

The Δ mass dependence of the M matrix and its influence on the $N\Delta \rightarrow NN$ cross-sections*

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Abstract: The Δ mass dependence of the M matrix and its influence on the $N\Delta \rightarrow NN$ cross-sections are investigated in the one-boson exchange model. Our calculations show that the Δ mass dependence of the momentum of the outgoing Δ and the M matrix affects the calculations of $\sigma_{N\Delta \rightarrow NN}$, especially around the threshold energy.

Keywords: heavy-ion collisions, $N\Delta \rightarrow NN$ cross-section, detailed balance

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1 Introduction

The production and absorption of the Δ resonance in heavy-ion collision around the threshold energy has attracted considerable attention in recent years because the ratio of charged pions which arise from the decay of the Δ resonance is assumed to be an observable sensitive to the symmetry energy at the suprasaturation density [1-4]. Different conclusions concerning the constraints of symmetry energy have been obtained with different transport models [2-7], which stimulate further study of the Δ production and absorption mechanisms, as well as their sensitivity to the density region probed by the π^-/π^+ ratio. Recently, based on the IBUU calculations, Gao-Chan Yong [8] claimed that the π^-/π^+ ratio is sensitive to the symmetry energy around the normal density, rather than around the suprasaturation density. The debate concerning the constraints on the symmetry energy at suprasaturation density based on the π^-/π^+ ratio indicates that a more careful study of the Δ production and absorption cross-sections, as well as of the propagation of π in the reaction, are urgently needed.

Generally, in heavy-ion collision at intermediate energies, the production and propagation of pions follow three stages: 1) First, Δ s are produced in $NN \rightarrow N\Delta$ collisions; 2) after about 2 fm/c, which depends on the width of Δ -resonances, Δ s decay into nucleons and pions, and following them, Δ s are absorbed through $\pi + N \rightarrow \Delta$ pro-

cesses; 3) Δ s with a longer lifetime and higher energy participate in the $N\Delta \rightarrow NN$ reactions. The probability of these processes is directly related to their cross-sections or decay widths. Due to the complications in the high-dimensional transport models, most of the transport models adopt Monte-Carlo cascade method to solve the collision part where the nucleon-nucleon cross sections and decay widths are the key inputs. For the stages 1) and 2), the cross-sections and decay widths can be obtained from the experiments, and there is less ambiguity. However, the cross-sections of $N\Delta \rightarrow NN$ in stage 3) cannot be measured directly, and they have to be calculated using the detailed balance relationship.

One of the popular ways to obtain the $N\Delta \rightarrow NN$ cross-sections is to calculate them from the measured cross-sections of $NN \rightarrow N\Delta$ using the detailed balance [9-16]. The cross-sections of $NN \rightarrow N\Delta$ in free space have been measured [17-21], and are well explained by the one-boson exchange model (OBEM) and the relativistic Boltzmann-Uhling-Uhlenbeck approach [22-27]. The detailed balance means that the scattering matrix elements obtained from the time-reversal invariance are equal, i.e. $|\mathcal{M}|_{if}^2 = |\mathcal{M}|_{fi}^2$, where i and f are the initial and final states of the scattered particles.

Since Δ is a resonance with a broad mass distribution, it requires to consider the Δ -mass effects in the calculations of the $N\Delta \rightarrow NN$ cross-sections [9-11, 14, 28, 29]. For example, Danielewicz et al. [10] considered a linear m_Δ dependence of $|\mathcal{M}|^2$ (i.e. the Δ mass dependence of

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$|\mathcal{M}_D|^2$ is ignored¹⁾ in the $NN \rightarrow N\Delta$ process), and obtained the following relationship for the one- $\Delta(1232)$ absorption cross-section [10, 14, 16, 22, 30, 31],

$$\sigma_{N_3\Delta_4 \rightarrow N_1N_2}^D = \frac{1}{2} \frac{1}{1 + \delta_{N_1N_2}} \frac{|\mathbf{p}_{12}|^2}{|\mathbf{p}_{34}(m_\Delta)|} \sigma_{N_1N_2 \rightarrow N_3\Delta_4} \int_{m_N+m_\pi}^{\sqrt{s}-m_N} dm'_\Delta f(m'_\Delta) |\mathbf{p}_{34}(m'_\Delta)|. \quad (1)$$

