IOP Publishing Physica Scripta

Phys. Scr. 95 (2020) 035301 (4pp) https://doi.org/10.1088/1402-4896/ab553f

Quantum wave packet approach to oscillation of Z^0 decay neutrinos

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Received 2 September 2019, revised 31 October 2019
Accepted for publication 7 November 2019
Published 28 January 2020

Abstract

In this paper, we consider the neutrino and anti-neutrino emerging from the Z^0 decay and the wave packet quantum mechanical approach is used to obtain the probability of detecting a neutrino of flavor α and the corresponding anti-neutrino of flavor β at the positions L and $\bar{L}$, respectively. For this purpose, it is essential to construct a bipartite entangled state of neutrino and anti-neutrino wave packets. The result of this calculation is the same as the result obtained before in quantum field theory approach.

Keywords: neutrino oscillation, quantum mechanics approach, wave packet

(Some figures may appear in colour only in the online journal)

1. Introduction

Neutrino oscillation, as one of the most interesting quantum mechanical phenomena, provides an opportunity for studying some aspects of quantum mechanics such as entanglement and coherency [1–5]. In particular, there exist three kinds of entanglements:

- In the case of neutrinos emerging from two body decays, neutrinos and their accompanying charged leptons are entangled due to the energy-momentum conservation. Hence, in order to have neutrino oscillation, one needs to detect neutrinos as well as the corresponding charged leptons. It is in contrary to the realistic experiments. However, the quantum uncertainties of energy and momentum allow a disentangling between neutrinos and their copartner such that the oscillation condition according to realistic experiments is provided [1].
- One can stablish the entanglement statement between various modes of single particle, for instance see [6]. In the case of neutrinos, the entanglement properties between different modes of the neutrino flavors were investigated [2, 3].
- Neutrino and anti-neutrino emerging from neutral current interactions are entangled with respect to either their flavor modes or mass modes as well as the energy momentum conservation [7]. In this paper, we study the neutrino oscillation theory under this kind of entanglements in quantum mechanics framework.

In fact, the neutrino oscillation phenomena are observed if neutrinos have specified initial flavor. It is possible for neutrinos emerging from the charged current processes. However, since all flavors of neutrinos are created with equal probability in the Z^0 decay, it is impossible to determine the flavor eigenstate of neutrinos in these processes. Therefore, we cannot observe the oscillation pattern by detecting either the neutrino or anti-neutrino emerging from the Z^0 decay. Meanwhile, it was shown, nontrivially, if both neutrino and antineutrino are detected, one can have usual oscillation pattern between detectors [7]. It is a manifestation of the entanglement between neutrino and anti-neutrino.

On the other hand, there exist some debates on the basic issues of the theory of neutrino oscillations [8]. In particular, in a number of papers, neutrinos were treated as plane waves. If it is applied accurately, it does not lead to neutrino oscillation in general [9, 10]. In contrast, neutrino oscillations can be consistently described either in the quantum-mechanical (QM) wave packet approach, or within a quantum field theoretic (QFT) framework [10, 11]. In the QM approach,

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neutrinos are considered as a wave packet, and the amplitude and probability of neutrino oscillation are manually normalized. (This approach was first given in [12] and developed in [13].) But in the framework of QFT, the particle accompanying neutrinos is written in the form of a wave packet, and the neutrinos themselves play the role of a propagator, and the amplitude and probability of oscillation are normal in themselves. (This approach was first formulated by [14].) For neutrinos emerging from the Z^0 decay, the QFT treatment was considered in [15]. It was shown that the oscillation pattern ceases if the distance between the detectors is larger than the coherence length, even though both neutrino and antineutrino states may be coherent. In this paper, we reconsider this problem in QM wave packet approach. This study shows the consistency of these approaches.

In the following section, the probability of detecting a neutrino with flavor α in one detector and the corresponding anti-neutrino with flavor β in the other detector is calculated. In the last section we will discuss about our results.

