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Second Braking Index of Intermittent Pulsar and Nulling Pulsar*

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Abstract

The braking index parameter of intermittent pulsar would provide important insight into the pulsar slowdown and emission mechanism. In previous work, we presented the first braking index $n = \nu\ddot{\nu}/\dot{\nu}^2$ of the intermittent pulsar B1931+24, J1841−0500, J1832+0029 and nulling pulsar B0823+26, by using their observed spin frequency first derivatives $\dot{\nu}$ in "on" and "off" states. In this paper, to improve and justify our method for studying the braking mechanism of intermittent pulsars, we continue to formulate a method in which the second braking indices, $m = \nu^2\dddot{\nu}/\dot{\nu}^3$, of these pulsars can also be given with only the observed spin frequency first derivatives $\dot{\nu}_{\rm on}$ and $\dot{\nu}_{\rm off}$. This paper will discuss, according to the emission and spin-down property of intermittent pulsars, three different magnetospheric configuration changes in the polar cap region. We find our approach produces feasible values of the second braking index of intermittent pulsar. Our calculations of the second braking indices are in close agreement with predicted second braking indices $m = n(2n - 1)$, which are given by simple spin-down law of pulsar spin evolution, resulting in indirect observational support to our model.

Key words: pulsars: general—pulsars – individual (PSR B1931+24)—stars – neutron

Online material: color figures

1. Introduction

It is commonly argued that radio pulsars are powered by rotational kinetic energy, and lose energy by electromagnetic radiation and particle wind (Livingstone et al. 2007). To understand the pulsar energy losing mechanism, we always study its braking index, but unfortunately there exist very few pulsars with measured braking indices (Lyne et al. 2015). The timing analysis of pulsars showed that the higher-order rotational frequency derivatives $\ddot{\nu}$ cannot be measured accurately due to different noise processes from pulsar itself or its surroundings (Camilo et al. 1994; Hobbs et al. 2004, 2010; Shannon & Cordes 2010). For younger pulsars, the glitch activity (i.e., a sudden spin-up and following recovery process in pulsar spinning caused by interaction between the neutron star crust and its superfluid core) would cause dramatic rotational irregularities, if it is not resolved properly, no precise measurement of braking index can be obtained (Hobbs et al. 2004, 2010). For older pulsars which have slower spin-down in general, the long timing baseline is required to measure $\ddot{\nu}$, then the strength of noise contribution from the intrinsic timing noise by pulsar and from external noise process by interstellar scattering and proper motion or other unknown process is increasing, resulting in very strange braking indices which cannot be explained by the standard pulsar theory (Hobbs et al. 2004, 2010). Also, the timing precisions of young millisecond pulsar which has globular cluster origin are easily contaminated by its proper motion and the undetected background stochastic gravitational waves, while the older one is mostly recycled binary pulsar, in which, in addition to the above noise process, the complex mass transfer from its companion star would bring a significant timing irregularities (Camilo et al. 1994; Shannon & Cordes 2010). So, for the majority of pulsar population, it is difficult to measure an accurate braking index (Hobbs et al. 2004, 2010).

In general, the spin evolution of a pulsar is described simply by the power law equation as below

$$\dot{\nu} = -K\nu^n, \tag{1}$$

which we call the simple spin-down law of pulsar, where K is a positive parameter that related to the momentum of inertia I, magnetic dipole moment μ and inclination angle α (between magnetic pole and spin axis of the pulsar), n is the braking index, ν is the pulsar spin frequency (Lorimer & Kramer 2004; Zampieri et al. 2014; Lyne et al. 2015). Assuming K is a constant, the braking index can be defined by differentiating Equation (1)

$$n = \nu\ddot{\nu}/\dot{\nu}^2. \tag{2}$$

In theory, braking index $n = 1$ is attributed to the wind braking scenario (Michel & Tucker 1969) and the braking index of 3 is ascribed to pure magnetic dipole braking (Pacini 1968; Lorimer & Kramer 2004) or the braking by force-free

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pulsar magnetosphere (Spitkovsky 2006). However, until now, none of the observed braking indices strictly equals to 1 or to 3 (Lorimer & Kramer 2004; Ferdman et al. 2015; Lyne et al. 2015; Archibald et al. 2016). The deviation from our expected values might be explained by the combined effects of wind braking, electromagnetic dipole radiation, changes of I, B, α parameters, relativistic effects and/or other unknown mechanisms (Lorimer & Kramer 2004; Livingstone et al. 2007; Lyne et al. 2015; Archibald et al. 2016).

To confirm the pulsar spin-down law of Equation (1), we need to measure the second braking indices of pulsars. Differentiating Equation (1) further, yields

$$\dddot{\nu} = n(2n - 1)\dot{\nu}^3/\nu^2, \tag{3}$$

then, in an analogy with n, the second braking index is defined as

$$m = \nu^2\dddot{\nu}/\dot{\nu}^3. \tag{4}$$

By using Equations (3) and (4), we express the m with n as

$$m = n(2n - 1). \tag{5}$$

If the frequency third derivative is measured, m can be evaluated by Equation (4). Also the relation expressed in Equation (5) can predict the second braking index m by a known first braking index n, allowing us to compare the observed second braking index (from Equation (4)) with the predicted one (from Equation (5)) to determine the validity of the simple spin-down law (Kaspi et al. 1994). Until now, we have measured the second braking indices of two pulsars, i.e., Crab ($m = 10.23 \pm 0.03$, Lyne et al. 1993 and PSR B1509−58 ($m = 14.5 \pm 3.6$, Kaspi et al. 1994). These observed values of m are consistent with the predicted value of $m = 10.09 \pm 0.10$ for Crab and $m = 13.26 \pm 0.03$ for PSR B1509−58, which has been supporting the simple spin-down law of pulsar (Kaspi et al. 1994; Zampieri et al. 2014).

