Penetration Depth of Free Falling Intruder into a Particles Bed in Fluid-Immersed Two-Dimension Spherical Particle System

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Abstract. In a system of spherical particles which is entirely immersed in fluid, an intruder is let free fall into the particles bed consisted of 220 particles with height about 13 particles diameter and width about 15 particles diameter, both are in average. The system is placed in a container with height of 40 cm and width of 15 cm. Bed particles have the same diameter of 1 cm, while the intruder has 4 cm. Water is the fluid with density of 1 g/cm^3 and viscosity of 8.90×10^{-4} Pa·s, which is assumed to constant during the simulation. Intruder initial height is always the same and bed particles are let to be relaxed for about 2 s from its initial random configuration of 20×11 in height and width of bed particle diameter, before the intruder is dropped into the system. Bed particles has always the same density about 2 g/cm^3, while intruder density $\rho_{\rm int}$ is varied from 2 to 4.5 g/cm^3 with increment of 0.5 g/cm^3. It is observed that higher $\rho_{\rm int}$ gives higher penetration depth after the density of 2.5 g/cm^3. Density of 4.5 g/cm^3 and beyond will give similar final result, since the intruder already reached bottom of the container. An empirical model for penetration depth as function of $\rho_{\rm int}$ is proposed.

1. Introduction

Penetration of a larger object into a bed of grains is still interesting to investigage. It ranges from modeling of crater forming of due to meteor impact on planet surface [1], investigation of depth-dependent resistance of penetration process [2], study of confinement influence [3], observing unified force of granular impact cratering [4], formulating granular bed viscosity [5], until seperation of two grains by one grain above [6]. In this work a whole system of spherical particles in two-dimension is immersed in fluid, so that both bed particles and intruder will be affected by drag force due to fluid viscosity and also by buoyant force due to fluid density. The intruder is released from a certain height z_0 and gives impact to the bed. How far it can penetrate the bed is then reported.

2. Simulation

A spherical particle or grain i will have density of ρ_i, diameter of D_i, and mass of

$$m_i = \frac{\pi}{6} D_i^3 \rho_i , \tag{1}$$

where i = 1, 2, .., N, with index N is for the intruder, while 1 .. N – 1 are for the remaining particles. Due to its surrounding fluid environment with viscosity η_f and density ρ_f particle i will have viscous force (or drag)

$$\vec{D}_i = -6\pi\eta_f D_i \vec{v}_i \tag{2}$$

and buoyant force

$$\vec{B}_i = -\frac{\pi}{6} D_i^3 \rho_f \vec{g} \tag{3}$$

with $\vec{g}$ is earth gravity. Since the system is simulated on earth surface it will have also gravitational force

$$\vec{G}_i = \frac{\pi}{6} D_i^3 \rho_i \vec{g} \,, \tag{4}$$

which differs from Eq. (3) in the sign and also the density. Position of particle i relative to particle j is

$$\vec{r}_{ij} = \vec{r}_i - \vec{r}_j \,, \tag{5}$$

distance between two particles

$$r_{ij} = \left|\vec{r}_{ij}\right| = \sqrt{\vec{r}_{ij} \cdot \vec{r}_{ij}} \,, \tag{6}$$

and unit vector

$$\hat{r}_{ij} = \frac{\vec{r}_{ij}}{r_{ij}} \,, \tag{7}$$

where for relative velocity and its unit vector can also be obtained using similar way. Using Eq. (6) and diameter of two particles, D_i and D_j, an overlap

$$\xi_{ij} = \max\left[0, \left(D_i + D_j - r_{ij}\right)\right], \tag{8}$$

can be calculated with

$$\max(x, y) = \begin{cases} x, & x \geq y, \\ y, & x < y, \end{cases} \tag{9}$$

and

$$\dot{\xi}_{ij} = \frac{d\xi_{ij}}{dt} = -v_{ij} \operatorname{sign}\left(\xi_{ij}\right), \tag{10}$$

with

$$\operatorname{sign}(x) = \begin{cases} +1, & x > 0, \\ 0, & x = 0, \\ -1, & x < 0. \end{cases} \tag{11}$$

Using Eqs. (8) and (10) normal force on particle i due to particle j can be defined as [7]

$$\vec{N}_{ij} = k_N \xi_{ij} \hat{r}_{ij} + \gamma_N \dot{\xi}_{ij} \hat{v}_{ij} \,. \tag{12}$$