The superscript D means that the $N\Delta \rightarrow NN$ cross-section is calculated using the method proposed by Danielewicz et al. $f(m'_\Delta)$ is the Δ mass distribution, $\frac{1}{1+\delta_{N_1N_2}}$ takes into account identical of final two nucleons. If the Δ mass dependence of $|\mathbf{p}_{34}|$ is also ignored, this leads to the Wolf et al. formula [11, 12], which we express as,

$$\sigma_{N_3\Delta_4 \rightarrow N_1N_2}^W = \frac{1}{2N} \frac{1}{1 + \delta_{N_1N_2}} \frac{|\mathbf{p}_{12}|^2}{|\mathbf{p}_{34}(m_\Delta)|^2} \times \sigma_{N_1N_2 \rightarrow N_3\Delta_4}. \quad (2)$$

where the factor $N = \int_{m_N+m_\pi}^{\sqrt{s}-m_N} f(m'_\Delta) dm'_\Delta$. The two methods of calculating $\sigma_{N\Delta \rightarrow NN}$ based on Eq. (1) and Eq. (2) have been discussed in Refs. [11, 12, 14]. It was found that they can obviously influence the calculation of observables of heavy-ion collisions, such as the rapidity distribution and flow of pions, at beam energies from 0.8 A GeV to 1.35 A GeV. Since both methods ignore the Δ mass dependence of $|\mathcal{M}_D|^2$ or $|\mathbf{p}_{34}|$, which is thought to be very important near the threshold energy, it would be interesting to evaluate the precision of the two methods and to calculate the $N\Delta \rightarrow NN$ cross-sections taking into account the m_Δ dependence of the M matrix and $|\mathbf{p}_{34}|$.

In this work, we first investigate the Δ mass dependence of $|\mathcal{M}|^2$ and $|\mathbf{p}_{34}(m_\Delta)|$ in the framework of OBEM. The cross-section of $N\Delta \rightarrow NN$ in free space is directly obtained from the M matrix element. It is denoted as $\sigma_{N\Delta \rightarrow NN}^{th}$, and is chosen as a benchmark for the methods proposed in [10-12], i.e. Eqs. (1) and (2), for calculating $\sigma_{N\Delta \rightarrow NN}$ from $\sigma_{NN \rightarrow N\Delta}$. Finally, the exact results for $\sigma_{N\Delta \rightarrow NN}$ and the sampling function for the mass of Δ in the transport models are given in the framework of OBEM.

2 Theoretical model

We adopt the OBEM method with the effective Lagrangian density for nucleons and Δ baryons which interact through σ , ω , ρ , δ , and π mesons [22, 27, 32-34]. Different from Ref. [23], we also include the isovector

mesons ρ and δ in order to describe the isospin asymmetric nuclear matter and the isospin dependent in-medium $NN \rightleftharpoons N\Delta$ cross-sections. The cross-section of $NN \rightleftharpoons N\Delta$ can be calculated from their M matrix [22]. The elementary two-body $NN \rightarrow N\Delta$ cross-section for a given m_Δ reads

$$\tilde{\sigma}(m_\Delta) = \frac{1}{4F} \int \frac{d^3\mathbf{p}_3}{(2\pi)^3 2E_3} \frac{d^3\mathbf{p}_4}{(2\pi)^3 2E_4} \times (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) |\overline{\mathcal{M}(m_\Delta)}|^2 = \frac{1}{64\pi^2} \int \frac{|\mathbf{p}_{34}(m_\Delta)|}{\sqrt{s_{12}} \sqrt{s_{34}} |\mathbf{p}_{12}|} |\overline{\mathcal{M}(m_\Delta)}|^2 d\Omega, \quad (3)$$

where $|\overline{\mathcal{M}(m_\Delta)}|^2 = \frac{1}{(2s_1+1)(2s_2+1)} \sum_{s_1 s_2 s_3 s_4} |\mathcal{M}(m_\Delta)|^2$ is for the $N_1N_2 \rightarrow N_3\Delta_4$ process. \mathbf{p}_{12} and $\mathbf{p}_{34}(m_\Delta)$ are the center-of-mass momenta of the incoming (1 and 2) and the outgoing particles (3 and 4). $F = \sqrt{(p_1 p_2)^2 - p_1^2 p_2^2} = \sqrt{s_{12}} |\mathbf{p}_{12}|$ is the invariant flux factor, $s_{12} = (p_1 + p_2)^2$, and $s_{34} = (p_3 + p_4)^2$ ($s_{12} = s_{34} = s$ in free space). The total cross-section is the elementary two-body cross-section averaged over the mass distribution of Δ , i.e.,

$$\sigma_{N_1N_2 \rightarrow N_3\Delta_4} = \frac{1}{64\pi^2} \times \int dm_\Delta d\Omega \frac{|\mathbf{p}_{34}(m_\Delta)|}{\sqrt{s_{12}} \sqrt{s_{34}} |\mathbf{p}_{12}|} |\overline{\mathcal{M}(m_\Delta)}|^2 f(m_\Delta) \quad (4)$$

$f(m_\Delta)$ is the mass distribution of the Δ resonance,

$$f(m_\Delta) = \frac{2}{\pi} \frac{m_\Delta^2 \Gamma(m_\Delta)}{(m_{0,\Delta}^2 - m_\Delta^2)^2 + m_\Delta^2 \Gamma^2(m_\Delta)}. \quad (5)$$

Here, $m_{0,\Delta}$ is the pole mass of Δ . The decay width $\Gamma(m_\Delta)$ is taken in a parameteric form [23].