It can be noted from Equations (2) and (4) that, to measure the first and second braking index, we are required to measure the rotational frequency, its first, second and third derivatives. As referred to above, unresolved timing noise and rotational glitches in the source may never allow a precise measurement of the frequency second and third derivatives of normal pulsars (Hobbs et al. 2004, 2010). For the intermittent pulsar, it is more difficult to give an accurate measurement to those parameters due to its regular state switching property, which causes the absence of data bounding a given transition into (or out of) a radio-on phase (Kramer et al. 2006; Lorimer et al. 2012; Young et al. 2013, 2012). However, the special importance of the spin-down mechanism of the intermittent pulsar drives many researches about its braking index in "on" (radio-on phase) and "off" (radio-quite phase) state, but both in the observation and theory, no reliable measurements or calculations are provided yet (Yue et al. 2007; Young et al. 2013; Li et al. 2014). Unlike other studies in which the "off" state is not fully considered (Kramer et al. 2006; Yue et al. 2007; Li et al. 2014), we have simultaneously settled the "on" and "off" state first braking index of few intermittent pulsars in a previous paper (Rusul et al. 2014), and primarily confirmed that the "off" state is not purely governed by magnetic dipole braking (Jason et al. 2012; Rusul et al. 2014). To better understand the spin-down propriety of intermittent pulsar and to further confirm and complete our approach, this paper will extend our earlier method to study the second braking index of intermittent pulsar. A method description is given in Section 2, results and discussions are given in Section 3 and conclusions are presented in Section 4.

2. A Method

The observed spin-down rates of intermittent pulsars in "on" and "off" state ($|\dot{\nu}_{\rm on}| > |\dot{\nu}_{\rm off}|$) revealed evidence that the wind torque associated with the "on" state radio emission has a major contribution to the pulsar's spin-down (Kramer et al. 2006; Camilo et al. 2012; Lorimer et al. 2012; Young et al. 2013). In the light of the emission characteristics and spin-down property of the intermittent pulsar, the pulsar wind contribution is expressed as

$$\dot{E}_{\rm w} = \dot{E}_{\rm on} - \dot{E}_{\rm off} = 2T\pi\nu \tag{6}$$

(Kramer et al. 2006; Lorimer et al. 2012; Young et al. 2013, 2012), then

$$T = 2\pi(\dot{\nu}_{\rm off} - \dot{\nu}_{\rm on})I, \tag{7}$$

where $\dot{E}_{\rm on} = -4\pi^2 I\nu\dot{\nu}_{\rm on}$, $\dot{E}_{\rm off} = -4\pi^2 I\nu\dot{\nu}_{\rm off}$, $\dot{\nu}_{\rm on}$ is the "on" state rotational frequency derivative, $\dot{\nu}_{\rm off}$ is the "off" state rotational frequency derivative, T is the electromagnetic torque comes from the influence between polar gap magnetic field and the out-flowing particle wind. Harding et al. (1999) provided the torque T as below

$$T = 2jB_0R_{\rm pc}^2/3c, \tag{8}$$

where $j = c\pi R_{\rm pc}^2\rho$ is the horizontal electric current in the polar cap, B_0 is the magnetic field strength at pulsar surface, $R_{\rm pc}$ is polar cap radius. Near the stellar surface, the polar cap radius is not too large and can be expressed as $R_{\rm pc} = \sqrt{2\pi\nu R^3/c}$ for the dipole co-rotating magnetic field of pulsar surface, where R is neutron star radius and c is light speed (Lorimer & Kramer 2004), ρ is polar cap charge density near the magnetic pole. By using Equations (7) and (8) we can derive the following expression[4]

$$\dot{\nu}_{\rm off} - \dot{\nu}_{\rm on} = B_0\rho R_{\rm pc}^4/(3I). \tag{9}$$

[4] In pulsar theory, no analytic expression of pulsar spin-down for oblique rotator is given yet (Contopoulos et al. 1999). According to the numerical study of the current distribution in the pulsar polar cap (Contopoulos et al. 1999, 2014) and the spin-down formula in three-dimension (Spitkovsky 2006), it is noticed that $(\dot{\nu}_{\rm off} - \dot{\nu}_{\rm on}) \propto B_0\rho R_{\rm pc}^4\cos^2\alpha/I$. The observed evidences of the intermittent pulsar did not suggest the variation of inclination angle α is responsible for the state-switching between the "on" and "off" states (Kramer et al. 2006), so by convenience and for keeping consistency with previous approaches (Kramer et al. 2006; Rusul et al. 2014), we still applied the aligned form of Equation (9), which would not affect our method descriptions and results.