Then total force acted on particle i will be

$$\vec{F}_i = \vec{D}_i + \vec{B}_i + \vec{G}_i + \sum_{j=1}^{N} \left(1 - \delta_{ij}\right) \vec{N}_{ij} \tag{13}$$

and

$$\vec{a}_i = \frac{1}{m_i} \vec{F}_i \tag{14}$$

is the acceleration. At every time t Eq. (14) can be written in its full form as

$$\vec{a}_i = -6\pi\eta_f D_i \vec{v}_i + \frac{\pi}{6} D_i^3 \left(\rho_i - \rho_f\right) \vec{g} + \sum_{j=1}^{N} \left(1 - \delta_{ij}\right) \left[k_N \xi_{ij} \frac{\vec{r}_i - \vec{r}_j}{\left|\vec{r}_i - \vec{r}_j\right|} - \gamma_N \left(\vec{v}_i - \vec{v}_j\right) \operatorname{sign}\left(\xi_{ij}\right) \right], \tag{15}$$

that shows a coupled differential equation, which is difficult to solve it analytically. Using forward difference method velocity of particle i at time $t + \Delta t$ can be obtained

$$\vec{v}_i(t+\Delta t) = \vec{v}_i(t) + \vec{a}_i(t) \quad (16)$$

and also the position

$$\vec{r}_i(t+\Delta t) = \vec{r}_i(t) + \vec{v}_i(t), \quad (17)$$

with initial condition $\vec{r}_i(0)$ and $\vec{v}_i(0)$ must be known for all particles. And for interaction between particles and the container walls, it is formulated similar to Eq. (12), where a flat wall can approximated with a spherical grain with large diameter, so that only one grain diameter is used and unit vector of relative positition will become the normal vector of the wall. The simulation will be performed from $t = t_{\text{beg}}$ until $t = t_{\text{end}}$ time step Δt.

During the simulation three parameters are observed, which are vertical position of interuder z_{int}, maximum vertical position of bed particle z_{max}, and average vertical position of bed partices

$$z_{\text{avg}} = \frac{1}{N-1}\sum_{i=1}^{N-1} \hat{z}\cdot\vec{r}_i \,, \quad (18)$$

where vertical position of the intruder (with index N) is excluded.

3. Results and discussion

Values of parameters used in the simulation is shown in folowing Table 1, where all are represented in SI units.

Table 1. Parameters values using in simulation.

Symbol	Value	Unit
g	9.87	$\text{m}\cdot\text{s}^{-2}$
η_f	8.9×10^{-4}	Pa·s
ρ_f, ρ_{bed}	1000, 2000	$\text{kg}\cdot\text{m}^{-3}$
ρ_{int}	2000, 2500, 3000, 3500, 4000, 4500	$\text{kg}\cdot\text{m}^{-3}$
t_{beg}, t_{end}, Δt, t_{int}	0, 6, 0.001, 2	s
N, $N_y \times N_z$	221, 20×11	-
D_{bed}, D_{int}	0.01, 0.04	m
k_N, γ_N	400, 0.1	$\text{N}\cdot\text{m}^{-1}$, $\text{N}\cdot\text{s}\cdot\text{m}^{-1}$
h, w, z_0	0.4, 0.15, 0.18	m

Simulation begins at $t = t_{\text{beg}}$ with creating spherical particles in $N_y \times N_z$ grid with slightly pertubated randomly and let the system condensed until $t = t_{\text{int}}$. After that time an intruder is let free falling from height z_0, hit the bed, and begin to penetrate it, as shown in Fig. 1.

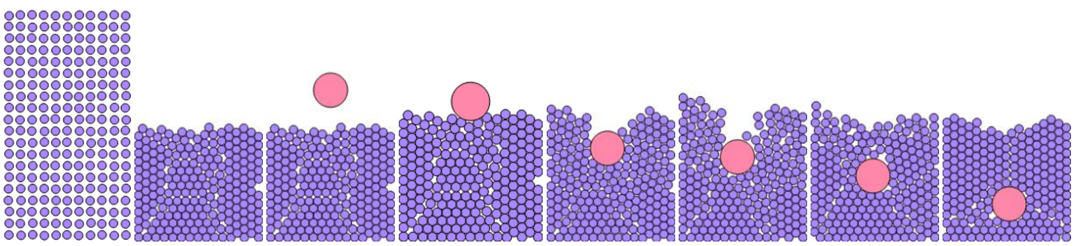

Figure 1. Initial configuration, condensation, free falling, contacting, penetrating and collapsing.

Figure 1 shows the system with ρ_{int} = 2000 kg/m^3 for time t at 0 s, 2 s, 2.02 s, 2.1 s, 2.21 s, 2.28 s, 2.48 s, and 6 s. A typical time series of z_{max}, z_{avg}, and z_{int} for different value of ρ_{int} are shown in following Fig. 2.

