New Journal of Physics

The open access journal at the forefront of physics

CrossMark

PAPER

Evolution of cooperation with individual diversity on interdependent weighted networks

Sicheng Liu[1,2], Lin Zhang[1,2,3] and Baokui Wang[4]

[1] School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, People's Republic of China
[2] Engineering Research Center of Complex Product Advanced Manufacturing Systems, Ministry of Education, Beijing 100191, People's Republic of China
[3] Beijing Advanced Innovation Center for Big Date-based Precision Medicine, Beihang University, Beijing 100083, People's Republic of China
[4] Joint Exercises and Training Center, Joint Operations College, National Defense University, Beijing 100091, People's Republic of China

E-mail: zhanglin@buaa.edu.cn

Abstract

The social networks as a complex set of networks fully reflect internal relations of individual interactions between them. Individual as an integral part of networks can show different functions under different levels. In the vast majority of current research realm of spatial evolutionary game, individuals who are often treated as identical peers interact with the local neighbours on a single, isolated same network, even the independent networks extended the content of spatial reciprocity. However, the individual diversity, including gender, wealth and social status and so on, usually is presented within the population. Individual heterogeneity impacts on the evolution of cooperation amongst selfish individuals. With this motivation, here we consider that two forms including Prisoner's Dilemma (PD) and Snowdrift Game (SG) take place on interdependent weighted networks via the mixed-coupling in which individuals participate in different networks of interactions, cooperative behaviour can be maintained. Remarkably, the numerical analysis shows that, as the network interdependence considering individual diversity increases, cooperation thrives on one network joining in PD, the other engaging in SG may be plagued by defectors. Meanwhile, there exists an optimal region of mixed-coupling between networks to persist in cooperation of one network. Furthermore, individual diversity may be a link between non-trivial systems across the network connection, thus probing in how to schedule heterogeneous competitive tasks and services in complex manufacturing systems.

1. Introduction

Cooperation is a key force in the process of biological evolution, from single-celled organisms to more complex multicellular life [1, 2]. But knowing that the elucidation of the emergence and sustentation of cooperative behaviour between selfish or unrelated individuals runs against Darwin's theory of evolution becomes a major problem in system science [3]. Evolutionary game theory [4, 5] provides a unifying mathematical framework to reach win–win in practical problems [6, 7], for example, the social welfare, water resource allocation, service composition and so on. Such Prisoner's Dilemma (PD) and Snowdrift Game (SG) as metaphors or paradigm for studying the emergence of cooperation between selfish individuals remain influential to this day. The PD states the fact that cooperating individuals are prone to exploitation and natural selection can favour cheaters. Thus, defection is the evolutionarily stable strategy, even though every individual would be better off if each chooses to collaborate. So the social dilemma is created. Then, the SG, as a kind of beneficial supplement to discuss cooperative behaviour, has a reverse order of payoffs of the PD, in which the best strategy to adopt depends on what the co-player is doing. Whether or not individuals adopt cooperation is a wait-and-see attitude lack of subjective initiative. Similarly, it represents a social dilemma. To investigate this issue, several novel approaches,

besides studies addressing known rules [8], have been proposed to explain the appearance and survival of cooperation above so-called social dilemma, such as diversity [9–12], social influence [13–15], reputation [16–19], conformity [20],co-evolution [21–24], punishment [25, 26], reward [27, 28], rubust [29], spatial structure [30], to name but a few. Accordingly, spatial structure, referred to as network reciprocity, is known to have an impact on the evolution of cooperation [31–34].

