - h_{\mu\nu} \nabla^2) \nabla^2 G_4(x, y). \quad (31)$$

Since $h_{\mu\nu}$ is taken to be Minkowski, $\Sigma_h = \tilde{\Sigma}_h = 0$, and therefore $\Gamma_i(\Omega) = \tilde{\Gamma}_i(\Omega)$. As a result, the equations of motion obtained in the two gauges are identical.

3. Quantum decay of vacuum energy

Thus far we have only required that the physical metric be Weyl equivalent to the Minkowski metric. The equations derived in the previous section are valid generally as long as the Weyl tensor of the physical metric vanishes. In a spatially-flat Robertson–Walker spacetime there is further simplification because the scale factor of the physical metric is a function of only the conformal time⁸ τ . With our gauge choice of $h_{\mu\nu} = \eta_{\mu\nu}$, we can write

$$a(\tau) = e^{\Omega(\tau)}. \quad (32)$$

The momentum tensors (21) and (22) now simplify further and the integro-differential equation (23) reduce to an ordinary differential equation of the usual Friedmann–Lemaître type but for an effective quantum fluid with an unusual equation of state.

⁷ Note that $\tilde{\eta}_{\mu\nu} = e^{2\tilde{\Sigma}_h} \eta_{\mu\nu}$ and $\Sigma_g = \tilde{\Sigma}_g + \Sigma_{\tilde{\eta}}$. As a result, the integrated anomalous gravitational dressing functions in the two gauges are related by a shift: $\tilde{\Gamma}_i(\tilde{\Sigma}_g) = \Gamma_i(\tilde{\Sigma}_g + \Sigma_{\tilde{\eta}})$.

⁸ Conformal time τ is related to comoving cosmological time t by $d\tau = \frac{dt}{a(t)}$.

3.1. Effective quantum fluid

For the Robertson–Walker metric, the explicit form of $G_4(x, y)$ discussed in appendix B is actually not needed because its contribution to the momentum tensors is of the form

$$\int dy F[\Omega] (\nabla_\mu \nabla_\nu - h_{\mu\nu} \nabla^2) \nabla^2 G_4(x, y). \quad (33)$$

The differential operator in the parenthesis vanishes when $\mu = \nu = \tau$, so it does not contribute to the energy density. For all other components, the first term in the parenthesis vanishes after integration by parts, and for the components $\mu, \nu = i$, the second term in the parenthesis can be identified with the Green equation of $\Delta_4[\eta]$. It follows that the quantum momentum tensors (21) and (22) correspond to perfect fluids, consistent with isotropy and homogeneity, although they are not separately conserved.

The density and pressure of the vacuum fluid are given by

$$\rho_\Lambda(t) = \frac{\Lambda}{\kappa^2} e^{\Gamma_K - \Gamma_\Lambda}, \quad p_\Lambda(t) = w_\Lambda(t) \rho_\Lambda(t), \quad (34)$$

$$w_\Lambda(t) = \left(-1 + \frac{\Gamma_\Lambda^{(1)}}{3} \right). \quad (35)$$

The density and pressure of the gravifluid *after using* the equations of motion are given by

$$\rho_K(t) = \frac{\Lambda}{\kappa^2} \frac{\Gamma_K^{(1)}}{1 - \Gamma_K^{(1)}} e^{\Gamma_K - \Gamma_\Lambda}, \quad p_K(t) = w_K(t) \rho_K(t), \quad (36)$$

$$w_K(t) = \left(-1 + \frac{\Gamma_\Lambda^{(1)} - 1}{3} - \frac{\Gamma_K^{(2)}}{3 \Gamma_K^{(1)} (1 - \Gamma_K^{(1)})} \right). \quad (37)$$

We have written the expressions above in the ‘barotropic’ form with the effective pressure proportional to the effective density, but the anomalous gravitational dressings $\Gamma_i(\ln a(t))$ are in general nontrivial functions of the comoving time. As a result, the barotropic indices w_Λ and w_K are in general *time-dependent*⁹ and should be regarded as a convenient parametrization.

Combining the two contributions one obtains the total momentum tensor on the right hand side of the equation (20). It is a perfect fluid with the effective density and pressure given by

$$\rho_e(t) = \frac{\Lambda}{\kappa^2} \frac{1}{1 - \Gamma_K^{(1)}} e^{\Gamma_K - \Gamma_\Lambda}, \quad p_e(t) = w_e(t) \rho_e(t) \quad (38)$$

$$w_e(t) = \left(-1 + \frac{\gamma}{3} \right), \quad \gamma = \left(\Gamma_\Lambda^{(1)} - \Gamma_K^{(1)} - \frac{\Gamma_K^{(2)}}{(1 - \Gamma_K^{(1)})} \right). \quad (39)$$

With this effective density, the equation of motion reduces to the first Friedmann equation

$$H^2 = \frac{\kappa^2 \rho_e}{3} \quad (40)$$

⁹ Recall that in classical cosmology the commonly encountered fluids have the barotropic index -1 for the cosmological constant, 0 for matter, and $1/3$ for radiation.

where $H := \dot{a}/a$ as usual. Note that our conclusions thus far follow purely from the symmetry considerations of isotropy, homogeneity, and spatial flatness.