To PSR B1931+24, the calculation of ρ is remarkably close to Golreich-julian charge density, $\rho_{\mathrm{GJ}} = \Omega B/(2\pi c)$ (Kramer et al. 2006; Young et al. 2013), which is the charge density near the magnetic pole of force-free pulsar magnetosphere (Goldreich & Julian 1969; Lorimer & Kramer 2004). In three other intermittent pulsars, the charge densities are also within an order of magnitude of ρ_{GJ} (Camilo et al. 2012; Lorimer et al. 2012; Young et al. 2012), the small discrepancies might be from the parameters of R, I, B and α. Therefore we assumed that the charge densities of these pulsars can be studied in ρ_{GJ} regime.

Except for the periodic state switching, no other timing noise factors are detected in timing analysis of the intermittent pulsars (Kramer et al. 2006; Camilo et al. 2012; Lorimer et al. 2012; Young et al. 2012, 2013). Observations also suggested that the state switching and the accompanied different spin-down rates of intermittent pulsar are ascribed to the changes in pulsar magnetospheric dynamics (Kramer et al. 2006; Camilo et al. 2012; Lorimer et al. 2012). So the modification of the polar cap magnetosphere is considered, which has direct relation to the state switches of pulsar (see Equation (9) and Timokhin 2010). In order to calculate the second braking index, we need to take the time derivative of Equation (9) for two different times. Due to no glitch or X-ray flux increasing during the state switches (Kramer et al. 2006; Camilo et al. 2012; Lorimer et al. 2012), it is assumed that B_0 and I are constant in "on" and "off" state, then in Equation (9), we only have the variables R_{pc} and ρ to discuss. With these two variables, we are going to construct three different cases in pulsar polar cap magnetosphere changes.

Case 1: R_{pc} is variable and ρ is constant, and therefore

$$\dddot{\nu}_{\mathrm{on}} - \dddot{\nu}_{\mathrm{off}} = 2(\dot{\nu}_{\mathrm{on}} - \dot{\nu}_{\mathrm{off}})(\dot{\nu}_{\mathrm{on}}^2 + \ddot{\nu}_{\mathrm{on}}\nu)/\nu^2. \quad (10)$$

Case 2: R_{pc} is constant and ρ is variable, and therefore

$$\dddot{\nu}_{\mathrm{on}} - \dddot{\nu}_{\mathrm{off}} = (\dot{\nu}_{\mathrm{on}} - \dot{\nu}_{\mathrm{off}})\ddot{\nu}_{\mathrm{on}}\nu/\nu^2. \quad (11)$$

Case 3: both R_{pc} and ρ are variable, and therefore

$$\dddot{\nu}_{\mathrm{on}} - \dddot{\nu}_{\mathrm{off}} = 3(\dot{\nu}_{\mathrm{on}} - \dot{\nu}_{\mathrm{off}})(2\dot{\nu}_{\mathrm{on}}^2 + \ddot{\nu}_{\mathrm{on}}\nu)/\nu^2. \quad (12)$$

For convenience, we expressed the first and second braking indices as $n_{\mathrm{on}} = \nu\ddot{\nu}_{\mathrm{on}}/\dot{\nu}_{\mathrm{on}}^2$ and $m_{\mathrm{on}} = \nu^2\dddot{\nu}_{\mathrm{on}}/\dot{\nu}_{\mathrm{on}}^3$ for "on" state respectively, and $n_{\mathrm{off}} = \nu\ddot{\nu}_{\mathrm{off}}/\dot{\nu}_{\mathrm{off}}^2$ and $m_{\mathrm{off}} = \nu^2\dddot{\nu}_{\mathrm{off}}/\dot{\nu}_{\mathrm{off}}^3$ for "off" state respectively. Therefore, the Equations (10), (11) and (12) can be written with n_{on}, m_{on} and m_{off} respectively as following

$$m_{\mathrm{on}} - m_{\mathrm{off}}\dot{\nu}_{\mathrm{off}}^3/\dot{\nu}_{\mathrm{on}}^3 = 2(1 - \dot{\nu}_{\mathrm{off}}/\dot{\nu}_{\mathrm{on}})(n_{\mathrm{on}} + 1) \quad (\mathrm{case1}). \quad (13)$$

$$m_{\mathrm{on}} - m_{\mathrm{off}}\dot{\nu}_{\mathrm{off}}^3/\dot{\nu}_{\mathrm{on}}^3 = (1 - \dot{\nu}_{\mathrm{off}}/\dot{\nu}_{\mathrm{on}})n_{\mathrm{on}} \quad (\mathrm{case2}). \quad (14)$$

$$m_{\mathrm{on}} - m_{\mathrm{off}}\dot{\nu}_{\mathrm{off}}^3/\dot{\nu}_{\mathrm{on}}^3 = 3(1 - \dot{\nu}_{\mathrm{off}}/\dot{\nu}_{\mathrm{on}})(n_{\mathrm{on}} + 2) \quad (\mathrm{case3}). \quad (15)$$

When ignoring the "off" state magnetosphere, the second braking index m_{on} can be predicted, under the assumption that the "off" state is magnetic dipole dominated, in which $n_{\mathrm{off}} = 3$ and $m_{\mathrm{off}} = 15$ as it was consumed by many others (Kramer et al. 2006; Yue et al. 2007; Li et al. 2014). To avoid dismissing valuable information about the pulsar magnetosphere in the "off" state, we do not presume $m_{\mathrm{off}} = 15$ in calculation.