$\sigma_{N_3\Delta_4 \rightarrow N_1N_2}$ for a given value of m_Δ can be exactly calculated as,

$$\sigma_{N_3\Delta_4 \rightarrow N_1N_2}^{th} = \frac{1}{4F} \int \frac{d^3\mathbf{p}'_2}{(2\pi)^3 2E'_2} \frac{d^3\mathbf{p}'_1}{(2\pi)^3 2E'_1} \times (2\pi)^4 \delta^4(p'_1 + p'_2 - p'_3 - p'_4) |\overline{\mathcal{M}_{N\Delta \rightarrow NN}}|^2 = \frac{1}{1 + \delta_{N_1N_2}} \frac{1}{64\pi^2} \int \frac{|\mathbf{p}'_{12}|}{\sqrt{s_{34}} \sqrt{s_{12}} |\mathbf{p}'_{34}(m_\Delta)|} \times |\overline{\mathcal{M}_{N\Delta \rightarrow NN}}|^2 d\Omega, \quad (6)$$

\mathbf{p}'_{34} is the momentum of the incoming N or Δ , and \mathbf{p}'_{12} is the momentum of the outgoing N in the center-of-mass frame. The M matrix is,

$$|\overline{\mathcal{M}_{N\Delta \rightarrow NN}}|^2 = \frac{(2s_1+1)(2s_2+1)}{(2s_3+1)(2s_4+1)} |\overline{\mathcal{M}(m_\Delta)}|^2, \quad (7)$$

where the Δ mass is the same for both processes. The ra-

1) The definition of $|\mathcal{M}|^2$ in this work is different from that in Danielewicz et al. For the convenience, we named the M -matrix from Danielewicz's work as \mathcal{M}_D . There is a following relationship between ours and Danielewicz's, i.e. $4m_\Delta m_N^3 |\mathcal{M}_D|^2 = |\mathcal{M}|^2$ in the following discussions. Thus, the mass independence of $|\mathcal{M}_D|^2$ means the $|\mathcal{M}|^2$ in this work should linearly increase with the mass of Δ .

tio between Eq. (6) and Eq. (4) gives an exact relationship between the cross-sections of $NN \rightarrow N\Delta$ and $N\Delta \rightarrow NN$. Thus, $\sigma_{N\Delta \rightarrow NN}^{th}$ for given m_Δ can be written as,

$$\sigma_{N_3\Delta_4 \rightarrow N_1N_2}^{th} = \frac{\sigma_{N_1N_2 \rightarrow N_3\Delta_4} (2s_1 + 1)(2s_2 + 1)}{1 + \delta_{N_1N_2} (2s_3 + 1)(2s_4 + 1)} \times \frac{\int d\Omega |\mathbf{p}_{12}|^2 |\mathcal{M}(m_\Delta)|^2}{\int d\Omega |\mathbf{p}'_{34}(m_\Delta)| \int |\mathbf{p}_{34}(m'_\Delta)| f(m'_\Delta) |\mathcal{M}(m'_\Delta)|^2 dm'_\Delta} \quad (8)$$

One should note that \mathbf{p}'_{34} and \mathbf{p}'_{12} are the momenta of N_3 (or Δ_4) and N_1 (or N_2) in the center-of-mass of $N\Delta \rightarrow NN$, while \mathbf{p}_{12} and \mathbf{p}_{34} are the momenta of N_1 (or N_2) and N_3 (or Δ_4) in $NN \rightarrow N\Delta$. For a given center-of-mass energy \sqrt{s} , $|\mathbf{p}_{12}| = |\mathbf{p}'_{12}|$ for the ingoing and outgoing nucleons, but $|\mathbf{p}_{34}|$ may not be equal to $|\mathbf{p}'_{34}|$, depending on whether the mass of Δ is equal in the production and absorption processes.