2. Calculation of oscillation probability by QM wave packets approach

Generally, we can write the one particle state with mass m in momentum space as follows:

$$|\mathbf{A}\rangle = \int [\mathrm{d}p] f_A(\mathbf{p}, \mathbf{p_i})|\mathbf{p}\rangle_A, \tag{1}$$

where $f_A(\mathbf{p}, \mathbf{p_i})$ is the wave function in momentum space or momentum distribution function with the mean momentum $\mathbf{p}_i$ taken at time $t = 0$, and $|\mathbf{p}\rangle_A$ belongs to the one-particle states of momentum $\mathbf{p}$, also at time $t = 0$. The following notation is used

$$[\mathrm{d}p] = \frac{\mathrm{d}^3 p}{(2\pi)^3\sqrt{2E_A(\mathbf{p})}}, \tag{2}$$

where $E_A(\mathbf{p}) = \sqrt{\mathbf{p}^2 + m_A^2}$. The normalization of free states is chosen as follows

$$\langle \mathbf{p}|\mathbf{p}'\rangle_A = 2E_A(\mathbf{p})(2\pi)^3\delta^3(\mathbf{p} - \mathbf{p}'), \tag{3}$$

so that $\langle \mathbf{A}|\mathbf{A}\rangle = 1$ if

$$\int \frac{\mathrm{d}^3 p}{(2\pi)^3} |f_A(\mathbf{p})|^2 = 1. \tag{4}$$

The coordinate-space wave function $\Psi_A(t, \mathbf{x})$ is the Fourier transform of $\langle \mathbf{p}|\mathbf{A}(t)\rangle$:

$$\Psi_A(t, \mathbf{x}) \equiv \langle \mathbf{x}|\mathbf{A}(t)\rangle = \int \frac{\mathrm{d}^3 p}{(2\pi)^{3/2}} f_A(\mathbf{p}, \mathbf{p}_i) \mathrm{e}^{\mathrm{i}\mathbf{p}\mathbf{x} - \mathrm{i}E_A(\mathbf{p})t}.$$

Wave packet centered in $\mathbf{x}_0$ at time t_0 is built with the help of the space-time translation operator $\exp(\mathrm{i}p.x_0)$, where $x_0 = (t_0, \mathbf{x}_0)$. If a wave packet in momentum space is given by

$$F(\mathbf{p}, \mathbf{p}_i, \mathbf{x}_0, t_0) = f(\mathbf{p}, \mathbf{p}_i)\exp[\mathrm{i}E(\mathbf{p})t_0 - \mathrm{i}\mathbf{p}.\mathbf{x}_0], \tag{5}$$

for the corresponding wave packet in coordinate space centered at the space and time $\mathbf{x}_0$ and t_0, we will have

$$\Psi(t, \mathbf{x}, \mathbf{p}_i, t_0, \mathbf{x}_0) = \int \frac{\mathrm{d}^3 p}{(2\pi)^{3/2}} f(\mathbf{p}, \mathbf{p}_i)\mathrm{e}^{\mathrm{i}\mathbf{p}.(\mathbf{x}-\mathbf{x}_0) - \mathrm{i}E(\mathbf{p})(t - t_0)}. \tag{6}$$

Usually, the wave packets are taken to be of the Gaussian form

$$f(\mathbf{p}, \mathbf{p}_i) = \left(\frac{2\pi}{\sigma_p^2}\right)^{3/4} \exp\left(-\frac{(\mathbf{p} - \mathbf{p}_i)^2}{4\sigma_p^2}\right), \tag{7}$$

where σ_p is the momentum uncertainty of neutrino state and it is assumed to be very smaller than the corresponding mean momentum. If the energy is expanded up to second order around the average momentum $_{pi}$ we have

$$E(\mathbf{p}) \simeq E(\mathbf{p}_i) + \mathbf{v}_i(\mathbf{p} - \mathbf{p}_i), \tag{8}$$

where $E(\mathbf{p}_i) = \sqrt{\mathbf{p_i}^2 + \mathbf{m_i}^2}$ and $\mathbf{v}_i = \dfrac{\mathbf{p_i}}{E_i}$, the wave function in coordinate space is thus given by

$$\Psi(t, \mathbf{x}) \simeq \exp\left[\mathrm{i}\mathbf{p}_i.\mathbf{x} - \mathrm{i}E_i(\mathbf{p}_i)t - \frac{(\mathbf{x} - \mathbf{v}_i t)^2}{4\sigma_x^2}\right]. \tag{9}$$