With the advent of the era of big data, the complex network has been studied intensively in network science [35, 36]. The robustness of interacting networks reveals that seemingly irrelevant changes in one network can have catastrophic and very much unexpected consequence in another network [37–40]. The multilayer networks [41–44] helps us understand these phenomena to explore the impact of interdependency on the cooperation. Although evolutionary game on interdependent networks has recently received substantial scholarly attention [45, 46], research mainly focuses on the study of evolutionary dynamics on networks [47–52], interdependent structured populations [53, 54] and game on graph. There are two types of connection mode on networks. The main differences between them depend on whether or not the connection is a physical one. Regarding no physical connection, there are two types, one is utility functions to connect and disconnect between networks [14, 24, 28, 55–63], and the other relies on the coupling of interaction and learning graphs [64]. However, research still focuses on the limited case of a single-mode connection interacting network. Modern systems are coupled together, and therefore should be modelled as mixed-mode connection interdependent networks. To the best of our knowledge, previous works have implemented the same game (typically PD) played on both networks. Different network played different games however can represent distinct environments, characterised by their game rules, which can interact or be coupled. Consider the example of two enterprises and the joint network of business contacts between their employees. The productivity growth in each enterprise may be different, depending on its internal management. Nevertheless, employees from one enterprise may interact with employees from the other enterprise, because they know each other personally or for instance when establishing a business transaction. By connecting with an acquaintance from the other enterprise, each employee can acquire new efficient strategies, which can afterwards be imitated by their contact inside each enterprise. Another natural example is the interaction between transportation of different customer requirements. Moreover, it is common practice to use the identical player to interact with every other on interdependent networks. Individual heterogeneity has great influence on individual interaction as well as individual decision- making process. Furthermore, how to represent this kind of difference among individuals is also an interesting question [11, 14].
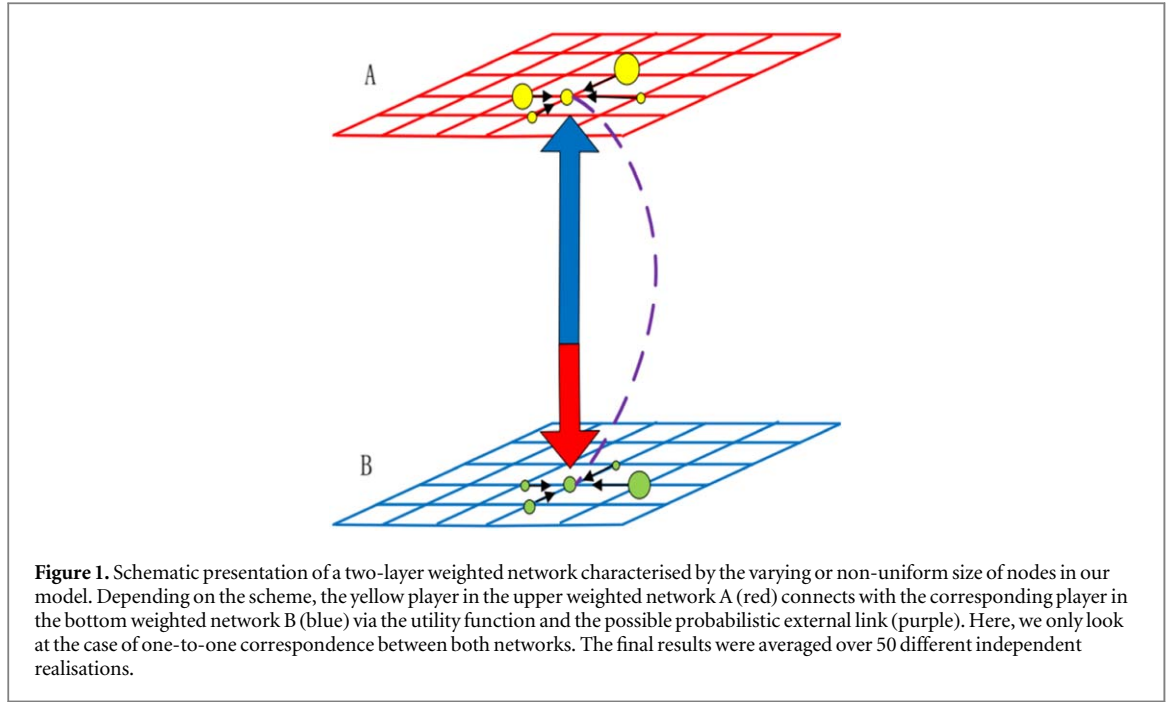
In this paper, here we consider that evolutionary dynamics of Prisoner's Dilemma (PD) and Snowdrift Game (SG) takes place on two-layered lattices via the mixed coupling (i.e. utility and probability) in which each individual endowed as a specific weight take part in different lattices of interactions simultaneously. Meanwhile, strategy imitation is possible between players residing on different networks. The players can adopt two types of distinct strategy imitation characterising the individual behaviour heterogeneity and diversity. Remarkably, the numerical analysis shows that, as the network interdependence considering individual diversity increases, cooperation thrives on one network playing PD, the other engaging in SG may be plagued by defectors. The results indicate that there exists an optimal region of mixed coupling between networks for the growth rates of cooperation to be promoted on one network.

This paper is organised as follows. Section 2 describes different evolutionary game models considering interdependency between weighted networks. Section 3 presents simulation results. Finally, section 4 concludes the paper and discuss the application of Network Physiology.

2. Model

In the classic two-person game, each player can choose either cooperation (*C*) or defection (*D*) strategy, respectively. Allow us to describe below two different evolutionary games, that is the PD and the SG. For PD, each player facing a cooperator yields the reward $R = 1$ or the temptation $T = 1 + r$ when playing as a cooperator or a defector, respectively. On the contrary, if a player meets a defector, it will not receive any payoff ($P = S = 0$) regardless of its strategy. Whereby $1 < 1 + r < 2$ ensures a proper payoff ranking [65]. Here is the weak version of the PD which can capture all relevant aspects of the social dilemma [31]. While in SG, the game follows the same parametrisation except for the situation in which a cooperator when interacting a defector ($S = 1-r$). The payoff matrix of the PD is expressed as follows

$$\begin{array}{c} \\ C \\ D \end{array} \begin{pmatrix} C & D \\ 1 & 0 \\ 1 + r & 0 \end{pmatrix}. \tag{1}$$

Figure 1. Schematic presentation of a two-layer weighted network characterised by the varying or non-uniform size of nodes in our model. Depending on the scheme, the yellow player in the upper weighted network A (red) connects with the corresponding player in the bottom weighted network B (blue) via the utility function and the possible probabilistic external link (purple). Here, we only look at the case of one-to-one correspondence between both networks. The final results were averaged over 50 different independent realisations.