The momentum tensor for the gravifluid (36) is proportional to the cosmological constant after using the equations of motion in a spatially-flat Robertson–Walker spacetime. As a result the total momentum tensor for the effective fluid is proportional to the cosmological constant. This implies that in the absence of the cosmological constant, the Minkowski metric continues to be an exact solution of the new equation (20) in vacuum. On the other hand, for positive cosmological constant, the classical de Sitter solution is no longer a solution of the quantum equation (20) as we describe below.

The conservation equation for the effective fluid is

$$\dot{\rho}_e = -3(p_e + \rho_e)H. \quad (41)$$

A useful consistency check is that the expressions (38) and (39) satisfy the conservation equation. It is of course guaranteed by the fact that the nonlocal action (1) is coordinate invariant and follows from the Bianchi identity. Note, however, that the gravifluid and the vacuum fluid are not conserved separately for nonzero Γ_K .

3.2. Cosmology of the decaying vacuum energy

The expressions (38) and (39) for the effective density and pressure already give their functional dependence on the scale factor. As discussed above, they automatically solve the conservation equation (41). Our task is then reduced to solving the equation (40) to obtain the scale factor as a function of the cosmological time. Even though (40) is much simpler than an integro-differential equation, it is nevertheless a complicated ordinary differential equation. In general, the integrated anomalous dressings Γ_K and Γ_Λ are nontrivial functions of the scale factor and this equation can be solved only numerically.

Analytic solutions are possible when Γ_K and Γ_Λ are both linear functions of Ω :

$$\Gamma_K(\Omega) = \gamma_K \Omega(x), \quad \Gamma_\Lambda(\Omega) = \gamma_\Lambda \Omega(x), \quad (42)$$

where γ_i are constants¹⁰. The barotropic index for both the vacuum fluid (35) and the gravifluid (37) becomes constant. It is useful to consider this case to gain some understanding of the resulting solutions. From (35) and (37) we obtain

$$w_\Lambda = -1 + \frac{\gamma_\Lambda}{3} \quad w_K = -1 + \frac{\gamma_\Lambda - 1}{3}. \quad (43)$$

More interestingly, the effective fluid appearing on the right-hand side of the Einstein equations becomes also barotropic with index

$$w_e = -1 + \frac{\gamma}{3}, \quad \text{with} \quad \gamma = \gamma_\Lambda - \gamma_K. \quad (44)$$

The cosmological solution to (20) is then given by

$$\rho_e(t) = \rho_{e*} \left(\frac{a}{a_*} \right)^{-\gamma}, \quad a(t) = a_* \left(1 + \frac{\gamma}{2} H_* t \right)^{\frac{2}{\gamma}}, \quad (45)$$

¹⁰ In two dimensions, $\Gamma_K(\Omega)$ and $\Gamma_\Lambda(\Omega)$ are indeed linear functions with $\gamma_K = 0$ and $\gamma_\Lambda = 2\beta^2$ [3]. This is a consequence of the conformal invariance of the timelike Liouville theory. In general, this need not be true, but it may be possible to approximate the integrated anomalous gravitational dressings by linear functions for long enough time intervals during the evolution of the universe.

where ρ_{e*}, H_*, a_* are the initial values at the beginning of universe at time $t = 0$. The densities of the vacuum fluid and the gravifluid are given by

$$\rho_\Lambda(t) = (1 - \gamma_K) \rho_e(t), \quad \rho_K(t) = \gamma_K \rho_e(t). \quad (46)$$

The conformal time as a function of the comoving time is given by

$$\tau = \tau_* \left(1 + \frac{\gamma}{2} H_* t\right)^{\frac{\gamma-2}{\gamma}} \quad \text{where} \quad \tau_* := \frac{2}{(\gamma-2)H_* a_*}. \quad (47)$$

The range of t is $0 \leq t < \infty$ with the universe starting with scale factor a_* . The range of τ is

$$|\tau_*| \leq \tau < \infty \quad \text{for} \quad \gamma > 2, \quad (48)$$

$$-|\tau_*| \leq \tau < 0 \quad \text{for} \quad 0 \leq \gamma < 2. \quad (49)$$

As a function of τ , the scale factor and the density are given by

$$a(\tau) = a_* \left(\frac{\tau}{\tau_*}\right)^{\frac{2}{\gamma-2}}, \quad \rho_e(\tau) = \rho_{e*} \left(\frac{\tau}{\tau_*}\right)^{-\frac{2\gamma}{\gamma-2}}. \quad (50)$$

In the semiclassical approximation both anomalous dressings are expected to be small. A spatially-flat Robertson–Walker solution is only compatible with $\Lambda \geq 0$. For positive γ , our model describes an expanding universe driven by an effective fluid with a barotropic index that is slightly larger than its classical value -1 . We arrive at the conclusion that the vacuum energy density decays from its initial value ρ_{e*} which could be of the order of the string scale or the scale of supersymmetry breaking. The classical exponential expansion of de Sitter spacetime is slowed down to a power law expansion as a result of the quantum anomalous gravitational dressings. In the limit of vanishing γ , one recovers de Sitter spacetime with constant density.