In the case of Z^0 decay, we know that it is blind to the neutrino flavors. This means that all of three flavors and also three mass eigenstates are created with the same probability amplitude, which are manifested respectively by the following relations:

$$|\nu_Z\rangle = \frac{1}{\sqrt{3}} \sum_{\alpha=e,\mu\tau} |\bar{\nu}_\alpha\rangle|\nu_\alpha\rangle, \tag{10}$$

$$|\nu_Z\rangle = \frac{1}{\sqrt{3}} \sum_{i=1,2,3} |\bar{\nu}_i\rangle|\nu_i\rangle. \tag{11}$$

Neutrino and anti-neutrino are produced during the time interval $\Delta t \sim \frac{1}{\Gamma}$ where Γ is the decay width of Z^0 and in a region with the uncertainty $\Delta x \sim v_Z\Delta t$ in which v_Z is the Z^0 group velocity. Therefore, one must describe the neutrino and anti-neutrino by a localized wave function with the coordinates of space-time (x, t) and $(\bar{x}, \bar{t})$, respectively. Moreover, the neutrino and anti-neutrino must be in a bipartite entangled state due to the conservation of momentum. Hence, the bipartite neutrino and anti-neutrino wave function is written as follows:

$$|\nu_Z; \mathbf{x}, t; \bar{\mathbf{x}}, \bar{t}\rangle = \frac{1}{\sqrt{3}} \sum_{i=1}^{3} \Psi_i^S(\mathbf{x}, t; \bar{\mathbf{x}}, \bar{t})|\nu_i\rangle|\bar{\nu}_i\rangle, \tag{12}$$

where

$$\begin{aligned}\Psi_i^S(\mathbf{x}, t; \bar{\mathbf{x}}, \bar{t}) &= \mathcal{N}\int \frac{\mathrm{d}^3 p}{(\sqrt{2\pi})^3} \int \frac{\mathrm{d}^3\bar{p}}{(\sqrt{2\pi})^3} f^S(\mathbf{p}, \mathbf{p}_i) \\ &\times \bar{f}^S(\bar{\mathbf{p}}, \bar{\mathbf{p}}_i)\delta^3(\mathbf{p} - \bar{\mathbf{p}}) \times \exp[-\mathrm{i}E(\mathbf{p})(t - t_P) \\ &+ \mathrm{i}\mathbf{p}(\mathbf{x} - \mathbf{x}_P) - \mathrm{i}\bar{E}(\bar{\mathbf{p}})(\bar{t} - t_P) + \mathrm{i}\bar{\mathbf{p}}(\bar{\mathbf{x}} - \mathbf{x}_P)]. \end{aligned} \tag{13}$$

Here, $f^S(\mathbf{p}, _{pi})$ and $\bar{f}^S(\bar{\mathbf{p}}, \bar{\mathbf{p}}_i)$ are the momentum distribution functions of neutrino and anti-neutrino with $_{pi}$ and $\bar{\mathbf{p}}_i$ being the corresponding mean momenta and $E(\mathbf{p}) = \sqrt{m^2 - \mathbf{p}^2}$. The superscript S at $f^S(\mathbf{p}, _{pi})$ and $\bar{f}^S(\bar{\mathbf{p}}, \bar{\mathbf{p}}_i)$ indicates that they correspond to the neutrino and antineutrino produced in source. The factor of Dirac delta is added to make sure that the momentum is conserved (we chose the Z^0 rest frame). We put the Gaussian momentum wave function similar to (7) in (13). The momentum widths of $f^S(\mathbf{p}, _{pi})$ and $\bar{f}^S(\bar{\mathbf{p}}, \bar{\mathbf{p}}_i)$ are assumed to be σ_{p_S} and $\bar{\sigma}_{\bar{p}_S}$, respectively. Using (8) and performing the corresponding integrations with disregarding the normalization coefficient we obtain from (13)

$$\Psi_i^S(\mathbf{x}, t; \bar{\mathbf{x}}, \bar{t}) \propto \exp\left[-\mathrm{i}E(\mathbf{p}_i)(t+\bar{t}) + \mathrm{i}\mathbf{p}_i(\mathbf{x}+\bar{\mathbf{x}}) - \frac{(\mathbf{x}+\bar{\mathbf{x}}-\mathbf{v}_i(t+\bar{t}))^2}{4\sigma_{xS}^2}\right]. \tag{14}$$

