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Relativistic electron energy conversion in one photon in crystals

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Abstract: Interaction of relativistic electrons with crystal axis is traditionally described in terms of channeling model — i.e. as a motion in some homogeneous averaged axial potential. However, real crystals are made of atoms, positioned periodically. Interaction of electrons with periodical potential must be accompanied with transitions of discrete portions of momentum $\Delta p_{\|} = 2\pi n\hbar/d$, $n = 1, 2, 3, \ldots$, defined by the potential period d. Transmission of sufficient portion of longitudinal momentum to the lattice may be accompanied by the emission of photon with correspondingly high energy. In case of relativistic electrons with energies $E_1 \sim$ GeV or more, nearly all the electron's energy may be converted into just one photon.

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Contents

1 Introduction

Interaction of relativistic electrons with crystals is usually described in channeling model as a motion in certain averaged homogeneous potential of a crystal axis or plane (see for example [1, 2]). However real crystal axis or plane consists of positioned periodically atoms. Interaction with periodical potential (with period d) shall be accompanied by transitions of strictly discrete portions of momentum to the crystal lattice:

$$\Delta p_{\|} = 2\pi n\hbar/d, \qquad n = 1, 2, 3, \ldots. \tag{1.1}$$

If electron is moving along certain crystal axis OZ, the period d coincides with the period of crystal lattice along OZ-axis a_z ($d = a_z$). If electron is moving in a planar channeling mode (along YZ-plane) under small angle θ to atomic Z-rows, the period of inhomogeneity of planar potential along the electron's trajectory may be defined as $d = a_Y/\theta \gg a_Y$, a_z (provided axial channeling critical angle $\theta_{\text{crit}} \ll \theta \ll 1$)

Interaction with crystal lattice, accompanied by discrete changes of electron momentum (1.1) may lead to radiating photons with certain discrete frequencies under certain angles. This effect is known as "resonance bremsstrahlung". In multiple publications (see for example [4–8]) it was considered for low and moderate energy electrons and positrons $E_1 <\sim mc^2$.

In this note we want to draw attention to specific features of resonance bremsstrahlung in ultra-relativistic case $E_1 \gg mc^2$, when electrons are moving along some crystal axis. In this case, considering resonance bremsstrahlung with discrete momentum transitions (1.1), we may even neglect the channeling effect. The final state of electron after strong momentum transition (1.1) hardly will be channeling. The initial state of electron can also be considered as practically free. The most important feature of resonance bremsstrahlung at high energy case ($E_1 \sim$ Gev or more) — as we will demonstrate - is the possibility to emit photons with energy, comparable to E_1. This effect probably was responsible (or partly responsible) for the high energy radiation observed in [9, 10].

2 Photon emission kinematics

In any bremsstrahlung process (see for example [11]) the emitted photon frequency is defined by energy and longitudinal momentum conservation laws.

Energy:

$$E_1 = E_2 + \hbar\omega, \tag{2.1}$$

here E_1 is initial electron energy, E_2 — electron energy after emission of photon with energy $\hbar\omega = E_1 - E_2$.

Longitudinal (z-component) momentum:

$$\begin{gathered} P_{1\parallel} = P_{2\parallel} + (\hbar\omega/c) \times \cos\theta_\gamma - \Delta p_\parallel, \text{ or} \\ (E_1^2 - m^2c^4)^{1/2} = (E_2^2 - P_{2\mathrm{tr}}^2 c^2 - m^2c^4)^{1/2} + \hbar\omega \times \cos\theta_\gamma + c\Delta p_\parallel \end{gathered} \tag{2.2}$$

here mc^2 — electron rest energy, θ_γ — photon emission angle, $P_{2\mathrm{tr}}$ — transversal momentum of electron after emission of photon. In ultra-relativistic limit ($E_{1,2} \gg mc^2$) condition (2.2) with respect to (2.1) can be rewritten as:

$$m^2c^4\hbar\omega/2E_1E_2 + \hbar\omega \times \theta_\gamma^2/2 = c\Delta p_\parallel - E_{2\mathrm{tr}}, \tag{2.3}$$

here $\theta_\gamma \sim m^2c^4/E_1E_2 \ll 1$, $E_{2\mathrm{tr}} = P_{2\mathrm{tr}}^2 c^2/2E_2 \sim m^2c^4/E_2$ — transversal energy of electron after emission of photon.