3 Results and discussion

We first check the mass dependence of the extracted M matrix, i.e. $|\mathcal{M}(m_\Delta)|^2$, in free space, based on OBEM. The details of the M matrix calculations can be found in our previous work [27], and the parameters in the expression for $|\mathcal{M}|^2$ were determined by fitting the measured cross-sections of $pp \rightarrow n\Delta^{++}$ [17]. Several groups have published measured cross-sections of $NN \rightarrow N\Delta$ [17, 19–21]. As shown in Fig. 1, there is still an uncertainty of 3–5 mb in the measured cross-sections of $pp \rightarrow n\Delta^{++}$ around $\sqrt{s} \sim 2.2$ GeV and above 3.0 GeV. Two typical values of the cross-section of $pp \rightarrow n\Delta^{++}$, CERN8401 (blue triangles) [20] and Landolt-Börnstein [17] (red circles), were chosen to adjust the parameters of the M matrix, and to understand the uncertainties due to the experimental

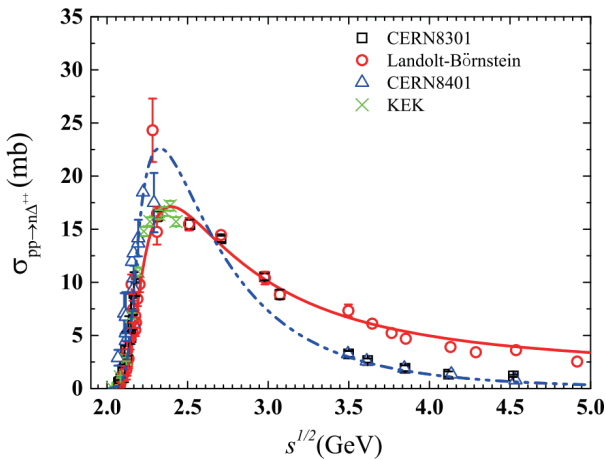


Fig. 1. (color online) $\sigma_{pp \rightarrow n\Delta^{++}}^*$ as a function of $s^{1/2}$ in free space. The experimental data are from [17, 19–21]. The blue dash line is the fit of CERN8401 [20] and the red dot line of the Landolt-Börnstein data [17].

errors, as they are the two extreme cases in the published data for $pp \rightarrow n\Delta^{++}$.

In Fig. 2 (a), we plot the angular integrated isospin independent M matrix as a function of m_Δ for the total energies $s^{1/2} = 2.1, 2.5$ and 3.0 GeV. The shadow regions correspond to the M matrix with the experimental uncertainties obtained from CERN8401 [20] and the Landolt-Börnstein data [17]. The range of m_Δ is from $m_N + m_\pi$ to $\sqrt{s} - m_N$, where the maximum value of m_Δ depends on the energy of the process of $NN \rightarrow N\Delta$. The isospin independent M matrix is obtained by normalizing the M matrix with the isospin factors, i.e.,

$$\mathcal{M} = \frac{1}{I_i^2} \int \sum_s |\mathcal{M}|^2 d\Omega = \frac{(2s_1 + 1)(2s_2 + 1)}{I_i^2} \int |\mathcal{M}|^2 d\Omega. \quad (9)$$

$I_{i=d,e}$ is the isospin factor as in Refs. [27, 32], and $I_{d,e}^2(n\Delta^{++} \rightarrow pp) = I_{d,e}^2(p\Delta^- \rightarrow nn) = 2$ and $I_{d,e}^2(\text{other channels}) = 2/3$. As shown in the left panel of Fig. 2, the behavior of \mathcal{M} as a function of m_Δ in OBEM clearly shows that $|\mathcal{M}(m_\Delta)|^2$ depends on m_Δ irrespective of the experimental data used. In order to understand the assumption of mass dependence of the M matrix in the method of Danielewicz et al. [10], we also show $\mathcal{M}_D = \frac{1}{I_i^2} \int |\mathcal{M}_D|^2 d\Omega$ in the inset of Fig. 2(a), which has the same convention as in Ref. [10] in GeV^4 . For the energy range selected, our calculations show that $|\mathcal{M}_D|^2$ clearly depends on m_Δ in all mass regions where Δ can be produced. This can be understood from the formula for the M matrix, Eq. (22) in Ref. [32]. For example, the exponent of m_Δ in the M -matrix is roughly m_Δ^2 . For higher energies, the Δ mass dependence of the M matrix becomes weak, which means that the assumption for calculating $N\Delta \rightarrow NN$ in Ref. [10] is reasonable.