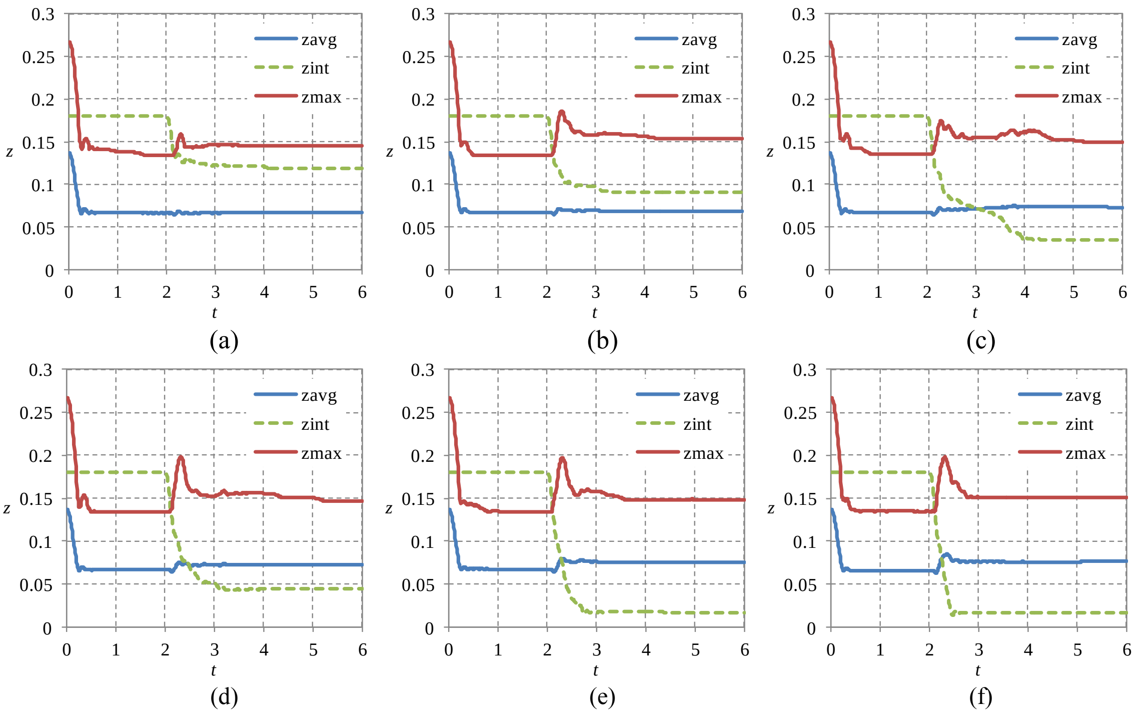

Figure 2. Time series of z_{max}, z_{avg}, z_{int} for various value of intruder density ρ_{int}: (a) 2000 kg/m^3, (b) 2500 kg/m^3, (c) 3000 kg/m^3, (d) 3500 kg/m^3, (e) 4000 kg/m^3, and (f) 4500 kg/m^3.

Solid red line indicates position of most top bed particles, while solid blue line shows average vertical position of bed particles. Motion of intruder penetrating the bed is given by dashed blue line, which is steeper for larger intruder density. Final depth of intruder mostly achieved after 3-4 s. Intruder has already contact with bottom of container for interuder density 4000 kg/m^3 and 4500 kg/m^3. Peak between 2-3 s is due to collision of free falling intruder with the bed, which makes some bed particles rise their position higher than usual.