The quantum decay of vacuum energy and the dynamics of the Omega field provides a new mechanism to drive slow-roll inflation in the early universe. It is convenient to define slow-roll parameters as usual in terms of fractional change in the Hubble parameter and its derivative:

$$\varepsilon_H := -\frac{\dot{H}}{H^2} = -\frac{d \ln H}{H dt} \quad \text{and} \quad \eta_H := \frac{\dot{\varepsilon}_H}{H \varepsilon_H} = \frac{d \ln \varepsilon_H}{H dt}. \quad (51)$$

For the solution

$$\varepsilon_H = \frac{\gamma}{2} \quad \text{and} \quad \eta_H = 0. \quad (52)$$

The condition for accelerated expansion ($\ddot{a} > 0$) requires that ε_H should be less than one. Slow-roll inflation requires that $\varepsilon_H \ll 1$. It is also necessary that $\eta_H \ll 1$ so that inflation lasts long enough. Since γ is small in the semiclassical approximation, all these conditions would be satisfied. A generic prediction is that $\eta_H = 0$. Thus, the quantum decay of vacuum energy can drive slow-roll inflation in the early universe. For small γ , the scale factor expands almost exponentially as a power law with a very high exponent. Nonzero ε_H measures the deviation from exact exponential expansion but the parameter η_H vanishes as in exact de Sitter spacetime.

In this model of inflation without an inflation or matter fields, inflation would last forever, the energy density would eventually vanish and the universe would end up empty. In order to end inflation, more realistic models with matter fields need to be considered. Matter fields would already be present during the inflationary epoch (whose quantum fluctuations integral gives rise to the low-energy effective action (1)), but their contribution to the evolution of the

background metric would be negligible due to the effective density being dominant during this period. One possibility for an inflationary end would be a phase transition triggered by a massive scalar field with m^2 turning negative. If the transition had high latent heat, it could reheat the universe and trigger an exit into a hot big bang.

More generally, the motivation for the phenomenological action (1) and the general lesson from this work is that renormalisation of composite operators leads to Weyl anomalies that become encoded in metric-dependent, and hence position-dependent, gravitational dressings of these operators as dictated by the renormalisation group. These dressings can be interpreted as effectively enducing a position-dependence in the coupling constants of the corresponding operators. In a model with scalars and fermions living on the background considered above, this would lead to time-dependent physical coupling constants. This time-dependence could then induce phase transitions, or particle-production as considered in models with parametric resonances. It would be very interesting to explore concrete models of these mechanisms.

For a general functional form of the anomalous dressings Γ_K and Γ_Λ , the equation (40) represents a novel generalization of the usual Friedmann equation because the equation of state of the effective fluid is rather unusual. It is conceivable that this has interesting consequences for early cosmology. Numerical integration may be necessary to find the time-dependence of the scale factor. However, we see from (38) that as long as $\Gamma_\Lambda - \Gamma_K$ is positive during the cosmological history, vacuum energy will decay. For negative γ , the *null energy condition* would be violated. In this case, the effective fluid could provide a model of phantom energy [22].

3.3. Broken time translation symmetry and stability

This novel mechanism for the decay of the vacuum energy raises the following puzzle. Unlike the classical de Sitter solution, our quantum corrected slow-roll solution (45) breaks the global time translation symmetry

$$t \rightarrow t + \pi \quad (53)$$

of the action (1) for a constant π . If a solution breaks a global symmetry of an action, the symmetry-transform of a given solution generates a new solution. This implies that if one now considers a position dependent symmetry parameter $\pi(x)$ then the effective action for $\pi(x)$ must be derivatively coupled so that there is a flat direction and arbitrary constant π is a solution of the equations of motion that follow from this effective action. Correspondingly, one expects a Nambu–Goldstone like scalar fluctuation mode. In usual inflationary models this scalar mode can be identified with a gauge-invariant combination of the inflaton and the metric. This idea is the basis of effective field theories of inflation [23, 24]. Where is this additional scalar degree of freedom? One could pose the puzzle slightly differently. Time translation symmetry is part of the diffeomorphism group. How can quantum effects break this symmetry?

The resolution of this puzzle is as follows. The scale factor of our solution has an initial value a_* at the initial value surface $t = 0$. Since we are using semiclassical gravity, a_* can be taken to be of the order of the short-distance cutoff scale a little larger than the Planck length. This means that, unlike the eternal de Sitter solution, one cannot continue this solution to times earlier than $t = 0$. The global time translation symmetry is thus explicitly broken by the fact that one must cutoff the evolution with an initial value surface in the early universe and impose initial conditions. Even though the action is invariant under the time translation symmetry, the initial conditions are not. Thus, one cannot apply the argument above to generate new solutions from a given solution to deduce the existence of a propagating scalar degree of freedom.