The observational evidence shows that the braking index is constant in a relatively short period of time. For example, for PSR B0531+21 (Crab pulsar), 0.5% variation in 23 yr from 1969 to 1993 (Lyne et al. 1993; Allen & Horvath 1997) and a small variation in more than 40 yr (Zampieri et al. 2014; Lyne et al. 2015); and for PSR B1509−58, 1.5% variation in 21 yr of observation, which is nearly constant within the sub-observation intervals (Livingstone et al. 2005). So, for the intermittent pulsar which have highly stable magnetosphere configurations in the "on" and "off" state (Young et al. 2013), we assume that the first and second braking index of neighboring observations (within one or two years interval) are constant, namely $n_{\mathrm{on}(i)} = n_{\mathrm{on}(i+1)}$, $n_{\mathrm{off}(i)} = n_{\mathrm{off}(i+1)}$ and $m_{\mathrm{on}(i)} = m_{\mathrm{on}(i+1)}$, $m_{\mathrm{off}(i)} = m_{\mathrm{off}(i+1)}$. As an example, Equation (13) is discussed in detail. We denote $A = \dot{\nu}_{\mathrm{off}}^3/\dot{\nu}_{\mathrm{on}}^3$ and $B = 2(1 - \dot{\nu}_{\mathrm{off}}/\dot{\nu}_{\mathrm{on}})$ and substitute them into Equation (13). With the set of neighboring observing data of $\dot{\nu}_{\mathrm{off}(i)}/\dot{\nu}_{\mathrm{on}(i)}$ and $\dot{\nu}_{\mathrm{off}(i+1)}/\dot{\nu}_{\mathrm{on}(i+1)}$, we can write equations as below

$$m_{\mathrm{on}(i)} - m_{\mathrm{off}(i)}A_i = B_i(n_{\mathrm{on}(i)} + 1), \quad (16)$$

$$m_{\mathrm{on}(i+1)} - m_{\mathrm{off}(i+1)}A_{i+1} = B_{i+1}(n_{\mathrm{on}(i+1)} + 1). \quad (17)$$

Then by using the assumption of $n_{\mathrm{on}(i)} = n_{\mathrm{on}(i+1)}$, $m_{\mathrm{on}(i)} = m_{\mathrm{on}(i+1)}$ and $m_{\mathrm{off}(i)} = m_{\mathrm{off}(i+1)}$, we derive $m_{\mathrm{on}(i)}$ (or $m_{\mathrm{off}(i)}$) from Equations (16) and (17)

$$m_{\mathrm{on}(i)} = (n_{\mathrm{on}(i)} + 1)(B_i - A_iB_{i+1}/A_{i+1}) \times (1 - A_i/A_{i+1})^{-1}. \quad (18)$$

By the same procedure, we derive the following equations

$$m_{\mathrm{on}(i)} = n_{\mathrm{on}(i)}(B_i - A_iB_{i+1}/A_{i+1}) \times (1 - A_i/A_{i+1})^{-1}, \quad (19)$$

$$m_{\mathrm{on}(i)} = (n_{\mathrm{on}(i)} + 2)(B_i - A_iB_{i+1}/A_{i+1}) \times (1 - A_i/A_{i+1})^{-1}, \quad (20)$$

for case 2 and case 3 respectively. It can be seen from the above equations that there are no canonical parameters, like pulsar mass or radius, are used during calculation, thus the obtained results are free from a large uncertainty. When we apply Equations (18)–(20) to the data in Table 1, each time we will use two sets of observing data of $\dot{\nu}_{\mathrm{on}(i)}$ and $\dot{\nu}_{\mathrm{off}(i)}$ in neighboring

Table 1
$\dot{\nu}_{\rm on}(10^{-15}\,{\rm s}^{-2})$ and $\dot{\nu}_{\rm off}(10^{-15}\,{\rm s}^{-2})$ of Intermittent Pulsar B1931+24

Epochs (MJD)	51869.7	52842.3	53782.6	53987.1	54340.7	54676.6	54875.0	55198.2
$\dot{\nu}_{\rm on}(10^{-15}\,{\rm s}^{-2})$	−15.2(2)	−16.0(2)	−17.6(5)	−19(1)	−16.2(4)	−17.1(8)	−15.8(2)	−14.3(1)
$\dot{\nu}_{\rm off}(10^{-15}\,{\rm s}^{-2})$	−11.13(9)	−10.78(7)	−10.4(2)	−10.2(2)	−10.7(1)	−10.3(3)	−11.07(8)	−11.40(3)

Note. $\dot{\nu}_{\rm on}$, $\dot{\nu}_{\rm off}$ are tabulated from column one (epoch 1, 51869.7 MJD) to column eight (epoch 8, 55198.2 MJD), the epochs here are the reference epoch of the timing analysis in a performed data span, which includes the data from both "on" and "off" states (for details see Table 4 from Young et al. 2013).