In case of low energy photon $\hbar\omega \ll E_1, E_2$, condition (2.3) can be reduced to:

$$\hbar\omega \times (m^2c^4/E_1^2 + \theta_\gamma^2)/2 = c\Delta p_\parallel - E_{2\mathrm{tr}} \tag{2.4}$$

If $\Delta p_\parallel = 0$ (motion in smooth homogeneous averaged potential) — photon emission is possible only if $E_{2\mathrm{tr}} < 0$ i.e. if after photon emission the electron is captured in quasi-bound channeling state. This kind of radiation was considered in [3]. Strictly speaking the condition is $\Delta E_{\mathrm{tr}} = E_{1\mathrm{tr}} - E_{2\mathrm{tr}} > 0$ — i.e. initial state may also be channeling, but not necessarily. Radiation in channeling mode was thoroughly studied by plenty of authors (see for example review in [2]). Hereafter we will not consider this well-studied type of radiation. We will focus on processes with $c\Delta p_\parallel \sim 2\pi\hbar c/d >\sim 10^3$ eV. This value is 2–3 orders of magnitude higher than typical transversal energy changes ΔE_{tr} within channeling mode. Thus in this case the energy of emitted photon is defined completely by longitudinal momentum transition and equals

$$\hbar\omega = 2c\Delta p_\parallel/(m^2c^4/E_1^2 + \theta_\gamma^2) \sim 4\pi n\hbar E_1^2/dm^2c^3 \tag{2.5}$$

Energies (2.5) relate to ranges of hard X-ray and γ-ray radiation. Corresponding wavelengths $\lambda \sim (mc^2/E)^2 d \ll d$ — are much shorter than distances between atoms.

The distinguishing features of considered radiation are:

1. It is possible without any participation of channeling effect;

2. Emitted photons will have extremely high energies, exceeding those from channeling particles due to the fact that $c\Delta p_\parallel = 2\pi\hbar c/d \gg U_0$.

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When $E_1 \sim dm^2c^3/4\pi\hbar \sim 100$ MeV or more, the energies of emitted photons may be comparable to the initial electron energy E_1. Even if the longitudinal momentum transition is minimal possible $c\Delta p_{\parallel} = 2\pi\hbar c/d$. For this case expressions (2.4) and (2.5) shall be correspondingly corrected . In case of small photon emission angles $\theta_\gamma < m/E_1$ condition (2.3) can be reduced to:

$$m^2c^4 \times \hbar\omega/2E_1(E_1 - \hbar\omega) = 2\pi n\hbar c/d \Rightarrow \hbar\omega = E_1/(1 + dm^2c^3/4\pi n\hbar E_1) \sim E_1 \tag{2.6}$$

The energy (2.6) defines the upper limit of radiation spectrum line, as photons emitted at non-zero angles $\theta_\gamma > 0$ will have lower energies. However expression (2.6) adequately defines the order of magnitude for emitted photons energies. Very probably this effect is responsible for high energy radiation, observed in experiments [5] and especially [6].

3 Photon emission probability estimation

We may assume that if to neglect the channeling effect, the integral cross-section for radiation in the range defined by condition (2.6), shall be approximately the same in crystals as in amorphous media with the same type of atoms and density of matter. The expected difference is the redistribution of bremsstrahlung over frequencies. In amorphous media bremsstrahlung has the evenly decreasing with frequency spectrum $\sim 1/\omega$. In crystal the spectral density of resonance bremsstrahlung will be concentrated close to the frequency (2.6), thus inevitably exceeding in this range the background level of normal bremsstrahlung.

The effective cross-section of coherent bremsstrahlung can be quantitatively estimated, if to consider it in the so called accompanying reference system (ARS), moving along axis Z with velocity, equal to the z-component of the electron's velocity v_z. where initial electron rests. In ARS the periodical potential of a crystal axis can be presented as a Fourier row

$$U(z + v_z t, r) = (E/m)U_0(r) + (E/m)\Sigma U_n(r)\cos[2\pi n(E/m)((z + v_z t)/d)] \tag{3.1}$$

here $(E/m) > 1$ — is the Lorentz-factor, strongly shortening in ultra-relativistic case the period of potential (3.1) and simultaneously multiplying amplitudes of all the periodical harmonics U_n as well as the amplitude of the averaged axial potential $U_0(r)$.