One can state the result slightly differently. The nonlocal expression (2) for the Weyl factor follows from inverting (7) only if one discards all solutions of the homogeneous equation

$$\Delta_4 \Sigma_g = 0. \quad (54)$$

These solutions correspond precisely to the would-be Nambu–Goldstone scalar fluctuations. The initial conditions on Σ_g on the initial value surface ensure that Σ_g is determined entirely in terms of the metric and is not an additional propagating field.

It is possible to reformulate the argument above using a local action. One can recast the nonlocal action (1) in a local form [11, 25] by introducing two auxiliary scalar fields $\Sigma(x)$ and $\Psi(x)$ with the action

$$S[g, \Sigma, \psi] = \frac{M_p^2}{2} \int d^4x \sqrt{-g} \left[R[g] e^{-\Gamma_K(\Sigma)} - 2\Lambda e^{-\Gamma_\Lambda(\Sigma)} + \Psi \left(\Delta_4 \Sigma - \frac{1}{4} F_4[g] \right) \right]. \quad (55)$$

The equations of motion for the two auxiliary fields are

$$\Delta_4 \Psi(x) = R[g] \Gamma_K^{(1)} e^{-\Gamma_K(\Sigma)} - 2\Lambda \Gamma_\Lambda^{(1)} e^{-\Gamma_\Lambda(\Sigma)}; \quad \Delta_4 \Sigma(x) = \frac{1}{4} F_4[g]. \quad (56)$$

The field $\Psi(x)$ acts therefore as a Lagrange multiplier for the condition $\Sigma = \Sigma_g$, and we recover (1) upon using its equation of motion in (55). This local action will reduce to the original nonlocal action only if the homogeneous solutions of (56) are eliminated by imposing an initial condition for Ψ and Σ that is similar to the initial condition for Σ_g . This ensures that the only propagating degrees of freedom are the usual tensor fluctuations of the metric and there are no additional scalar fluctuations.

If a Lagrangian depends on higher time derivatives of the fields, then one should also worry about the possibility of the Ostrogradsky instability [26]. We do not carry out the stability analysis of our action in this paper but refer the reader to the stability analysis for a class of nonlocal actions [10, 11] similar to the one we consider in this paper in the R-flat gauge in section 2.4.

These conclusions are physically reasonable from the point of view of the original quantum path integral. The action (1) is the quantum 1PI-effective action for the background metric obtained by a semiclassical evaluation of the path integral at weak coupling. It would be strange if one were to discover an extra scalar degree of freedom or unphysical instability in this IR effective action if the starting point is a well-defined path integral.

3.4. Quantum gravity and De Sitter spacetime

The idea of vacuum energy decay caused by infrared quantum effects has been explored earlier in four-dimensional gravity by several physicists. There is considerable divergence in the literature about the final result [27–35] and more generally about infrared effects in nearly de Sitter spacetime [36–48]. One of the new ingredients in the present work is to summarize the quantum effects in terms of a gauge-invariant nonlocal action. This way of organizing the analysis may be useful for future explorations since it separates the computation of the anomalous dressings from the classical evolution. Since the effective action (1) is a solution to the local renormalization group equation, it effectively sums up the leading logarithms. To see this explicitly, one can expand the solution (50) for the scale factor for small γ :

$$a(\tau) = a_* \left(\frac{\tau}{\tau_*} \right)^{\frac{2}{\gamma-2}} = a_* \left(\frac{\tau_*}{\tau} \right) \left(\frac{\tau}{\tau_*} \right)^{\frac{-\gamma}{2-\gamma}} \sim \frac{a_* \tau_*}{\tau} \left[1 - \frac{\gamma}{2} \log \left(\frac{\tau}{\tau_*} \right) + \dots \right]. \quad (57)$$

One obtains the usual de Sitter solution when $\gamma = 0$. For nonzero γ there are logarithmic corrections which add up to a small exponent which slows down the de Sitter expansion.

As it stands (1) should be regarded as a phenomenological parametrization in terms of the integrated anomalous gravitational dressings. These anomalous dressings are computable in a given microscopic theory. Renormalization of Newton's constant and the cosmological constant has been considered earlier in the literature [49–51]. One can extract the precise logarithmic running from these results. The order of magnitude of the anomalous dimensions is expected to be $G\Lambda$. If the UV cutoff is M_0 then the vacuum energy is of order M_0^4 , Λ is of the order M_0^4/M_p^2 , and the Hubble scale is $H = M_0^2/M_p$. The anomalous gravitational dressings would thus be of order H^2/M_p^2 . In very early universe, for example if H is of order $0.1M_p$ then these estimates suggest that γ and the slow-roll parameter would be of order 0.01. One can thus obtain slow-roll inflation driven entirely by slowly decaying vacuum energy through the nontrivial effective classical dynamics of the Omega field. This provides an example of ‘inflation without inflaton’ or what is termed ‘omflation’ in [1, 2]. A high value of H is ruled out by current bounds on primordial gravitational waves but it is interesting that an alternative mechanism for inflation is possible. It is worth exploring if this mechanism can be embedded in a realistic cosmology and if there are other ways to enhance anomalous dressing, for example, by thermal effects. A systematic analysis of these effects will be presented in [2, 52].