Meanwhile the payoff matrix for the SG is obtained as follows

$$\begin{array}{cc} & \begin{array}{cc} C & \quad D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} 1 & 1-r \\ 1+r & 0 \end{pmatrix}. \end{array} \tag{2}$$

The PD on network A and the SG on another network B are staged on two square lattices with periodic boundary conditions and von-Neumman neighbourhoods, each of size $L \times L$, where each player is thus connected with its $k = 4$ nearest neighbours, as seen in figure 1 for schematic representation. Likewise, a fraction p of players on network A is randomly selected and permitted to generate an external link with a corresponding player on network B. When studying a two-layer weighted network for two different games, we assume that players obtain their accumulated payoff by interacting with their four nearest neighbours on the same network, in addition to that no player has a right to have more than one external link between both networks.

Initially, an equal percentage of strategies (cooperators or defectors) is randomly distributed across networks. P_x and $P_{x'}$ denote the accumulated payoff of player x playing the PD and accumulated payoff of its partner x' playing the SG, respectively. Here, player interacting with its neighbours can only occupy sites on the same network.

To quantitatively consider the utility of each player on interdependent networks, we will take the following way

$$\begin{cases} U_x = \alpha P_x \lambda_1 + (1-\alpha) P_{x'} \lambda_2 \\ U_{x'} = \alpha P_{x'} \lambda_1 + (1-\alpha) P_x \lambda_2 \end{cases}, \tag{3}$$

where $\alpha\ (0 < \alpha < 1)$ is a utility coupling coefficient determining the strength of the external link, i.e. small its value the higher the potential increase of utility coupling of two players that may be connected by the external link. In particular, $\alpha = 0$ leads to the fact that U_x is dominated by $P_{x'}\lambda_2$ while U_x is determined by $P_x\lambda_1$ for $\alpha = 1$. Without loss of generality [55], we here use a fixed value $\alpha = 0.5$. $\lambda_1\ (0 < \lambda_1 \leqslant 1)$ denotes the individual weight of focal player x, i.e. λ_1 is set to be 1.0 for any player on the same network and λ_1 is random number for the players on another network. λ_2 has the same implication as λ_1.

Then, player x selects one of its nearest neighbours y (including the corresponding neighbour on the other network from the possible external link) at random, and imitates its strategy S_y with a probability based on the Fermi function. Here the strategy transfer between different networks can be permitted.

$$W(S_y \rightarrow S_x) = w_y \frac{1}{1 + \exp[(U_x - U_y)/K]}, \tag{4}$$

where K quantifies the amplitude of noise [65], which is usually related to the errors in decision making or imperfect information transfer over players. Based on the previous research [66], we fix $K = 0.1$ throughout this

work. The scaling factor w_y represents the individual behaviour diversity and depicts the impact of strategic diffusion of player y in heterogeneous layers

$$w_y = \begin{cases} 1, & \text{if } y \in A \\ 1 - \beta\dfrac{N_x}{G}, & \text{if } y \in B \end{cases}, \tag{5}$$

where y belonging to network A (i.e. $y \in A$) is regarded as an influential player who can convince its neighbours to adopt its strategy with a higher payoff when compared to network B (i.e. $y \in B$). N_x is the number of players in that group $G = k + 1$ centred on the corresponding one x' on network B, which adopts the same strategy as player x on network A. k is four nearest neighbours of a player on network. β is the multiplicative factor $0 < \beta < 1$. Different from the previous research [14, 24, 67], we define the multiplicative factor β related to strategy transfer process of individual heterogeneity as follow

$$\beta = F\chi, \tag{6}$$

where χ denotes a uniformly distributed number in [0, 1], and satisfies simultaneously $\int_0^1 F\chi \, \mathrm{d}\chi = 0$ which ensures that the average value of the multiplicative factor across both network is zero. F is the tunable factor, reflecting a kind of heterogeneity attribute. Obviously, $F = 0$ will turn it back into a traditional form [67]. Based on the existing research [59], we merely consider $F = 1.0$ in this work. Therefore, all players on two networks will be divided into two types of players regarding the strategic diffusion, to reflect the difference in individual behaviour within the real-world populations as far as possible.

The linear system size was varied from $L = 100–300$ in order to avoid finite size effects. We check that the presented results do not qualitatively change for reasonable variations. Also, simulations of the model were performed using the synchronous update, namely each player on different networks had a chance to interact in their respective neighbourhood and then all sites are updated simultaneously through competition with a randomly chosen neighbour once on average during a Monte Carlo step (MCS). If not stated before, the equilibration is required up to MCS $= 2 \times 10^4$ steps and then sampled by another 2×10^3 steps. These final results were averaged over 50 different independent realisations to further improve accuracy.

3. Results

Here we first explore how the network interdependence influences the evolution of cooperation on two weighted networks. Figure 2 presents the frequency of cooperators on interdependent networks as a function of r for different p. Defection is the best choice for the rational individual in the PD while the best action depends on the opponent in SG: to defect if the other cooperates, but to cooperate if the other defects. As a result, evolution under different payoff matrix leads to an equilibrium frequency for cooperators on both networks. For small r, interdependence promotes cooperation on network A and B. However, for intermediate and high r, the fraction of cooperators on network B ρ_{bc} shown in figure 2(b) is higher than that on network A ρ_{ac} presented in figure 2(a), which presents a step-like behaviour. One reason is the sharing information lessens their propensity to change for identical strategies (high payoff). Another reason is that the fraction of cooperators decreases with increasing r due to spatial structure. Hence the effect and influence of interdependence are mutual and dialectical on two weighted networks. Figure 3 shows the frequency of cooperators on interdependent networks as a function of p for different r. The fraction of cooperators in the PD presents a trend of rapid increase for small p, and then increases slowly for intermediate and high r, as shown in figure 3(a). While that in the SG shows an opposite trend, which slightly decreases in the entire range of p, as shown in figure 3(b). The main reason is also believed that the difference in the interaction of each game leads to this. Thus, imbalance occurs in two weighted networks, one party enhances cooperation while the other party inhibits.