Because this calculation is done in Z^0 rest frame, the group velocities of the neutrino and antineutrino are equal and we denote it by $\mathbf{v}_i$ (subscript i indicates the mass eigenstate tag). Due to the momentum conservation, we assume $\sigma_{p_S} = \bar{\sigma}_{\bar{p}_S}$ and take $\sigma_{xS} = 1/\sigma_{p_S}$.

On the other hand, since the detection processes are essentially time independent, detected states have no time dependence. Therefore, the wave function of detected neutrino and antineutrino states is described by

$$|\nu_\alpha; \mathbf{x}-\mathbf{L}; \bar{\nu}_\beta; \bar{\mathbf{x}}-\bar{\mathbf{L}}\rangle = \sum_{i=1}^{3}\sum_{j=1}^{3} U_{\alpha i}^* U_{\beta j} \times \Psi_i^D(\mathbf{x}-\mathbf{L})\bar{\Psi}_j^{\bar{D}}(\bar{\mathbf{x}}-\bar{\mathbf{L}})|\nu_i\rangle|\bar{\nu}_j\rangle, \tag{15}$$

where $\Psi_i^D(\mathbf{x}-\mathbf{L})$ and $\bar{\Psi}_j^{\bar{D}}(\bar{\mathbf{x}}-\bar{\mathbf{L}})$ are the wave functions of the detected neutrino at position L and of the detected anti-neutrino at position $\bar{L}$, respectively. Again, we assume a localized Gaussian wave function in momentum space similar to (7) for both detected neutrino and detected anti-neutrino. Therefore, one can obtain the corresponding wave function in position space as follows:

$$|\nu_\alpha; \mathbf{x}-\mathbf{L}; \bar{\nu}_\beta; \bar{\mathbf{x}}-\bar{\mathbf{L}}\rangle = \sum_{i=1}^{3}\sum_{j=1}^{3} U_{\alpha i}^* U_{\beta j} \exp\left[\mathrm{i}\mathbf{p}'_i(\mathbf{x}-\mathbf{L}) + \mathrm{i}\bar{\mathbf{p}}'_j(\bar{\mathbf{x}}-\bar{\mathbf{L}}) - \frac{(\mathbf{x}-\mathbf{L})^2}{4\sigma_{xD}^2} - \frac{(\bar{\mathbf{x}}-\bar{\mathbf{L}})^2}{4\bar{\sigma}_{x\bar{D}}^2}\right]|\nu_i\rangle|\bar{\nu}_j\rangle, \tag{16}$$

where σ_{xD} and $\bar{\sigma}_{x\bar{D}}^2$ are the uncertainty of the detecting neutrino and anti-neutrino processes. In general, the mean momenta of the produced particles, $_{pi}$ and $\bar{\mathbf{p}}_i$, are different from one of the detected particles, $\mathbf{p}_i'$ and $\bar{\mathbf{p}}_i'$ [11]. However, we assume that they coincide.

The probability amplitude of detecting ν_β at the coordinate $\mathbf{L}$ and the time t and $\bar{\nu}_\beta$ at the coordinate $\bar{\mathbf{L}}$ and the time $\bar{t}$ is as follows:

$$\mathcal{A}_{\alpha\beta} = \int \mathrm{d}^3x \int \mathrm{d}^3\bar{x}\, \langle \nu_\alpha; \mathbf{x}-\mathbf{L}; \bar{\nu}_\beta; \bar{\mathbf{x}}-\bar{\mathbf{L}}|\nu_Z; \mathbf{x}, t, \bar{\mathbf{x}}, \bar{t}\rangle. \tag{17}$$