In the present era, the value of γ is expected to be of order $G\Lambda$, which would be too small to be observationally interesting. Nevertheless, one is led to the conclusion that, in principle, the dark energy will eventually decay even if it happens extremely slowly. At a purely theoretical level, this provides a new perspective on quantum gravity in de Sitter spacetime. In a quantum theory of gravity there are no gauge-invariant local observables because the metric itself is dynamical. An important question, independent of any ultraviolet completion of Einstein gravity, is to define gauge invariant observables for the three possible asymptotically maximally symmetric spaces with zero, negative, and positive curvature. In asymptotically Minkowski spacetime, the observables are the S-matrix elements. In asymptotically Anti de Sitter spacetime the observables are the boundary correlation functions. In both cases, string theory provides a consistent framework and precise prescriptions for obtaining well-defined finite answers for these quantities. On the other hand, quantum gravity in asymptotically de Sitter spacetime presents a number of conceptual difficulties. Considerations of de Sitter entropy [53, 54] suggest that the Hilbert space might be finite-dimensional [55–57] but it is not entirely clear what the gauge-invariant observables on this Hilbert space might be [58–60]. Moreover, it has proved to be difficult to accommodate asymptotically de Sitter spacetime within the framework of string theory. According to a no-go theorem, it is impossible to obtain a de Sitter compactification in the classical supergravity limit [61, 62]; and all known constructions in string theory [63–65] correspond to metastable vacua which decay nonperturbatively.

Our results imply that perhaps it is not necessary to try to make sense of asymptotically de Sitter spacetime if we take quantum effects into account even at the perturbative level. If γ is positive, the universe will be asymptotically Minkowski in the future. Hence the Penrose diagram looks like a ‘house’ with a sloping roof in the asymptotic future and with a floor on the initial spacelike surface in the asymptotic past where one must impose an appropriate cutoff to avoid the initial singularity. In this case, the observables are the usual cosmological observables computed for a given wavefunction of the universe defined on the initial value surface.

Acknowledgments

A major part of this work was conducted within the framework of the ILP LABEX (ANR-10-LABX-63) supported by French state funds managed by the Agence National de la Recherche within the Investissements d’Avenir programme under reference ANR-11-IDEX-0004-02, and by the project QHNS in the program ANR Blanc SIMI5.

TB and AG thank the HECAP group at the ICTP for hosting their visits. AD thanks the Department of Theoretical Physics at the Tata Institute of Fundamental Research where this work was initiated; and acknowledges the hospitality of the Aspen Center for Physics, the Benasque Center and the Theory Division at CERN during part of this work.

We thank Paolo Creminelli, Alba Grassi, Jeff Harvey, and Ashoke Sen for useful discussions.

Appendix A. Nearly static coordinates

Static coordinates of de Sitter spacetime are useful for studying the thermodynamic properties of the spacetime [66, 67]. Even though there is no global timelike Killing vector in de Sitter spacetime, the static coordinates provide a timelike future-oriented Killing vector in the static patch. These are also the natural coordinates for a Schwarzschild-de Sitter solution. Is there an analog of the static coordinates for our new solution?

Since our solution violates the de Sitter symmetry, we do not expect exactly static coordinates. Indeed, it can be shown that no such exact static coordinates exist for our cosmological solution (45). However, since any two Robertson–Walker metrics are Weyl equivalent, our solution admits nearly static coordinates in which it is conformal to static de Sitter:

$$ds^2 = \left(\frac{e^{2hT}}{1 - h^2 R^2} \right)^{\frac{\gamma}{\gamma-2}} \left[-(1 - h^2 R^2) dT^2 + \frac{1}{1 - h^2 R^2} dR^2 + R^2 d\Omega_2^2 \right], \quad (\text{A.1})$$

where constant h is defined as

$$h := \frac{2 - \gamma}{2} H_* a_*^{\gamma/2}. \quad (\text{A.2})$$

In the limit of vanishing γ , one recovers the static de Sitter metric.