To study the specific changes of cooperation on each weighted network with probability p, we define a set of the growth rate of cooperation G_c:

$$G_c = \mathrm{d}\rho_c / \mathrm{d}p, \tag{7}$$

where $\mathrm{d}\rho_c$ and $\mathrm{d}p$ are the gradient of ρ_c and p, respectively. Hence, G_{ac} and G_{bc} denote the growth rate of cooperation on network A and B correspondingly. A valuable insight is that G_{ac} increases above baseline over the entire range of p while G_{bc} decreases generally under baseline below, as shown in figure 4. This illustrates that network interdependence can promote persistence of cooperation on network A. However, network B is not so lucky that it apparently tends to reduce the proportion of cooperators for the full range of p. But interestingly, different r for the same model also have different impacts on the growth rate of cooperation with increasing p. One reason is that the sharing information inspires their aspiration to keep identical strategies for high payoff. Another reason is that the spatial structure is not in favour of cooperative behaviour in SG (network B) for high r.
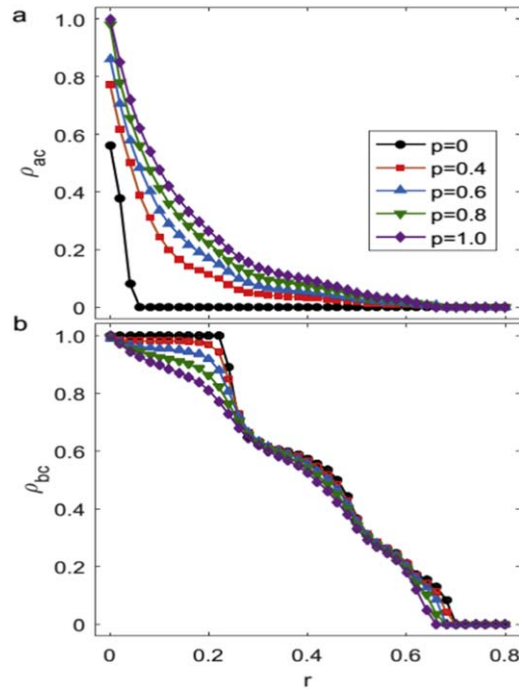
Figure 2. The frequency of cooperators (ρ_c) on interdependent weighted networks for different values of p. Panel (a) shows the frequency of cooperators (ρ_{ac}) for the PD played in network A as functions of r. Panel (b) shows the corresponding results (ρ_{bc}) for the SG played in network B. Cooperation is dramatically enhanced on for both games, dominating over the entire ranges of r. The final results were averaged over 50 different independent realisations.
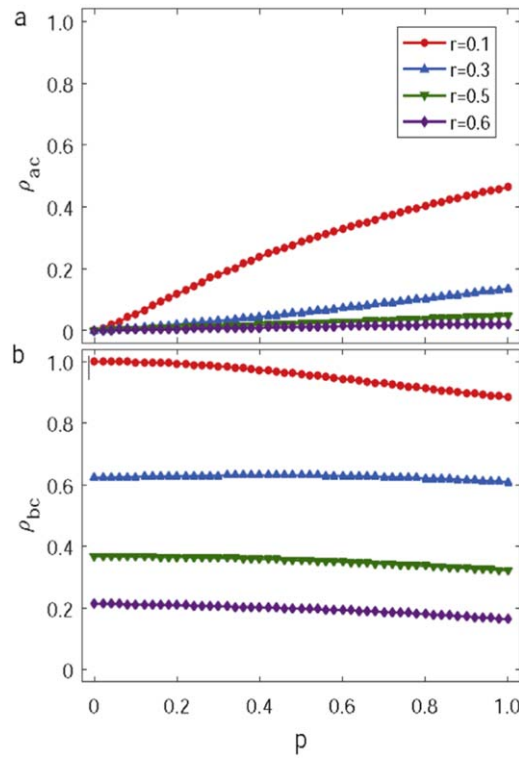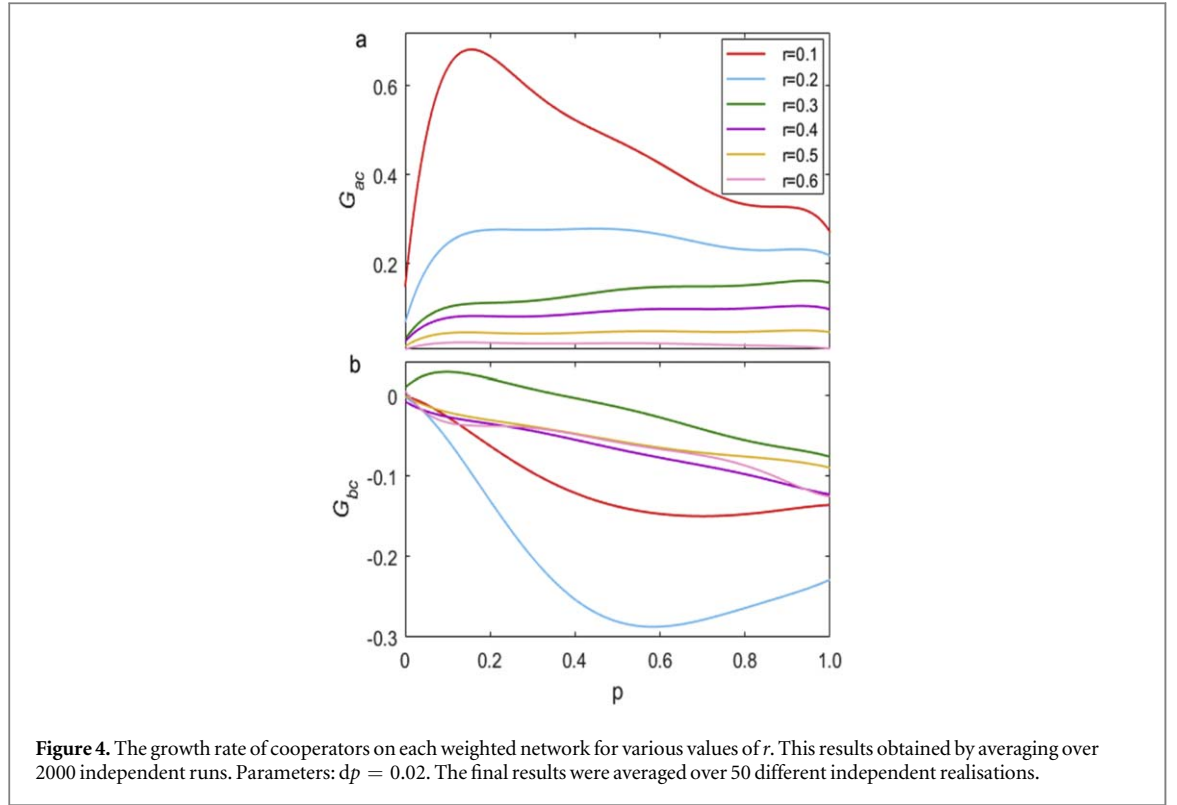