Another point which needs to be investigated is the mass dependence of $|\mathbf{p}_{N\Delta}(m_\Delta)|$ (here $|\mathbf{p}_{N\Delta}(m_\Delta)|$ is $|\mathbf{p}_{34}(m_\Delta)|$) in Eq. (8). In Fig. 2(b), we present the mass dependence of $|\mathbf{p}_{N\Delta}(m_\Delta)|$ for different energies, where one can see that $|\mathbf{p}_{N\Delta}(m_\Delta)|$ decreases with the mass of Δ , and that the mass dependence is much sharper for lower energies than for higher energies. The panels in Fig. 2 show that both $|\mathcal{M}(m_\Delta)|^2$ and $|\mathbf{p}_{N\Delta}(m_\Delta)|$ in Eq. (8) depend on the mass of Δ , especially for lower energies.

Clearly, Fig. 2 shows that the Δ mass dependence (which in turn depends on the system energy) of the M matrix and $|\mathbf{p}_{N\Delta}(m_\Delta)|$ cannot be ignored, and can influence the accuracy of calculations of the $N\Delta \rightarrow NN$ cross-sections based on the detailed balance using Eq. (1) or Eq. (2). We select three typical values of m_Δ to understand the precision of the different methods of estimating $\sigma_{N\Delta \rightarrow NN}$: the minimum mass of Δ ($m_{\min,\Delta} = 1.077$ GeV), the pole mass ($m_{0,\Delta} = 1.232$ GeV); and $m_\Delta = 1.387$ GeV, which corresponds to the maximum mass of Δ produced in heavy-ion collisions with the beam energy of 1 GeV. Since other data give a similar m_Δ dependence of

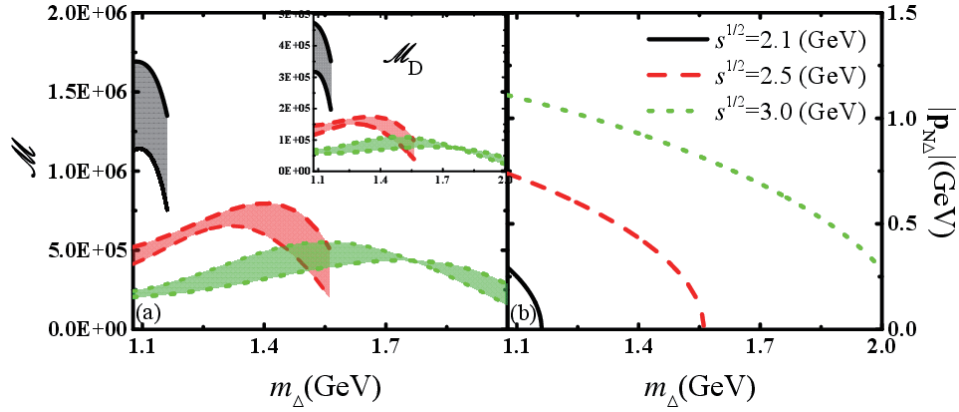


Fig. 2. (color online) (a) $\mathcal{M} = \frac{1}{f_i} \int \sum_s |\mathcal{M}|^2 d\Omega$ as a function of m_Δ in free space for the energies $s^{1/2}$ of 2.1, 2.5 and 3 GeV. The insert in the figure is $\frac{1}{f_i} \int \sum_s |\mathcal{M}_D|^2 d\Omega$ (GeV $^{-4}$) as in Ref. [10]. (b) $|p_{N\Delta}|$ as a function of m_Δ in free space.

the M matrix as shown in Fig. 2, we use in the following the M matrix parameters extracted from the Landolt-Börnstein data [17].

Figure 3 (a)-(c) present the results of $\sigma_{n\Delta^{++} \rightarrow pp}$ as a function of $s^{1/2}$ in free space for $m_\Delta = 1.077$, 1.232 and 1.387 GeV. The black solid lines are $\sigma_{n\Delta^{++} \rightarrow pp}^{th}$ which are directly calculated from the M matrix element of $N\Delta \rightarrow NN$ based on the scattering theory. The red dashed lines are $\sigma_{n\Delta^{++} \rightarrow pp}^W$ obtained with the method adopted by Wolf et al. [11-13], i.e. Eq. (2), without taking into account the mass dependence of $|p_{N\Delta}|$. The green dotted lines are $\sigma_{n\Delta^{++} \rightarrow pp}^D$ obtained by Danielewicz et al. [10], i.e. Eq. (1), where the mass dependence of $|M|^2$ is neglected. All methods predict a large Δ absorption cross-section around the threshold energy of the $n\Delta^{++} \rightarrow pp$ process

which increases with m_Δ , while $\sigma_{n\Delta^{++} \rightarrow pp}$ decreases with energy. However, the methods of Wolf [11-13] and Danielewicz [10] do not reproduce well $\sigma_{n\Delta^{++} \rightarrow pp}$ around the threshold energy if the Δ mass is far from the pole $m_\Delta = m_{0,\Delta} = 1.232$ GeV. Other channels of $N\Delta \rightarrow NN$ give similar results since the differences come only from the isospin factor.