Appendix B. Green function in Minkowski spacetime

The quartic Green equation (5) is Weyl invariant. Hence the Green function in a conformally flat spacetime is the same as the Green function in Minkowski spacetime:

$$G(t, \vec{x}; t', \vec{x}') = \int \frac{dE}{2\pi} \frac{d^3k}{(2\pi)^3} \left(\frac{1}{(E^2 - k^2)^2} \right) e^{-iE\Delta t + i\vec{k} \cdot \Delta \vec{x}} \quad (\text{B.1})$$

where $\Delta t := t - t'$ and $\Delta \vec{x} := \vec{x} - \vec{x}'$. As usual that the retarded Green function is obtained by lowering the poles off the real axis in the E integral. The E integral gives

$$\int dE \frac{e^{-iE\Delta t}}{(E - k)^2 (E + k)^2} = \frac{2\pi i}{4} \theta(\Delta t) \left[\frac{i\Delta t}{k^2} (e^{-ik\Delta t} + e^{ik\Delta t}) + \frac{1}{k^3} (e^{-ik\Delta t} - e^{ik\Delta t}) \right]. \quad (\text{B.2})$$

The angular integrals can be readily performed to obtain

$$\int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^\pi \frac{d\theta}{2\pi} \sin(\theta) e^{i\vec{k} \cdot \Delta \vec{x}} = \frac{1}{2\pi} \int_{-1}^1 d(\cos \theta) e^{ik|\Delta \vec{x}| \cos \theta} = \frac{1}{2\pi i |\Delta \vec{x}|} \left(\frac{e^{ik|\Delta \vec{x}|} - e^{-ik|\Delta \vec{x}|}}{k} \right) \quad (\text{B.3})$$

and we are left with the integral

$$G_{ret}(t, \vec{x}; t', \vec{x}') = \frac{\theta(\Delta t)}{8\pi|\Delta\vec{x}|} \int_0^\infty \frac{dk}{2\pi} \left[\frac{1}{k^2} \left(e^{ik|\Delta\vec{x}|} - e^{-ik|\Delta\vec{x}|} \right) (e^{-ik\Delta t} - e^{ik\Delta t}) + \frac{i\Delta t}{k} \left(e^{ik|\Delta\vec{x}|} - e^{-ik|\Delta\vec{x}|} \right) (e^{-ik\Delta t} + e^{ik\Delta t}) \right]. \quad (\text{B.4})$$

Since the integrand is even, one can extend the integral: $\int_0^\infty dk \rightarrow \frac{1}{2} \int_{-\infty}^\infty dk$. The integrand has no poles so one can deform the contour slightly below the real axis and write the integrand as a sum of exponentials with poles

$$G_{ret}(t, \vec{x}; t', \vec{x}') = \frac{\theta(\Delta t)}{16\pi|\Delta\vec{x}|} \int_{-\infty}^\infty \frac{dk}{2\pi} \left[\frac{1}{k^2} \left(e^{ik(|\Delta\vec{x}|-\Delta t)} - e^{ik(|\Delta\vec{x}|+\Delta t)} - e^{ik(-|\Delta\vec{x}|-\Delta t)} + e^{ik(-|\Delta\vec{x}|+\Delta t)} \right) + \frac{i\Delta t}{k} \left(e^{ik(|\Delta\vec{x}|-\Delta t)} + e^{ik(|\Delta\vec{x}|+\Delta t)} - e^{ik(-|\Delta\vec{x}|-\Delta t)} - e^{ik(-|\Delta\vec{x}|+\Delta t)} \right) \right]. \quad (\text{B.5})$$

This can be readily evaluated using the Cauchy residue theorem to obtain

$$G_{ret}(t, \vec{x}; t', \vec{x}') = \frac{\theta(t - t' - |\vec{x} - \vec{x}'|)}{8\pi}. \quad (\text{B.6})$$

One can verify explicitly that it satisfies the Green equation:

$$(\partial_t^4 - \nabla_x^2 \nabla_x^2) \frac{\theta(t-r)}{8\pi} = \frac{1}{8\pi} \left[2\delta(t-r) \nabla^2 \left(\frac{1}{r} \right) \right] = \delta(t-r) \delta^{(3)}(\vec{x}) = \delta(t) \delta^{(3)}(\vec{x}). \quad (\text{B.7})$$

Note that this retarded Green function receives contribution not only from the points on the light cone but also from the ‘wake’ inside the light cone in four dimensions. This phenomenon occurs also in two dimensions for the Green function of the d’Alembertian which is given by

$$G_{ret}^{(2)}(t, x; t', x') = \frac{1}{2} \theta(t - t' - |x - x'|). \quad (\text{B.8})$$

It is interesting to note that the Green function for the d’Alembertian receives contributions from within the light-cone for all odd dimensions but gets contributions from points precisely on the light cone in all even dimensions except in two dimensions [68].

ORCID iDs

Teresa Bautista  <https://orcid.org/0000-0001-7633-9842>

References

- [1] Dabholkar A 2016 Quantum Weyl invariance and cosmology *Phys. Lett. B* **760** 31–35
- [2] Dabholkar A Quantum modification of Einstein gravity at long distances (to appear)
- [3] Bautista T and Dabholkar A 2016 Quantum cosmology near two dimensions *Phys. Rev. D* **94** 044017
- [4] Osborn H 1989 Derivation of a four-dimensional c theorem *Phys. Lett. B* **222** 97
- [5] Jack I and Osborn H 2014 Constraints on RG flow for four dimensional quantum field theories *Nucl. Phys. B* **883** 425–500
- [6] Osborn H 1991 Weyl consistency conditions and a local renormalization group equation for general renormalizable field theories *Nucl. Phys. B* **363** 486–526