Substituting equations (12) and (16) into the above equation yields

$$\mathcal{A}_{\alpha\beta} \propto \frac{1}{\sqrt{3}}\sum_{i=1}^{3} U_{\alpha i}^* U_{\beta i}\int \mathrm{d}^3x \times \int \mathrm{d}^3\bar{x} \exp\left[-\mathrm{i}E_i(t+\bar{t}) + \mathrm{i}\mathbf{p}_i(\mathbf{L}+\bar{\mathbf{L}}) - \frac{(\mathbf{x}-\mathbf{L})}{4\sigma_{xD}^2} - \frac{(\bar{\mathbf{x}}-\bar{\mathbf{L}})^2}{4\bar{\sigma}_{x\bar{D}}^2} - \frac{(\mathbf{x}+\bar{\mathbf{x}}-\mathbf{v}_i(t+\bar{t}))^2}{4\sigma_{xS}^2}\right]. \tag{18}$$

After the calculation of the integrals over the position coordinates, we have

$$\mathcal{A}_{\alpha\beta} \propto \frac{1}{\sqrt{3}}\sum_{i=1}^{3} U_{\alpha i}^* U_{\beta i} \exp\left[-\mathrm{i}E_i(t+\bar{t}) + \mathrm{i}\mathbf{p}_i(\mathbf{L}+\bar{\mathbf{L}}) - \frac{(\mathbf{L}+\bar{\mathbf{L}}-\mathbf{v}_i(t+\bar{t}))^2}{4\sigma_x^2}\right], \tag{19}$$

where $\sigma_x^2 = \sigma_{xD}^2 + \bar{\sigma}_{x\bar{D}}^2 + \sigma_{xS}^2$.

The probability of detecting a neutrino with flavor α at the position L and the corresponding anti-neutrino with flavor β at the position $\bar{L}$ and vice versa is given by

$$P_{\alpha\beta} = |\mathcal{A}_{\alpha\beta}|^2. \tag{20}$$

Because the neutrino and the antineutrino emission times and the corresponding arrival times are not measured, we must integrate over the time during which a neutrino travels from a detector to the other detector. Hence, we have

$$P_{\alpha\beta} \propto \frac{1}{3}\sum_{i,j=1}^{3} U_{\alpha i}^* U_{\alpha j} U_{\beta j}^* U_{\beta i} \int \mathrm{d}T \exp\left[\mathrm{i}(\mathbf{p}_i-\mathbf{p}_j)(\mathbf{L}+\bar{\mathbf{L}}) - \mathrm{i}(E_i-E_j)T - \frac{(\mathbf{L}+\bar{\mathbf{L}}-\mathbf{v}_iT)^2}{4\sigma_x^2} - \frac{(\mathbf{L}+\bar{\mathbf{L}}-\mathbf{v}_jT)^2}{4\sigma_x^2}\right], \tag{21}$$

where T stands for $t+\bar{t}$. Therefore, the following expression for $P_{\alpha\beta}$ is obtained:

$$P_{\alpha\beta} \propto \frac{1}{3}\sum_{i,j=1}^{3} U_{\alpha i}^* U_{\alpha j} U_{\beta j}^* U_{\beta i} \times \exp\left[\mathrm{i}(\mathbf{p}_i-\mathbf{p}_j)(\mathbf{L}+\bar{\mathbf{L}}) - \mathrm{i}(E_i-E_j)(\mathbf{L}+\bar{\mathbf{L}})\frac{(\mathbf{v}_i+\mathbf{v}_j)}{(\mathbf{v}_i^2+\mathbf{v}_j^2)} - (E_i-E_j)^2\frac{4\sigma_x^2}{(\mathbf{v}_i^2+\mathbf{v}_j^2)} - (\mathbf{L}+\bar{\mathbf{L}})^2\frac{(\mathbf{v}_i-\mathbf{v}_j)^2}{4\sigma_x^2(\mathbf{v}_i^2+\mathbf{v}_j^2)}\right]. \tag{22}$$