Figure 3. The frequency of cooperators on interdependent weighted networks for different values of r. Results are shown as functions of p for the PD (upper panel) and the SG (lower panel). The final results were averaged over 50 different independent realisations.

An intriguing phenomenon is found that there exists an optimal region of p maximising the growth rate of cooperation on network A, which has a nonlinear characteristic rather than monotony. There is with nothing to break the monotony for the growth rate of cooperation on network B, compared to network A.

Figure 4. The growth rate of cooperators on each weighted network for various values of *r*. This results obtained by averaging over 2000 independent runs. Parameters: dp = 0.02. The final results were averaged over 50 different independent realisations.

Information sharing (strategy choice) promotes prosocial behaviour. Moreover, given that spatial structure maintains cooperation in PD while it eliminates cooperation in SG, the optimal value of *p* that is a key point enhances growth-rate of cooperation in network A but suppresses it in network B. So the network interdependence manifested the unnecessarily strengthened benefit for cooperative behaviour. Figure 5 illustrates the frequency for different links at equilibrium for different values of *r* regardless of whether there is a connection between them or not. *C–C* and *D–D* stand for mutual cooperation strategy and mutual defection strategy of two corresponding players between networks respectively, while *C–D* represents the conflict strategy. It is impressive that the frequency for *C–C* links at $r = 0.1$ dramatically promotes or maintains the evolution of cooperation while that at $r = 0.6$ eliminates or inhibits the cooperative behaviour. By contrast, the frequency for *D–D* links is a bit similar to the state the frequency for *C–C* links exchanges position both $r = 0.1$ and $r = 0.6$. The frequency for *C–D* links at $r = 0.3$ dominates the entire range of *p*. However, it is useful to noting that the frequency for *C–D* links and *D–D* links at $r = 0.3$ can promote persistence of cooperation. The reason is that it is best to defect regardless of the opponent's decision in the PD while depending on the opponent is the optimal action in the SG. Analysis of this phenomenon shows that the individual diversity between interdependent weighted networks refers to coupling effect, which can significantly influence cooperative behaviour on network A directly. Further study to find out the role of individual diversity between weighted networks quantitatively, we apply the simplified correlation coefficient to elaborate the relationship between *C–C* strategies and networks A

$$RR_{ac} = (\rho_{cc} - \rho_{ac}^2)/(\rho_c - \rho_{ac}^2), \tag{8}$$

where ρ_{cc} denotes the fraction of *CC* strategies of two corresponding individuals between weighted networks. Hence when ρ_{ac} satisfies the range of 0 to 1, R_{ac} becomes greater than 0 for different *r*. So cooperative behaviour across interdependent weighed networks is spread through the individual diversity, which can be of certain signs to the evolution of cooperation.