- [7] Baume F, Keren-Zur B, Rattazzi R and Vitale L 2014 The local Callan–Symanzik equation: structure and applications *J. High Energy Phys.* **JHEP08(2015)152**
- [8] Deser S and Woodard R P 2007 Nonlocal cosmology *Phys. Rev. Lett.* **99** 111301
- [9] Deffayet C and Woodard R P 2009 Reconstructing the distortion function for nonlocal cosmology *J. Cosmol. Astropart. Phys.* **JCAP08(2009)023**
- [10] Deser S and Woodard R P 2013 Observational viability and stability of nonlocal cosmology *J. Cosmol. Astropart. Phys.* **JCAP11(2013)036**
- [11] Tsamis N and Woodard R 2014 A caveat on building nonlocal models of cosmology *J. Cosmol. Astropart. Phys.* **JCAP09(2014)008**
- [12] Woodard R P 2014 Nonlocal models of cosmic acceleration *Found. Phys.* **44** 213–33
- [13] Donoghue J F and El-Menoufi B K 2014 Nonlocal quantum effects in cosmology: quantum memory, nonlocal FLRW equations, and singularity avoidance *Phys. Rev. D* **89** 104062
- [14] Woodard R P 2015 Nonlocal metric realizations of MOND *Can. J. Phys.* **93** 242–9
- [15] Park S and Dodelson S 2013 Structure formation in a nonlocally modified gravity model *Phys. Rev. D* **87** 024003
- [16] Barreira A, Li B, Hellwing W A, Baugh C M and Pascoli S 2014 Nonlinear structure formation in nonlocal gravity *J. Cosmol. Astropart. Phys.* **JCAP09(2014)031**
- [17] Fradkin E S and Vilkovisky G A 1978 Conformal invariance and asymptotic freedom in quantum gravity *Phys. Lett. B* **77** 262
- [18] Paneitz S 2008 A quartic conformally covariant differential operator for arbitrary Pseudo–Riemannian manifolds (summary) *SIGMA* **4** 36
- [19] Riegert R 1984 A nonlocal action for the trace anomaly *Phys. Lett. B* **134** 56–60
- [20] Fradkin E S and Vilkovisky G A 1978 Conformal off mass shell extension and elimination of conformal anomalies in quantum gravity *Phys. Lett. B* **73** 209–13
- [21] Barvinsky A O, Mirzabekian A G and Zhytnikov V V 1995 Conformal decomposition of the effective action and covariant curvature expansion *Quantum gravity. Proc., 6th Seminar (Moscow, Russia, 12–19 June 1995)*
- [22] Weinberg S 2008 *Cosmology* (Oxford: Oxford University Press)
- [23] Cheung C, Creminelli P, Fitzpatrick A L, Kaplan J and Senatore L 2008 The effective field theory of inflation *J. High Energy Phys.* **JHEP03(2008)014**
- [24] Weinberg S 2008 Effective field theory for inflation *Phys. Rev. D* **77** 123541
- [25] Nojiri S and Odintsov S D 2008 Modified non-local-F(R) gravity as the key for the inflation and dark energy *Phys. Lett. B* **659** 821–6
- [26] Woodard R P 2015 The theorem of Ostrogradsky *Report No. UFIFT-QG-15-03* (arXiv:1506.02210 [hep-th])
- [27] Polyakov A M 1982 Phase transitions and the universe *Sov. Phys.—Usp.* **25** 187
Polyakov A M 1982 *Usp. Fiz. Nauk* **136** 538
- [28] Mottola E 1985 Particle creation in de Sitter space *Phys. Rev. D* **31** 754
- [29] Antoniadis I, Iliopoulos J and Tomaras T N 1986 Quantum instability of De Sitter space *Phys. Rev. Lett.* **56** 1319
- [30] Tsamis N C and Woodard R P 1997 The quantum gravitational back reaction on inflation *Ann. Phys.* **253** 1–54
- [31] Tsamis N C and Woodard R P 1996 Quantum gravity slows inflation *Nucl. Phys. B* **474** 235–48
- [32] Antoniadis I and Mottola E 1992 4-D quantum gravity in the conformal sector *Phys. Rev. D* **45** 2013–25
- [33] Polyakov A M 2008 De Sitter space and eternity *Nucl. Phys. B* **797** 199–217
- [34] Polyakov A M 2012 Infrared instability of the de Sitter space (arXiv:1209.4135 [hep-th])
- [35] Romania M G, Tsamis N C and Woodard R P 2013 Quantum gravity and inflation *Lect. Notes Phys.* **863** 375–95
- [36] Starobinsky A A and Yokoyama J 1994 Equilibrium state of a selfinteracting scalar field in the De Sitter background *Phys. Rev. D* **50** 6357–68
- [37] Weinberg S 2005 Quantum contributions to cosmological correlations *Phys. Rev. D* **72** 043514
- [38] Weinberg S 2006 Quantum contributions to cosmological correlations. II. Can these corrections become large? *Phys. Rev. D* **74** 023508
- [39] Senatore L and Zaldarriaga M 2010 On loops in inflation *J. High Energy Phys.* **JHEP12(2010)008**
- [40] Kitamoto H and Kitazawa Y 2010 Boltzmann equation in de Sitter space *Nucl. Phys. B* **839** 552–79
- [41] Rajaraman A 2010 On the proper treatment of massless fields in Euclidean de Sitter space *Phys. Rev. D* **82** 123522