In order to see the deviations clearly, we present in Fig. 3(d)-(f) the ratio defined as $R_d = \sigma_{\Delta N \rightarrow NN}^i / \sigma_{\Delta N \rightarrow NN}^{th}$ ($\sigma_{\Delta N \rightarrow NN}^W$ for $i = W$, $\sigma_{\Delta N \rightarrow NN}^D$ for $i = D$) for different masses of Δ . R_d has the same value in all channels of $N\Delta \rightarrow NN$ since the contributions from the isospin factor cancel in the ratio. $R_d = 1$ means the proposed detailed balance can well describe the $N\Delta \rightarrow NN$ cross section. Red dashed lines are the results of $\sigma_{\Delta N \rightarrow NN}^W / \sigma_{\Delta N \rightarrow NN}^{th}$, and green dotted lines $\sigma_{\Delta N \rightarrow NN}^D / \sigma_{\Delta N \rightarrow NN}^{th}$. For $m_\Delta = m_{0,\Delta}$, both methods

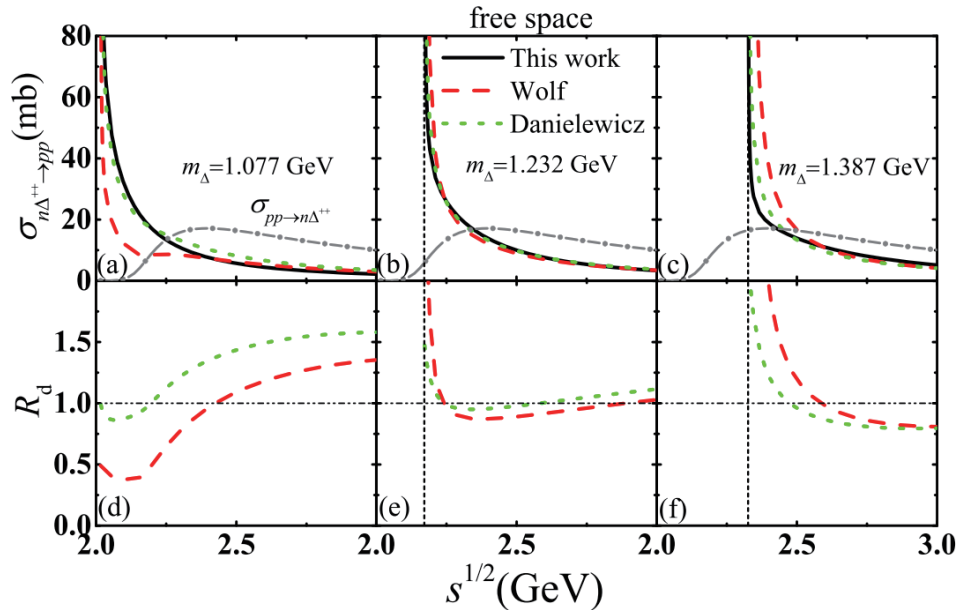


Fig. 3. (color online) Panels (a)-(c): $\sigma_{n\Delta^{++} \rightarrow pp}$ in free space as a function of $s^{1/2}$ for different detailed balance methods and for three values of m_Δ . Panels (d)-(f): R_d as a function $s^{1/2}$ corresponding to the upper panels.

reproduce well the values of $\sigma_{N\Delta \rightarrow NN}^{th}$, except for $s^{1/2} < 2.2$ GeV, where the Danielewicz method is much closer to the precise result than the Wolf method. If $m_\Delta = 1.076$ or 1.387 GeV, larger deviations are found near the threshold energy of the $N\Delta \rightarrow NN$ process (close to the vertical dashed lines). For example, if m_Δ is close to the minimum mass of Δ , both methods in Refs. [10, 11] underestimate the Δ absorption cross-section for $s^{1/2} < 2.2$ GeV, and the deviation is less than 20% for the Danielewicz method and larger than 50% for the Wolf approach. Both methods overestimate the Δ absorption cross-section, and the deviation is close to 50% for the Danielewicz method while the Wolf approach gives the deviation is less than 40% for $s^{1/2} > 2.5$ GeV. In the large mass region of Δ , both methods overestimate the Δ absorption cross-section for $s^{1/2} < 2.47$ GeV, but underestimate it for $s^{1/2} > 2.6$ GeV. Furthermore, the cross-sections of $N\Delta \rightarrow NN$ in free space can be changed by using different coupling constants [32]. The mass dependence of the M matrix should exist and cannot be simply ignored near the threshold energy, since the M matrix is determined by fitting the $NN \rightarrow N\Delta$ data. The above comparison suggests that the mass dependence of the M mat-

rix and $|p_{N\Delta}(m_\Delta)|$ should be taken into account for precise calculations of the $N\Delta \rightarrow NN$ cross-sections.