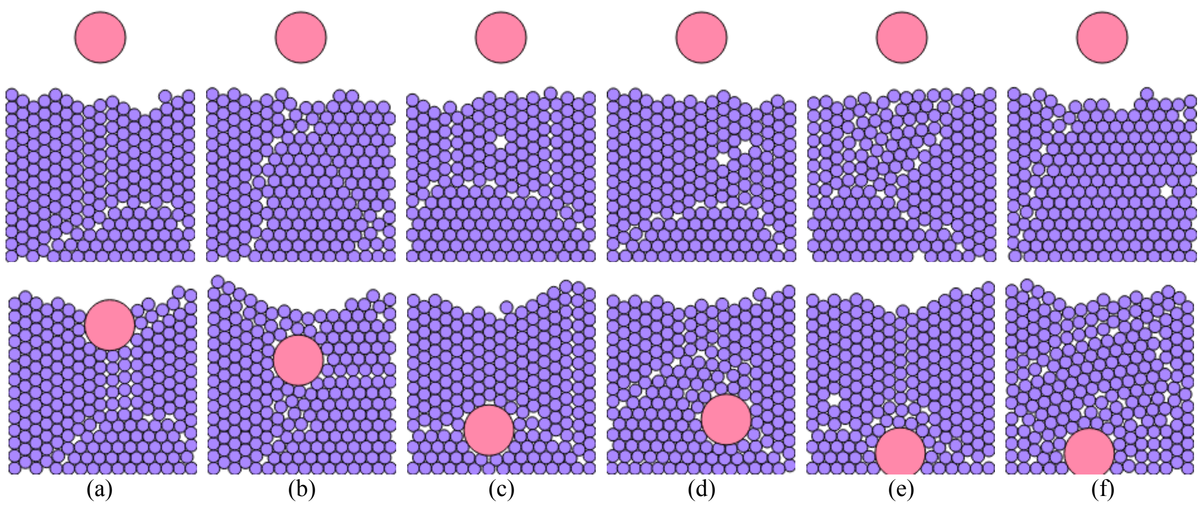

Figure 3. Condensed (top) and final (bottom) configuration for interuder density ρ_{int}: (a) 2000 kg/m^3, (b) 2500 kg/m^3, (c) 3000 kg/m^3, (d) 3500 kg/m^3, (e) 4000 kg/m^3, and (f) 4500 kg/m^3.

Since particles bed are randomly generated before it condensed for each density ρ_{int} there are five times repetition of simulation, which give average value of z_{avg}, z_{max}, and z_{int} as shown in Fig. 4.

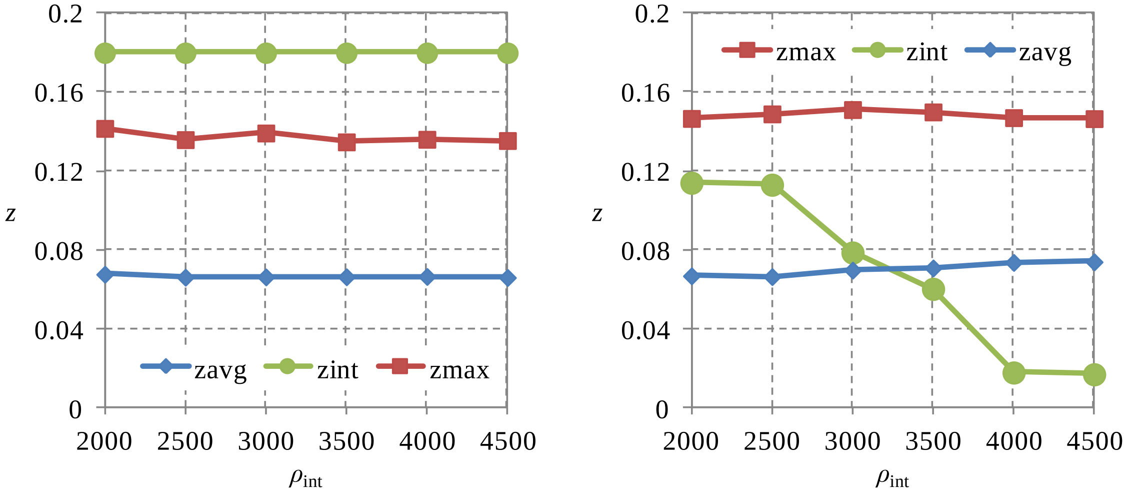

Figure 4. Condensed (left) and final (right) configuration for average value from five simulations.

Configuration in Fig. 3 (top) is one of the results contributing to average value in Fig. 4 (left), while Fig. 3 (bottom) is related to Fig. 4 (right). We can also calculate the difference between final and condensed configuration, as given in Fig. 5 with standard deviation from 1.2×10^{-4} until 1.7×10^{-2} for the right figure.