epochs (see Table 1) and the braking indices $n_{{\rm on}(i)}$ which are derived in previous work (see Equation (17) and Table 3 from Rusul et al. 2014), then we average them. For PSR J1832+0029, we take the limit at the "on"/"off" ratio point 1.77 (i.e., $\dot{\nu}_{{\rm off}(i)}/\dot{\nu}_{{\rm on}(i)} = 0.56$ and $\dot{\nu}_{{\rm off}(i+1)}/\dot{\nu}_{{\rm on}(i+1)} \to 0.56$) due to the one set of data (see the data descriptions and Table 2 in Rusul et al. 2014); this approach is the same to the newly discovered intermittent pulsar J1929+1357 which has different spin-down rates in "on" and "off" state (Lyne et al. 2017). To constrain the method application, we will use our method to the non-intermittent pulsar B0540+69 which has high spin-down rate $\dot{\nu}_{\rm on} = -2.528664(1) \times 10^{-10}\,{\rm s}^{-2}$ and low spin-down rate $\dot{\nu}_{\rm off} = -1.878173(9) \times 10^{-10}\,{\rm s}^{-2}$ (Ferdman et al. 2015; Marshall et al. 2015). The related data of PSR J1929+1357 and B0540+69 are given in Table 2.

Table 2
Parameters of Two Pulsars and Related References

PSR Name	P(ms)	$\dot{\nu}_{\rm on}/\dot{\nu}_{\rm off}$	References
B0540+69	50	1.36	Marshall et al. (2015)
J1929+1357	869.56	1.8	Lyne et al. (2017)

Note. The parameters are tabulated from column one to column four as pulsar name, period, ratios of $\dot{\nu}_{\rm on}/\dot{\nu}_{\rm off}$ and references.

3. Results And Discussions

Comparing other pulsars, PSR B1931+24 has relatively enough and complete observing data of $\dot{\nu}_{\rm on}$ and $\dot{\nu}_{\rm off}$ with their measurement errors. So, in calculation, to manifest the reliability of the approach, we give propagation errors of the first and second braking index to this pulsar in Table 1 (the $n_{\rm on}$ and $n_{\rm off}$ adopted from Rusul et al. 2014, $m_{\rm on}$ and $m_{\rm off}$ are calculated by Equations (18)–(20) and being averaged). It can be seen from Figure 1 that the relatively bigger measurement errors of $\dot{\nu}_{\rm on}$ and $\dot{\nu}_{\rm off}$ at 53987.1 MJD (see Table 1), cause the larger propagation errors at ∼537250 MJD and ∼54500 MJD in case 1, 2 and 3, the other propagation errors are considerable small (see in Table 1 for averages of the propagation errors). This indicates that our formulae work well to calculate and/or predict the braking index under the current measurement accuracy of $\dot{\nu}_{\rm on}$ and $\dot{\nu}_{\rm off}$ of PSR B1931+24. The plots in Figure 1 also show that $n_{\rm on}$, $n_{\rm off}$, $m_{\rm on}$ and $m_{\rm off}$ vary smoothly and have nearly the same episodes of "on" and "off" phases. These are consistent with the stable magnetospheric features of PSR B1931+24, which would provide a chance to accurately measure the braking index of this pulsar with the future observation (Young et al. 2013; Hobbs et al. 2014; Tauris et al. 2014).

Considering the predictable range of second braking indices of known pulsars which have observed first braking index ∼0.9 − ∼3.15 (Lyne et al. 1996; Livingstone et al. 2007; Espinoza et al. 2011; Archibald et al. 2016), most of the calculated second braking indices in Table 4 are within the plausible range. Take PSR B1931+24 as an example, the relatively small standard errors of the second braking indices in Table 4 imply that the spin-down mechanism and magnetosphere configuration in "on" and "off" states are considerably stable for 13 yr of observing period, which is consistent with the observing features of this pulsar (Young et al. 2013). The differences between $n_{\rm on}$ and $n_{\rm off}$, and $m_{\rm on}$ and $m_{\rm off}$ are the reflection of fluctuating torque acting on pulsar rotation in "on" and "off" states. In other point of view, the closeness between "on" and "off" state first and second braking index (see Tables 3 and 4), mostly suggest that the discrepancy between "on" and "off" state is just due to the small variation of the braking torque which is associated with the change of particle flow or of polar cap radius but not due to the global magnetospheric state change as proposed by Hermsen et al. (2013).

To check the consistency of our method, we predicted $m_{\rm on}$ and $m_{\rm off}$ (see Table 4) by using Equation (5) with the $n_{\rm on}$ and $n_{\rm off}$ which are calculated in previous paper (pleases see Table 3 in Rusul et al. 2014). It is found that the results of $m_{\rm on}$ and $m_{\rm off}$ in Table 4 almost close to their predicted counterparts. This indicates that our methods are self-consistent. The results in case 2 of PSR B0823+26 are relatively different from the predicted one. It is because our approach and Equation (5) have different variation features as the $\dot{\nu}_{\rm off}/\dot{\nu}_{\rm on} \to 1$ or $n \to 0.5$, i.e., $m_{\rm on} = m_{\rm off} = 0.17$ as $\dot{\nu}_{\rm off}/\dot{\nu}_{\rm on} \to 1$ ($n_{\rm on} = n_{\rm off} = 0.5$) under our method, while $m_{\rm on} = m_{\rm off} = 0$ under the prediction of Equation (5). These results may wait for an observation to discriminate which of the value is more appropriate. For the newly discovered PSR J1929+1357, the first braking indices in three cases are as follows: $n_{\rm on} = 1.44$ and $n_{\rm off} = 1.79$ in case 1; $n_{\rm on} = 0.72$ and $n_{\rm off} = 0.89$ in case 2; $n_{\rm on} = 2.17$ and