Using the isospin independent matrix \mathcal{M} in Eq. (9), the cross-section of $N\Delta \rightarrow NN$ can be expressed as,

$$\sigma_{N\Delta \rightarrow NN}^{th} = \frac{1}{64\pi^2 s} \frac{|p_{12}|}{|p_{N\Delta}(m_\Delta)|} \frac{I_i^2 \mathcal{M}}{(2s_3 + 1)(2s_4 + 1)} \frac{1}{1 + \delta_{N_1 N_2}}. \quad (10)$$

The form of \mathcal{M} can be found in Ref. [35]. For the $n\Delta^{++} \rightarrow pp$ and $p\Delta^- \rightarrow nn$ channels, $I_i^2 = 2$, while for the $n\Delta^+ \rightarrow np$, $n\Delta^0 \rightarrow nn$, $p\Delta^+ \rightarrow pp$, $p\Delta^0 \rightarrow np$ channels, $I_i^2 = 2/3$. Hence, the ratio $\sigma_{n\Delta^{++} \rightarrow pp} : \sigma_{p\Delta^- \rightarrow nn} : \sigma_{n\Delta^+ \rightarrow np} : \sigma_{p\Delta^0 \rightarrow np} : \sigma_{n\Delta^0 \rightarrow nn} : \sigma_{p\Delta^+ \rightarrow pp}$ is 3:3:2:2:1:1. Since the mass dependence of the M matrix is considered, the mass of Δ in the process $NN \rightarrow N\Delta$ should be sampled by considering the mass dependence of $|M|^2$. Therefore, the Δ mass should be sampled as,

$$P(m_\Delta) = \frac{\int_{m_N+m_\pi}^{m_\Delta} |p_{N\Delta}(m'_\Delta)| \times I_i^2 \mathcal{M} \times f(m'_\Delta) dm'_\Delta}{\int_{m_N+m_\pi}^{\sqrt{s}-m_N} |p_{N\Delta}(m_\Delta)| \times I_i^2 \mathcal{M} \times f(m_\Delta) dm_\Delta}. \quad (11)$$