Figure 6 shows the result of our simulations, namely, the frequency of cooperators on both weighted networks as a function of *p*, for several *r*. The condition that a cooperator on network A (B) has an external link with the corresponding individual on network B (A) for *r* is represented by r_e. Otherwise, the condition that a cooperator on network A (B) has not an external link with the corresponding player on network B (A) for *r* is denoted as r_i. Note some similarity between figures 3(b) and 6(b), hardly any variation with changing *p*. The spatial structure promotes the evolution of cooperation for the PD while it eliminates cooperation if the cost-to-benefit ratio of cooperation *r* is high for the SG. However, since sharing information about strategy choice
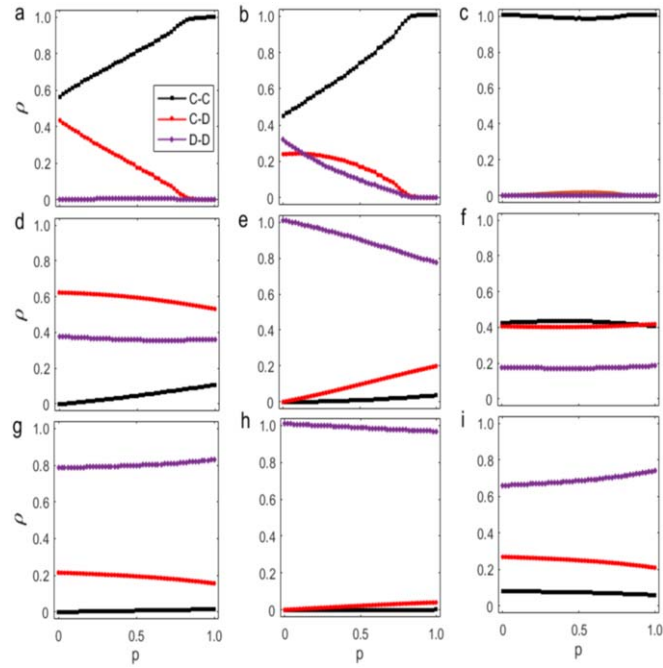
Figure 5. The frequency for different links at $r = 0.1, 0.3$ and 0.6 from top to bottom. Panels (a)–(d)–(g) correspond to results obtained between interdependent weighted networks. While panels (c)–(f)–(i) and panels (b)–(e)–(h) correspond to results obtained on network A and B, respectively. The final results were averaged over 50 different independent realisations.
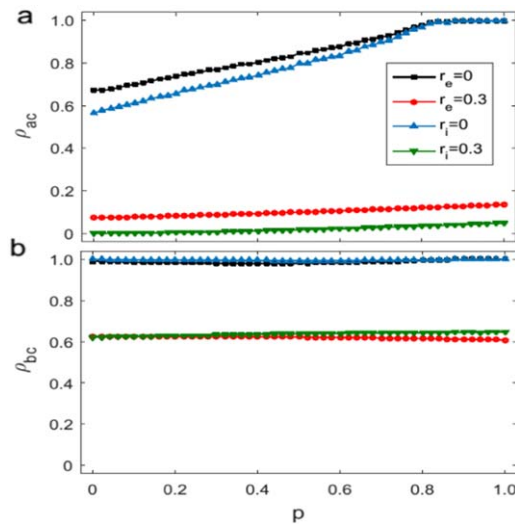

Figure 6. The frequency of cooperators on both weighted networks as a function of p, for various values r of r_e and r_i with corresponding individuals on another network. Results correspond to those presented in figure 3. The frequency of cooperators for the network A where PD is being played is shown in the upper panel, while the corresponding curves for the network B where SG is being played is presented in the bottom panel. For simplicity, we include only some of the r values studies in figure 3. The final results were averaged over 50 different independent realisations.

between individuals residing on both different networks reinforces the interactive relationship between the two networks for cooperative behaviour, it brings the positive effect that whole levels of cooperation on network B persist at fixed r. So, that is why the network A where PD is being played is affected by the probability p of interconnection between both networks but network B where SG is played is unaffected by varying p. It means that individual diversity is beneficial for cooperative behaviour between two networks to a certain extent.

Figure 7 shows characteristic snapshots in such a stationary state of given values, which provide an intuitive understanding of the positive effects of individual diversity between interdependent weighted networks on the evolution of cooperation. With the effect of individual diversity, the pattern tends of cooperators and defectors are partly similar in the two weighted networks for different MCS. Cooperators in the middle domain of figure 7
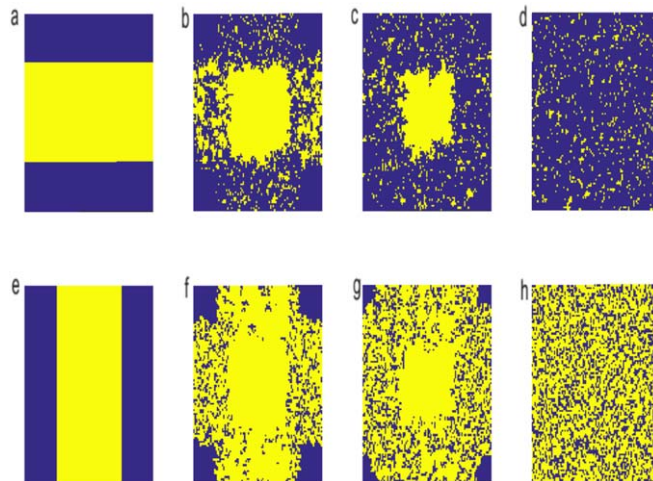
Figure 7. Snapshots of typical patterns of cooperators (yellow) and defectors (blue) on two interdependent weighted square lattices at 0, 20, 40 and 20 000 MCS from left to right. Panels (a)–(d) correspond to results obtained on network A, while panels (e)–(h) correspond to results obtained on network B. Parameter values are $r = 0.3$ and $p = 0.6$. The final results were averaged over 50 different independent realisations.