- [42] Kahya E O, Onemli V K and Woodard R P 2011 The Zeta–Zeta correlator is time dependent *Phys. Lett. B* **694** 101–7
- [43] Marolf D and Morrison I A 2010 The IR stability of de Sitter: loop corrections to scalar propagators *Phys. Rev. D* **82** 105032
- [44] Giddings S B and Sloth M S 2011 Semiclassical relations and IR effects in de Sitter and slow-roll space-times *J. Cosmol. Astropart. Phys.* **JCAP01(2011)023**
- [45] Higuchi A, Marolf D and Morrison I A 2011 de Sitter invariance of the dS graviton vacuum *Class. Quantum Grav.* **28** 245012
- [46] Marolf D and Morrison I A 2011 The IR stability of de Sitter QFT: physical initial conditions *Gen. Relativ. Gravit.* **43** 3497–530
- [47] Pimentel G L, Senatore L and Zaldarriaga M 2012 On Loops in inflation III: time independence of zeta in single clock inflation *J. High Energy Phys.* **JHEP07(2012)166**
- [48] Akhmedov E T 2014 Lecture notes on interacting quantum fields in de Sitter space *Int. J. Mod. Phys. D* **23** 1430001
- [49] Gibbons G W and Perry M J 1978 Quantizing gravitational instantons *Nucl. Phys. B* **146** 90
- [50] Christensen S M and Duff M J 1980 Quantizing gravity with a cosmological constant *Nucl. Phys. B* **170** 480
- [51] Kazakov K A, Pronin P I and Stepanyantz K V 1998 On the gravity renormalization off-shell *Grav. Cosmol.* **4** 17–27
- [52] Bautista T, Benevides A, Dabholkar A and Grassi A Semiclassical gravitational dressings (in preparation)
- [53] Figari R, Hoegh-Krohn R and Nappi C R 1975 Interacting relativistic Boson fields in the de Sitter universe with two space-time dimensions *Commun. Math. Phys.* **44** 265–78
- [54] Gibbons G W and Hawking S W 1977 Cosmological event Horizons, thermodynamics, and particle creation *Phys. Rev. D* **15** 2738–51
- [55] Banks T 1985 T C P, quantum gravity, the cosmological constant and all that..., *Nucl. Phys. B* **249** 332
- [56] Bousso R 1999 Holography in general space-times *J. High Energy Phys.* **JHEP06(1999)028**
- [57] Bousso R 2000 Positive vacuum energy and the N bound *J. High Energy Phys.* **JHEP11(2000)038**
- [58] Hellerman S, Kaloper N and Susskind L 2001 String theory and quintessence *J. High Energy Phys.* **JHEP06(2001)003**
- [59] Witten E 2001 Quantum gravity in de Sitter space *Strings 2001: Int. Conf. (Mumbai, India, 5–10 January 2001)*
- [60] Goheer N, Kleban M and Susskind L 2003 The trouble with de Sitter space *J. High Energy Phys.* **JHEP07(2003)056**
- [61] de Wit B, Smit D J and Hari Dass N D 1987 Residual supersymmetry of compactified $D = 10$ supergravity *Nucl. Phys. B* **283** 165
- [62] Maldacena J M and Nunez C 2001 Supergravity description of field theories on curved manifolds and a no go theorem *Int. J. Mod. Phys. A* **16** 822–55
- [62] Maldacena J M and Nunez C 2000 *Int. J. Mod. Phys. A* **16** 182
- [63] Kachru S, Kallosh R, Linde A D and Trivedi S P 2003 De Sitter vacua in string theory *Phys. Rev. D* **68** 046005
- [64] Douglas M R and Kachru S 2007 Flux compactification *Rev. Mod. Phys.* **79** 733–96
- [65] Silverstein E 2008 Simple de Sitter solutions *Phys. Rev. D* **77** 106006
- [66] Birrell N D and Davies P C W *Quantum Fields in Curved Space (Cambridge Monographs on Mathematical Physics)* (Cambridge: Cambridge University Press)
- [67] Spradlin M, Strominger A and Volovich A 2001 Les Houches lectures on de Sitter space *Unity from Duality: Gravity, Gauge Theory and Strings. Proc., NATO Advanced Study Institute, Euro Summer School, 76th session (Les Houches, France, 30 July–31 August 2001)* pp 423–53
- [68] Economou E 2006 *Green’s Functions in Quantum Physics (Springer Series in Solid-State Sciences)* (Berlin: Springer)