The axial periodical potential (without the averaged component) may be considered in ARS as a package of electromagnetic waves with frequencies nc/d, or — in quantum approach — as a flow of photons with corresponding frequencies and energies. The photons flow intensity is defined by the amplitudes of potential harmonics (3.1). The bremsstrahlung process in this case can be considered as the result of the Compton's back-scattering of photons on a resting electron.

We may expect that the most effective will be the first harmonic component with $n = 1$, which has the lowest frequency and shall have an amplitude comparable to the amplitude of the averaged component $\sim (E/m)U_0$. The density of the 1$^{\text{st}}$ harmonic photons flow is proportional to the squared amplitude of the 1$^{\text{st}}$ harmonic electric field, divided by the corresponding photon frequency $\sim E/mcd$. For electrons moving close to the crystal axis (channeling or close to channeling mode) the acting electric field can be estimated as the ratio of the 1$^{\text{st}}$ harmonic potential amplitude $\sim (E/m)U_0$ in ARS to the harmonic wave-length ~ md/E in ARS (hereafter the atomic units system $\hbar = c = 1$ is applied):

$$\Phi[\mathrm{m}^{-2}\mathrm{s}^{-1}] \sim [(EU_0/m)/(md/E)]^2/(E/md) \sim E^3U_0^2/dm^3 \tag{3.2}$$

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and the Compton back-scattering probability is equal in ARS to $P_{\mathrm{ARS}}[\mathrm{s}^{-1}] = \sigma\Phi$ where σ is the cross-section of Compton backscattering.

For non-channeling ('ree') electrons, evenly distributed over the impact parameters to crystal axis, the additional factor $(\pi R^2/a^2)$ shall be introduced in (3.2) to take into account the fast ddecrease of ionic axis potential at distances $r > R$ from closest axis (R is the radius of the atomic lattice ions — i.e. the radius of the axial potential, a^2 is the transversal cross-section of a crystal cell, perpendicular to considered axis).

The Compton back-scattering probability is equal in ARS to $P_{\mathrm{ARS}}[\mathrm{s}^{-1}] = \sigma\Phi$, where σ is the cross-section of Compton backscattering. In ultra-relativistic case this cross-section is well known (see for example [11]) and approximately equals (in ARS, in atomic units):

$$\sigma \sim= \pi r_e^2 (m/\omega) \ln(2\omega/m) \sim= (\pi e^4/m\omega) \ln(2\omega/m) \tag{3.3}$$

Here $r_e = e^2/m$ — the classical radius of electron, $e^2 =\sim 1/137$ — the thin structure constant, $\omega = 2\pi E/md$ — the frequency of the first harmonic wave (in our case) in ARS. Substituting (3.3) to (3.2) we obtain (in atomic units):

$$P_{\mathrm{ARS}} = \sigma\Phi =\sim (e^4/2) ln(4\pi E/m^2 d)(E^2 U_0^2/m^3) \tag{3.4}$$

In Lab reference system the scattering probability decreases by Lorentz-factor as a consequence of changing time duration of all the observed processes:

$$P_{\mathrm{Lab}} = (m/E) P_{\mathrm{ARS}} =\sim (e^4/2) \ln(4\pi E/m^2 d)(E U_0^2/m2) \tag{3.5}$$

Probability (3.5) defines how thick shall be a crystal target in which an electron will suffer the resonance bremsstrahlung with probability $P \sim 1$: $L \sim 1/P_{\mathrm{Lab}}$. With typical values $U_0 \sim 50\,\mathrm{eV}$, $R \sim 10^{-10}$ m the probability (3.5) secures the effective conversion length (in meters): $L \sim 10^3 (m/E) \sim 10$ cm for 5 GeV electrons.

It means that in a 1 cm thick crystal already a noticeable share of electrons may really convert their energy into practically one photon each. More accurate estimation of the effect probability requires numerical calculations with particular parameters and for certain particular ion lattice potential model. However, by the order of magnitude estimation (3.5) does not contradict with experimental evidences [9, 10],

In case of free electrons (no channeling effect) the estimation (3.5) shall be decreased by the factor $(\pi R^2/a^2)$.

4 Conclusion

The interaction of ultra-relativistic electrons with periodical inhomogeneity of crystal lattice potential lead to emission of extreme high energy photons (resonance bremsstrahlung). If electron's energy is comparable to or exceeding 1 GeV this effect lead to converting of nearly complete energy of electron into one photon. The estimated probability of the considered process does not exclude the possibility that resonance bremsstrahlung effect is responsible for the reported experimental results [9, 10].

Acknowledgments

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