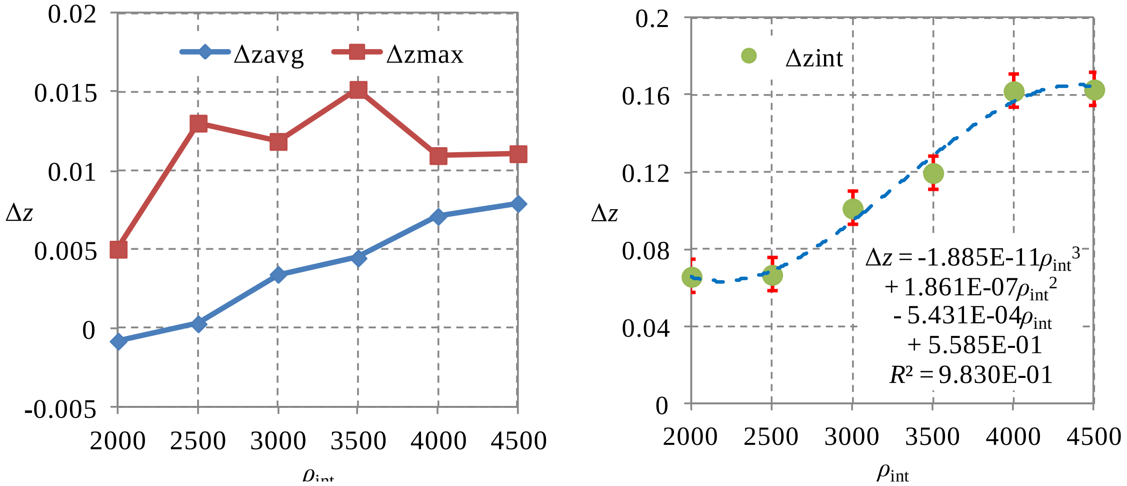

Figure 5. Difference of final and condensed configuration for: Δz_{max}, Δz_{avg} (left) and Δz_{int} (right).

It can seen from Fig. 5 (left) that Δz_{max} and Δz_{avg} has order of 10^{-2}, while from Fig. 5 (right) that Δz_{int} has order of 10^{-1}. It can understand since Δz_{max} and Δz_{avg} have not change much, but Δz_{int} is very dependent on initial configuration, that always be different. Monotonic increasing of Δz_{avg} as increasing of ρ_{int} indicates an interesting result since it is already average values.

Polynomial regression is used in fitting Δz_{int} as function of ρ_{int} and following relation

$$\Delta z_{\mathrm{int}} = -1.885\times10^{-11}\rho_{\mathrm{int}}^3 + 1.861\times10^{-17}\rho_{\mathrm{int}}^2 - 5.431\times10^{-4}\rho_{\mathrm{int}} + 5.585\times10^{-1}, \quad (18)$$

is obtained with $R^2 = 0.983$. From Fig. 5 (right) the fitting curve from Eq. (18) still lays in the range of error bar provided by standar deviation of simulation repetitions. Unfortunatelya, Eq. (18) does not hold for $\rho_{\mathrm{int}} < 2000$ and $\rho_{\mathrm{int}} > 4500$ as shown in Fig. 6.

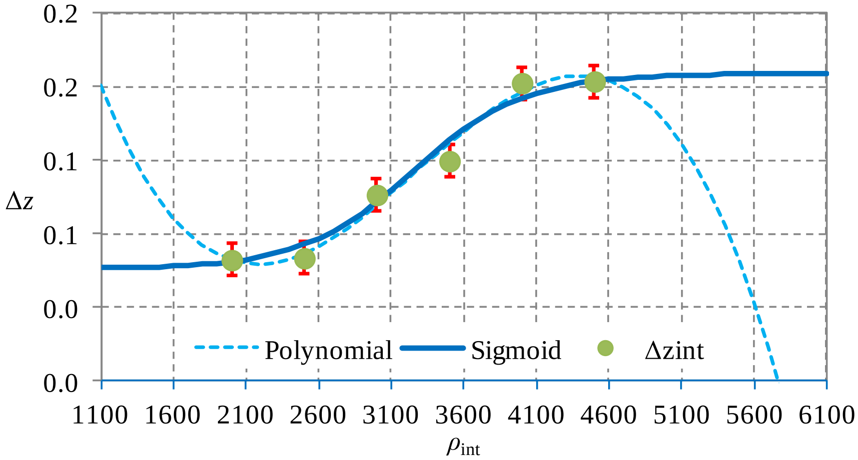

Figure 6. Fitting simulation data zint with two models: Polynomial (dashed line), sigmoid (solid line).

From experiment results, it can be deduced that smaller density will give smaller penetration depth and larger density will give constant value since the intruder already at the bottom of the container. Based on this physical facts another model is proposed using sigmoid function

$$\Delta z_{int} = \frac{\Delta z_{max} - \Delta z_{min}}{1 + \exp[-a(\rho_{int} - \rho_0)]} + \Delta z_{min} , \tag{19}$$

with $\Delta z_{min} = 0.061$ m, $\Delta z_{max} = 0.167$ m, $a = 0.0026$ m^3/kg, $\rho_0 = 3250$ kg/m^3, which hold for all range value of ρ_{int}.

4. Conclusions

Simulation of free falling intruder penetrating bed particles, where the whole system is immersed in fluid, has been performed. Larger density of intruder gives larger penetration depth with the limit of bed depth and container height. This feature can be accomodated by model using sigmoid function but not using polynomial function.

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