Since we are concerned with the relativistic neutrinos, we will use the approximation below

$$\mathbf{v}_j \simeq 1 - \frac{m_j^2}{2E^2}, \tag{23}$$

in which E will be the energy of neutrinos if we ignore their masses. We consider $E_i - E_j \ll E$ which corresponds to the relativistic and quasi degenerate neutrinos. Without using any other additional assumptions such as the same-energy and the same-momentum, one can simplify (22) to the following relation [8]:

$$P_{\alpha\beta} \propto \frac{1}{3}\sum_{i,j=1}^{3} U_{\alpha i}^* U_{\alpha j} U_{\beta j}^* U_{\beta i} \times \exp\left[-2\pi\mathrm{i}\frac{(\mathbf{L}+\bar{\mathbf{L}})}{\mathbf{L}_{ij}^{\mathrm{osc}}} - \frac{(\mathbf{L}+\bar{\mathbf{L}})^2}{(\mathbf{L}_{ij}^{\mathrm{coh}})^2} - \left(\frac{2\pi\sigma_x}{\mathbf{L}_{ij}^{\mathrm{osc}}}\right)^2\right], \tag{24}$$

with the oscillation lengths L_{ij}^{osc} and the coherence lengths L_{ij}^{coh}, for $i \neq j$, given by

$$L_{ij}^{\text{osc}} \equiv \frac{4\pi E}{\Delta m_{ij}^2}, \quad L_{ij}^{\text{coh}} \equiv 4\sqrt{2}\frac{E^2}{|\Delta m_{ij}^2|}\sigma_x, \tag{25}$$

where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$. The transition probability (24) is the same as the result obtained in [15] in which we used the QFT approach. It is noticeable that here we have manually imposed the entanglement of neutrino and antineutrino state via entering a delta function which grantees the momentum conservation. Meanwhile, in the QFT approach it is performed automatically by integrating over the Z^0 position.

Similar to [15], the first term in the exponential is the usual oscillating phase which gives the neutrino oscillation pattern between two detectors with distance $\mathbf{L} + \bar{\mathbf{L}}$. The second term causes the oscillation amplitude to be suppressed for $\mathbf{L} + \bar{\mathbf{L}} \geqslant \mathbf{L}_{ij}^{\text{coh}}$. In fact, this is the coherence condition which differs from the usual neutrino oscillation since it leads to suppression of the neutrino oscillation amplitude while the overlapping of various neutrino mass eigenstate may not diminish. The last term in the exponential of the transition probability suppresses the corresponding oscillatory term unless the localizations of the production and detection processes are much smaller than the oscillation length.

3. Conclusion

The standard neutrino oscillation is one of the most interesting physical phenomena which is confirmed experimentally. Indeed, neutrinos created by charged current have definite flavor in the source but a different flavor is detected in the detector, whose probability varies sinuously with distance between the source and the detector. However, there exist some debates about the theoretical explanation of these phenomena [8]. We can treat this problem via two approaches; QM and QFT approach. In QM approach, neutrinos are described by a wave packet which propagates from the source to the detector while in QFT approach neutrinos are treated as a mediator particle between the source and the detector [10, 11].

In the case of neutrinos emerging from the neutral current interactions, all three neutrino flavors are created with equal probability, hence, we cannot see the neutrino oscillation by detecting either neutrino or anti-neutrino. However, it was shown if both neutrino and anti-neutrino are detected, it is possible to have an oscillation pattern similar to the usual one [7]. In order to include all of the theoretical viewpoints, this problem was restudied in QFT approach [15]. In this paper, we have restudied the issue in QM approach. Through this approach, we attribute to each of the neutrino and anti-neutrino a wave packet whose localization is determined with the uncertainty of the source. These wave packets propagate to different detectors at positions L and $\bar{L}$. The conservation of momentum causes these wave packets to be entangled together. Hereby, we have used a Dirac delta function to guarantee the momentum conversation in the construction of the bipartite entangled state of neutrino and anti-neutrino wave packets. While we put manually the momentum conservation delta function in this approach, it appears automatically by integrating over Z^0 position in the QFT approach. Meanwhile, the oscillation probability which we have obtained is the same as the result of [15]. The entanglement property plays an important role in the final result; the coherency condition for the oscillation is $L + \bar{L} < L_{\text{coh}}$. This means that the overlapping of the wave functions of the neutrino and anti-neutrino corresponding to the various mass eigenstates may not diminish, while the amplitude of the oscillation ceases due to the decoherency effect.

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