(panels (b) and (c)) on network A can survive by forming large, compact clusters, thus contributing to reduce exploitation by defectors. Only a small number of cooperators occur in the active region of defectors. Meanwhile, the proportion of cooperators on networks B booms under favourable conditions that are built by network interdependence adding individual diversity. Because of generating small filament-like clusters, cooperators on network A have been struggling for survival in the siege of defectors over time. By contrast, cooperation on networks B goes through the whole networks with the network interdependence's help. As a consequence, the network interdependence adding individual diversity brings out significantly constraint, in turn, promotes the persistence of cooperation.

4. Summary

We have explored the evolutionary dynamics of cooperation in the two different evolutionary games (the PD Game and the SG) played within an interdependent weighted network via the mixed coupling (i.e. utility and probability) in which individuals take part in different networks of interactions simultaneously, cooperative behaviour is maintained. We demonstrated that as the network interdependence considering individual diversity increases, cooperation thrives on one network playing PD, the other engaging in SG may be plagued by defectors. Meanwhile, there exists an optimal region of mixed coupling between networks for the growth rates of cooperation to be promoted on one network. Information sharing (strategy choice) between two networks promotes prosocial behaviour. Moreover, given that spatial structure maintains cooperation in PD while it eliminates cooperation in SG, the optimal value of p that is a key point can enhance growth-rate of cooperation in network A but suppress it in network B. Besides the application of task-to-service interactions in complex manufacturing systems, our work may also contribute to the development of Network Physiology [68–71], where multiple interconnected networks comprise human organisms, and these robust interacting networks operate and generate different physiological states (e.g. light or deep sleep) characterised by the distinct network topology and function, the same way like the social network generating cooperation behaviours. To understand how diverse organ systems dynamically interact and collectively behave as a network is benefit for promoting health and combating disease (e.g. headaches) as a result of organ interactions. Furthermore, this diversity is intrinsically related to a non-trivial organisation of cooperation across the network layers, thus providing a new way out for cooperation in advanced manufacturing industry.