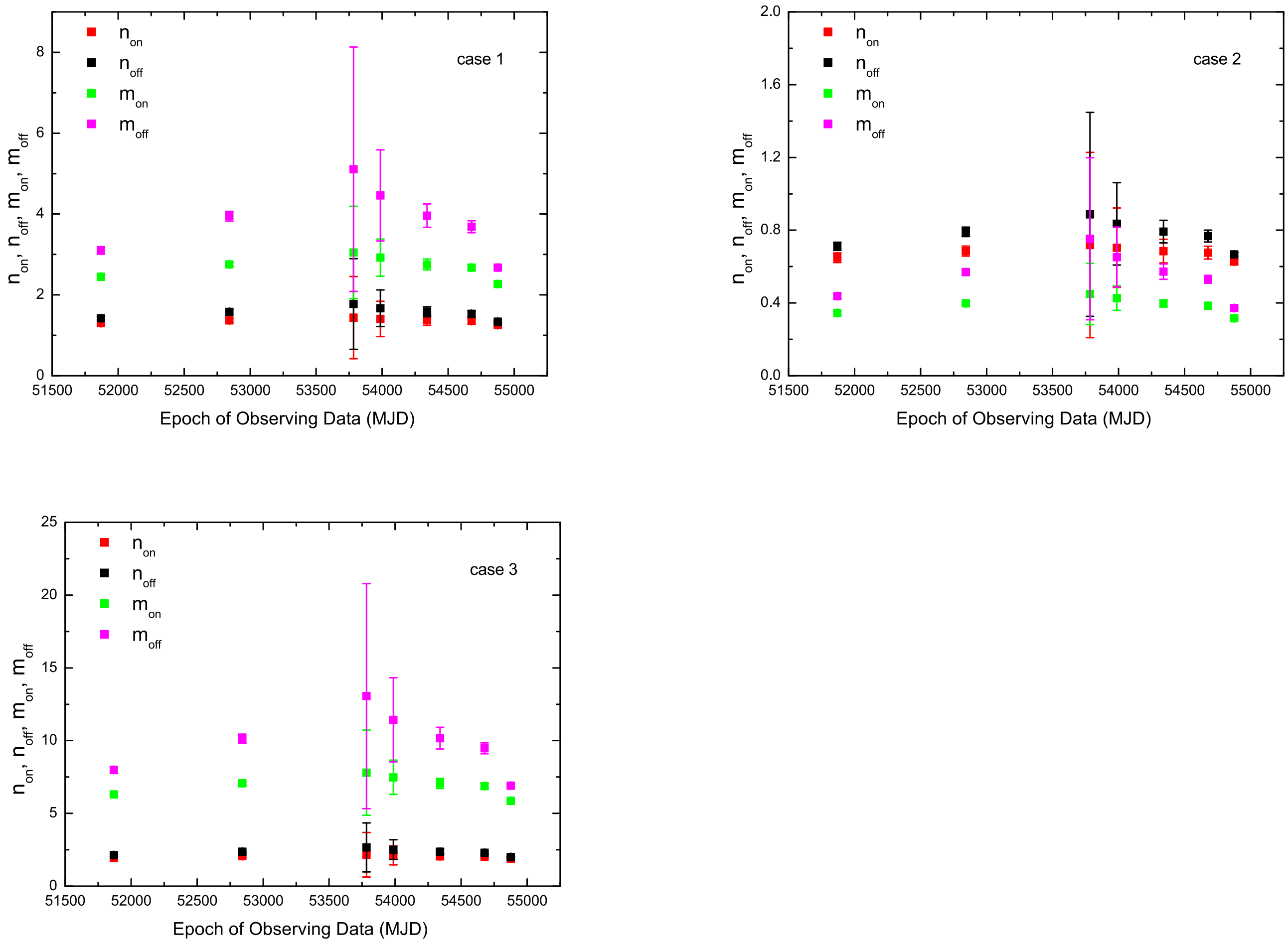

Figure 1. The first and second braking indices are shown with propagation errors in "on" and "off" states of PSR B1931+24 under three different cases. The red, black, green and pink squares are n_{on}, n_{off}, m_{on} and m_{off} respectively. It can be seen that the propagation errors are small in most of the points, the larger measurement errors of $\dot{\nu}_{on}$ and $\dot{\nu}_{off}$ at 53987.1 MJD causes relatively bigger uncertainty in its two neighboring points.
(A color version of this figure is available in the online journal.)

Table 3
The First and Second Braking Indices of PSR 1931+24

Case	n_{on}	n_{off}	m_{on}	m_{off}
1	1.35 ± 0.25	1.56 ± 0.27	2.70 ± 0.28	3.85 ± 0.69
2	0.68 ± 0.13	0.78 ± 0.13	0.39 ± 0.04	0.56 ± 0.10
3	2.03 ± 0.38	2.33 ± 0.40	6.92 ± 0.71	9.88 ± 1.77

Note. The results n_{on}, n_{off}, m_{on}, and m_{off} are given with propagation errors under three different cases.