Since the in-medium cross-sections are adopted in the

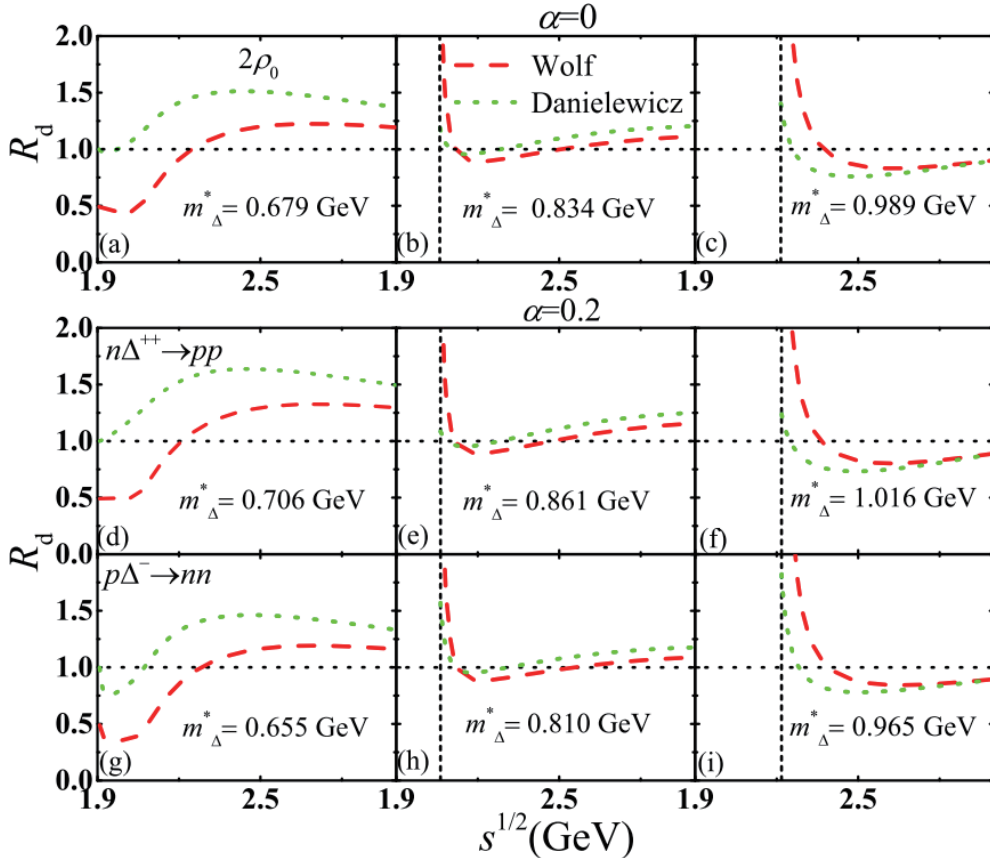


Fig. 4. (color online) Panel (a)-(c): R_d as a function of $s^{1/2}$ for $2\rho_0$, different types of detailed balance and three values of m_Δ^* , in symmetric nuclear matter $\alpha = 0$. Panel (d)-(f): R_d as a function of $s^{1/2}$ for $n\Delta^{++} \rightarrow pp$, and (g)-(i) for $p\Delta^- \rightarrow nn$, for $2\rho_0$ and different values of m_Δ^* , in asymmetric nuclear matter ($\alpha = 0.2$).

simulations of heavy-ion collisions, it is necessary to check the isospin effects and the precision of the calculations of the in-medium cross-sections of $N\Delta \rightarrow NN$ using the methods proposed in [10, 11]. The in-medium cross-sections of $NN \rightarrow N\Delta$ are calculated in the same way as in Refs. [27, 32], and depend on the coupling constants and isospin asymmetry. As a result of the isospin splitting of the in-medium $NN \rightarrow N\Delta$ cross-section, one can expect that R_d also depends on the isospin asymmetry and the channel of the $N\Delta \rightarrow NN$. In Fig. 4, we present R_d as a function of $s^{1/2}$ for two times normal density ($2\rho_0$) and for three m_Δ^* values. The upper panels are the results in symmetric nuclear matter, i.e. $\alpha = (\rho_n - \rho_p)/(\rho_n + \rho_p) = 0$. The panels (d)-(i) are the results in isospin asymmetric nuclear matter with $\alpha = 0.2$, where (d), (e) and (f) panels are for $n\Delta^{++} \rightarrow pp$, and (g), (h) and (i) panels are for $p\Delta^- \rightarrow nn$. The selected three values of m_Δ^* are the effective masses, which depend on the density, isospin asymmetry, and the charge state of Δ . Similarly to the results in free space, the deviations in the low and high Δ mass regions are larger than in the pole mass region. and the R_d values depend on m_Δ^* , $s^{1/2}$ and the channels of $N\Delta \rightarrow NN$. As shown in Fig. 4 (d) and (g), one can see that the m_Δ^* dependence of the M matrix influences the $p\Delta^- \rightarrow nn$ cross-section more than that of $n\Delta^{++} \rightarrow pp$ near the threshold energy in neutron-rich matter. The effect of isospin on R_d of in-medium $N\Delta \rightarrow NN$ can be understood from the ratio between Eq. (1) and Eq. (8), or between

Eq. (2) and Eq. (8), where the effective mass splitting of the Δ and nucleon in the M matrix leads to different R_d for different channels of $N\Delta \rightarrow NN$. Hence, the Δ mass dependence of the M matrix for in-medium $N\Delta \rightarrow NN$ cross-sections also depends on the channels of $N\Delta \rightarrow NN$.

4 Summary and outlook

In summary, we have evaluated the methods of calculating $\sigma_{N\Delta \rightarrow NN}$ from $\sigma_{NN \rightarrow N\Delta}$ in the framework of OBEM. Comparing $\sigma_{N\Delta \rightarrow NN}$ obtained with the methods proposed in [10] and [12] with $\sigma_{N\Delta \rightarrow NN}^{th}$, which is the exact result using the M matrix in OBEM, we showed that the methods in Refs. [10] and [12] underestimate the low mass and overestimate the large mass Δ absorption cross-sections near the threshold. We found that the mass dependence of the M matrix should be taken into account, especially around the threshold energy.

The role of accurate calculations of $\sigma_{N\Delta \rightarrow NN}$ in transport models of heavy-ion collisions near the threshold energy should also be investigated, since most of the Δ resonances participating in the processes $N\Delta \rightarrow NN$ are low mass Δ s. Another interesting work for the future is to evaluate the effect of the resonances with a much broader width, which could be useful to better understand the mechanisms of particle production in high energy heavy-ion collisions.

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