Acknowledgments

ORCID iDs

Baokui Wang https://orcid.org/0000-0003-2469-1531

References

[1] Szathmary E and Smith J M 1995 *Science* **374** 227
[2] Roca C P, Cuesta J A and Sánchez A 2009 *Phys. Rev.* E **80** 046106
[3] Pennisi E 2005 *Science* **309** 93
[4] Axelrod R and Hamilton W D 1981 *Science* **211** 1390
[5] Smith J M 1974 *J. Theor. Biol.* **47** 209
[6] Santos F C and Pacheco J M 2005 *Phys. Rev. Lett.* **95** 098104
[7] Santos F C, Pacheco J M and Lenaerts T 2006 *Proc. Natl Acad. Sci.* **103** 3490
[8] Nowak M A 2006 *Science* **314** 1560
[9] Santos F C, Santos M D and Pacheco J M 2008 *Nature* **454** 213
[10] Perc M and Szolnoki A 2008 *Phys. Rev.* E **77** 011904
[11] Zhu C-J, Sun S-W, Wang L, Ding S, Wang J and Xia C-Y 2014 *Physica* A **406** 145
[12] Qin J, Chen Y, Fu W, Kang Y and Perc M 2018 *IEEE Access* **6** 5003
[13] Portillo I G 2014 *Physica* A **394** 198
[14] Meng X-K, Xia C-Y, Gao Z-K, Wang L and Sun S-W 2015 *Phys. Lett.* A **379** 767
[15] Jiang L-L, Li W-J and Wang Z 2015 *Sci. Rep.* **5** 14657
[16] Milinski M, Semmann D and Krambeck H-J 2002 *Nature* **415** 424
[17] Wang C, Wang L, Wang J, Sun S and Xia C 2017 *Appl. Math. Comput.* **293** 18
[18] Yang H-X and Wang Z 2017 *J. Stat. Mech.* 023403
[19] Yang H-X and Yang J 2019 *Physica* A **523** 886
[20] Yang H-X and Chaos Tian L 2017 *Solitons Fract.* **103** 159
[21] Perc M and Szolnoki A 2010 *BioSystems* **99** 109
[22] Fu F, Wu T and Wang L 2009 *Phys. Rev.* E **79** 036101
[23] Szolnoki A and Perc M 2014 *Europhys. Lett.* **108** 28004
[24] Wang Z, Szolnoki A and Perc M 2014 *New J. Phys.* **16** 033041
[25] Wang Z, Xia C-Y, Meloni S, Zhou C-S and Moreno Y 2013 *Sci. Rep.* **3** 3055
[26] Yang H-X and Chen X 2018 *Appl. Math. Comput.* **316** 460
[27] Szolnoki A and Perc M 2010 *Europhys. Lett.* **92** 38003
[28] Wang Z, Szolnoki A and Perc M 2014 *J. Theor. Biol.* **349** 50
[29] Shao S, Huang X, Stanley H E and Havlin S 2014 *Phys. Rev.* E **89** 032812
[30] Yang H-X and Yang J 2019 *Europhys. Lett.* **124** 60005
[31] Nowak M A and May R M 1992 *Nature* **359** 826
[32] Hauert C and Doebeli M 2004 *Nature* **428** 643
[33] Sysi-Aho M, Saramäki J, Kertész J and Kaski K 2005 *Eur. Phys. J.* B **44** 129
[34] Roca C P, Cuesta J A and Sánchez A 2009 *Phys. Life Rev.* **6** 208
[35] Albert R and Barabási A-L 2002 *Rev. Mod. Phys.* **74** 47
[36] Boccaletti S, Latora V, Moreno Y, Chavez M and Hwang D-U 2006 *Phys. Rep.* **424** 175
[37] Buldyrev S V, Parshani R, Paul G, Stanley H E and Havlin S 2010 *Nature* **464** 1025
[38] Huang X, Gao J, Buldyrev S V, Havlin S and Stanley H E 2011 *Phys. Rev.* E **83** 065101
[39] Baxter G, Dorogovtsev S, Goltsev A and Mendes J 2012 *Phys. Rev. Lett.* **109** 248701
[40] Gao J, Buldyrev S V, Stanley H E and Havlin S 2012 *Nat. Phys.* **8** 40
[41] Boccaletti S, Bianconi G, Criado R, Del Genio C I, Gómez-Gardenes J, Romance M, Sendina-Nadal I, Wang Z and Zanin M 2014 *Phys. Rep.* **544** 1
[42] Kivelä M, Arenas A, Barthelemy M, Gleeson J P, Moreno Y and Porter M A 2014 *J. Complex Netw.* **2** 203
[43] Wang Z, Wang L, Szolnoki A and Perc M 2015 *Eur. Phys. J.* B **88** 124
[44] Gosak M, Marković R, Dolenšek J, Rupnik M S, Marhl M, Stožer A and Perc M 2018 *Phys. Life Rev.* **24** 118
[45] Li A, Cornelius S P, Liu Y-Y, Wang L and Barabási A-L 2017 *Science* **358** 1042
[46] Lu Z, Zhang L and Wang L 2018 *Int. J. Robust Nonlinear Control* **28** 2560
[47] Gómez-Gardenes J, Gracia-Lázaro C, Floría L M and Moreno Y 2012 *Phys. Rev.* E **86** 056113
[48] Gómez-Gardenes J, Reinares I, Arenas A and Floría L M 2012 *Sci. Rep.* **2** 620
[49] Perc M, Gómez-Gardeñes J, Szolnoki A, Floría L M and Moreno Y 2013 *J. R. Soc. Interface* **10** 20120997
[50] Gomez S, Diaz-Guilera A, Gomez-Gardenes J, Perez-Vicente C J, Moreno Y and Arenas A 2013 *Phys. Rev. Lett.* **110** 028701
[51] Szolnoki A and Perc M 2018 *New J. Phys.* **20** 013031
[52] Wang B, Pei Z and Wang L 2014 *Europhys. Lett.* **107** 58006
[53] Tang C, Wang Z and Li X 2014 *PLoS One* **9** e88412
[54] Zhang L, Ying L, Zhou J, Guan S and Zou Y 2016 *Phys. Rev.* E **94** 032307
[55] Wang Z, Szolnoki A and Perc M 2012 *Europhys. Lett.* **97** 48001
[56] Wang Z, Szolnoki A and Perc M 2013 *Sci. Rep.* **3** 1183
[57] Wang Z, Szolnoki A and Perc M 2013 *Sci. Rep.* **3** 2470
[58] Jin Q, Wang L, Xia C-Y and Wang Z 2014 *Sci. Rep.* **4** 4095
[59] Xia C-Y, Meng X-K and Wang Z 2015 *PLoS One* **10** e0129542
[60] Meng X, Sun S, Li X, Wang L, Xia C and Sun J 2016 *Physica* A **442** 388
[61] Luo C, Zhang X, Liu H and Shao R 2016 *Physica* A **450** 560
[62] Deng X, Zhang Q, Deng Y and Wang Z 2016 *Sci. Rep.* **6** 23024
[63] Luo C and Zhang X 2017 *Commun. Nonlinear Sci. Numer. Simul.* **42** 73
[64] Wang Z, Wang L and Perc M 2014 *Phys. Rev.* E **89** 052813
[65] Szabó G and Fath G 2007 *Phys. Rep.* **446** 97

[66] Szolnoki A, Perc M and Szabó G 2009 *Phys. Rev.* E **80** 056109

[67] Szolnoki A and Perc M 2013 *New J. Phys.* **15** 053010

[68] Ivanov P C and Bartsch R P 2014 *Network Physiology: Mapping Interactions Between Networks of Physiologic Networks in Networks of Networks: the last Frontier of Complexity* (Berlin: Springer) pp 203–22

[69] Bartsch R P, Liu K K, Bashan A and Ivanov P C 2015 *PLoS One* **10** e0142143

[70] Liu K K, Bartsch R P, Ma Q D and Ivanov P C 2015 Major component analysis of dynamic networks of physiologic organ interactions *J. Phys.: Conf. Ser.* **640** 012013

[71] Bashan A, Bartsch R P, Kantelhardt J W, Havlin S and Ivanov P C 2012 *Nat. Commun.* **3** 702