$n_{off} = 2.71$ in case 3; the second braking index are given in Table 4.

To further confirm the relation between our method and the simple spin-down model, we analyzed the variations features of m_{on} with respect to n_{on} in our model (Equation (18)–(20), and compared with relation Equation (5) (see Table 5 and Figure 2). For all cases, as n_{on} decreases (i.e., $\dot{\nu}_{off}/\dot{\nu}_{on} \to 1$) m_{on} decreases and vice versa (when $\dot{\nu}_{off}/\dot{\nu}_{on} \to 0$). It can be noted from the Table 5 that if the spin-down ratio is getting closer to 0, the m_{on} approximately equals to ∼6, ∼1, ∼15 for the three cases respectively and m_{off} is getting extremely large. The infinity of m_{off} probably indicates that, as $\dot{\nu}_{off}/\dot{\nu}_{on} \to 0$, the spin-down of a intermittent pulsar in radio-off state may be quite complex, it may not simply be a case of screening the acceleration field in pulsar magnetosphere. If the spin-down ratio is getting closer to 1, the m_{on} and m_{off} approximately equal to ∼1.33, ∼0.17, ∼3.5 for the three cases respectively. It can be seen from results Table 5 and Figure 2 that our model and simple spin-down model have perfectly matched with each other in most of the intervals. This largely supports the validity of our approach and confirms our results in some extent.

In our consideration, the physics behind the three different conditions as follows: in case 1, an acceleration field in the "on" state disappeared or damped in the "off" state due to the changes in configuration of polar cap emission region; in

Table 4
The Calculated Second Braking Indices of Five Pulsars Under Three Cases; the First Three are Given with Standard Error; Due to Only One Set of Data for the Last Two Pulsar, the Standard Error Cannot be Calculated

PSR Name	case 1		case 2		case 3	
	$m_{\rm on}$	$m_{\rm off}$	$m_{\rm on}$	$m_{\rm off}$	$m_{\rm on}$	$m_{\rm off}$
B1931+24	2.70 ± 0.18	3.85 ± 0.66	0.39 ± 0.03	0.56 ± 0.09	6.92 ± 0.45	9.88 ± 1.69
predicted	2.32 ± 0.34	3.29 ± 0.92	0.24 ± 0.09	0.43 ± 0.24	6.24 ± 0.77	8.58 ± 2.06
B0823+26	1.46 ± 0.02	1.47 ± 0.03	0.19 ± 0.00	0.19 ± 0.00	3.83 ± 0.06	3.84 ± 0.07
predicted	1.12 ± 0.03	1.12 ± 0.03	0.02 ± 0.01	0.02 ± 0.01	3.29 ± 0.07	3.30 ± 0.08
J1841−0500	3.83 ± 0.04	11.03 ± 0.48	0.59 ± 0.01	1.70 ± 0.08	9.72 ± 0.10	27.99 ± 1.20
predicted	3.54 ± 0.05	10.19 ± 0.51	0.48 ± 0.00	1.92 ± 0.14	9.16 ± 0.12	24.81 ± 1.16
J1832+0029	3.06	5.20	0.45	0.76	7.82	13.26
predicted	2.71	4.56	0.32	0.69	7.17	11.68
J1929+1357	3.07	5.26	0.45	0.76	7.87	13.47
predicted	2.71	4.62	0.32	0.69	7.25	11.98

Note. The predicted second braking index are listed in corresponding line and column, its errors are evaluated by using standard errors of first braking index. Parameters are tabulated from column one to column seven as pulsar name and second braking index of $m_{\rm on}$, $m_{\rm off}$ for three cases.

case 2, the polar cap acceleration region in "off" state is depleted of charged plasma to produce emission; the case 3 is the combining contribution of the above two cases. It is noticed that under the above three conditions, PSR B0540+69 has $n_{\rm on}^{\rm predicted} = 1.26, 0.63, 1.89$ and $n_{\rm off}^{\rm predicted} = 1.35, 0.68, 2.03$ for three cases respectively, we can see that $n_{\rm off}^{\rm predicted} = 2.03$ is nearly the same with $n_{\rm off}^{\rm observed} = 2.13$ (Ferdman et al. 2015), but corresponding $n_{\rm on}^{\rm predicted} = 1.89$ is quite different from $n_{\rm on}^{\rm observed} = 0.94$ (Ge et al. 2019). This shows that the above magnetospheric changes cannot explain the observed braking property of PSR B0540+69. However, in the light of the emission and spin-down features of PSR B0540+69 (Zhang et al. 2001; Ferdman et al. 2015; Marshall et al. 2015; Ge et al. 2019; Kim & An 2019), we continue to examine the changes in polar cap magnetic field (this will be studied in detail in the forthcoming papers), and find that $n_{\rm on}^{\rm predicted} = 0.65 - 0.97$, which can easily explain the current value of the braking index of this pulsar (Ge et al. 2019).

Attempts to measure the braking index of PSR B1931+24 failed even after a long observation baseline, due to the timing noise attributed to the unpredictable state switches of the source (Young et al. 2013), but the observing campaign aimed to uncover the state switching behavior of pulsar has not stopped (Hobbs et al. 2014; Tauris et al. 2014). Meanwhile the results and the constraints we obtained to the first and second braking indices of intermittent pulsars, urge us to further test our method observationally, and provide better understanding about pulsar braking mechanism and emission properties. Here we give a few tips for further observational study. First, it is necessary to measure the braking index of younger nulling pulsars such as PSR B0823+26 and PSR J1107−5907 (Young et al. 2013), which have almost the same spin-down rates in "on" and "off" phases, thus we can check our model in one extreme case; second, continue to observe a typical intermittent pulsar, such as PSR B1931+24, until the second and third frequency derivatives have been measured, thus providing confidence and credibility to our approach and allowing us to understand where our method breaks down and where it can be applied successfully; third and lastly, we continue to measure n and m for younger pulsars and keep the verification of spin-down relation of Equation (1), while Equation (1) backs up our method uniquely. To be noted that the emission features of intermittent pulsar require a frequent high-sensitivity observations to measure its braking index. This would be easily performed by the advanced new facilities such as FAST[5] (Hobbs et al. 2019), LOFAR[6] (Young et al. 2013) and the SKA (Young et al. 2013; Tauris et al. 2014), which have and will have high performance multi-beam capabilities to meet with observing demands. We wish the measurement of the braking index of intermittent pulsar will be realized in near future with such powerful telescopes.

Table 5
The Second Braking Indices of Intermittent Pulsars Under our Model and by the Spin-down Law; the $\dot{\nu}_{\rm off}/\dot{\nu}_{\rm on}$ Ratio Ranging from 0 to 1, and Corresponding Second Braking Index and Its Predicted Values are Given in Subsequent Lines

Spin-down Ratio	Case 1		Case 2		Case 3	
	$m_{\rm on}$	$m_{\rm off}$	$m_{\rm on}$	$m_{\rm off}$	$m_{\rm on}$	$m_{\rm off}$
$\dot{\nu}_{\rm off}/\dot{\nu}_{\rm on} \to 0$	6	∞	1	∞	15	∞
Predicted	6	∞	1	∞	15	∞
$\dot{\nu}_{\rm off}/\dot{\nu}_{\rm on} \to 1$	1.33	1.33	0.17	0.17	3.5	3.5
Predicted	1	1	0	0	3	3

Note. The parameters are tabulated from column one to column seven as $\dot{\nu}_{\rm off}/\dot{\nu}_{\rm on}$, the second braking indices of $m_{\rm on}$, $m_{\rm off}$ under three cases.

5 http://www.bao.ac.cn/en/
6 http://www.lofar.org/

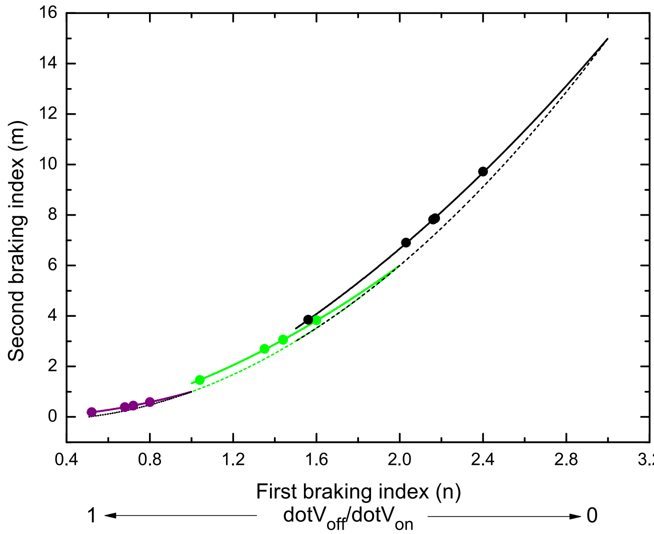

Figure 2. The comparison between our model and simple spin-down law. The filled lines represent case 1 (green), case 2 (purple) and case 3 (black); correspondingly the dashed lines are drawn by Equation (5); the filled circles are the pulsars studied in this paper, which represent "on" state braking index for the three cases.

(A color version of this figure is available in the online journal.)

4. Conclusions

In this paper, we have discussed the second braking indices of few intermittent pulsars under a simple model, in which we need only the observed spin-down rate $\dot{\nu}_{\mathrm{on}}$ and $\dot{\nu}_{\mathrm{off}}$, and compared our results and model with the pulsar spin-down law. It is found that our results are in proper ranges of measured braking indices, and there is a good agreement between our model and pulsar spin-down law. These indicate that the method in this paper has a potential application for studying the braking mechanism of intermittent pulsar and its magnetosphere changes. On the base of our results and discussion, we bring following conclusions.

(1) The methods we developed in calculating the first and second braking index of the intermittent pulsar is self-consistent.

(2) The agreement between the simple spin-down law and our model guarantees the validity of our approach and supports our results to some extent.

(3) The calculated first and second braking indices indicate that pulsar magnetosphere may never become entirely depleted of plasma current during "off" state, and the switches between "on" and "off" phases may not causes global change in magnetosphere.

(4) Under our model, the changes in charge density and polar cap size cannot explain the braking index jump of PSR B0540 +69; but the variation of magnetic field strength would be the best option to solve this problem. Acknowledgment

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