

Collective excitation of two-dimensional Bose–Einstein condensate in liquid phase with spin–orbit coupling

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Abstract

We have studied the collective excitation of Bose–Einstein condensate of short-range weak interacting atoms with spin–orbit coupling (SOC) in two dimensions in the liquid phase. In our study, we have included Rashba, Dresselhaus, as well as Raman SOC. The study of Bogoliubov excitation shows that in small interactions, only phonon modes are present, and for higher interactions, roton modes start to appear. Energy spectra contain two roton modes in relatively strong interacting atomic systems.

Keywords: Bose–Einstein condensation, spin–orbit coupling, collective excitations, quantum droplets

Liquid formation of the dilute ultracold atomic system [1, 2] is one of the most exciting topics in Bose–Einstein condensation (BEC). The droplets have been observed in the isotropic short-range interacting system of two species of cold atoms [2–4], as well as in the anisotropic long-range dipolar interacting systems of ^{164}Dy or ^{166}Er atoms [5]. In the mixture of two-component Bose atoms, spherical droplets have been observed under the competition between the effective short-range attractive interaction and the repulsive interaction due to the quantum fluctuation [6]. The two-component BEC may be the mixture of atoms of two different elements (different atomic mass) [7, 8] or possibly the mixture of atoms with two different internal degrees of freedom of a given element [9–11]. In the dipolar system, cigar-shaped droplets have been observed in the balanced attractive interaction due to the asymmetric dipolar and repulsive interactions due to quantum fluctuations. Three-body collisions limit the lifetime of the droplets. In the lower dimensions, it is expected that this lifetime can be extended because of the reduced phase space available to colliding atoms. This is why people have an interest in droplets in the lower dimension. There are already theoretical proposals of liquid states of BEC in the lower dimensions [12–16].

Spin–orbit coupling (SOC), the coupling between the spin angular momentum of a particle with its center of mass

motion, is present in atomic electrons and electrons in some solids [17]. Based on the symmetry of interaction, we have two types of 2D SOC in solid-state systems such as Rashba [18] and Dresselhaus SOC [19]. SOC in 2D solids produces the topological spin Hall effect; it has been observed in neutral ultracold atoms with the help of atom–light interaction [20, 21]. The study of BEC in the system of spin–orbit-coupled Bose gas became very important after the discovery of Lin et al. [20]. Chiquillo has theoretically addressed the liquid phase of two species of Bose atoms with Rabi coupling [22]. The self-trapped BEC with SOC has been investigated by Zhang et al. [23] in three dimensional system and Li et al. [24] in two dimensional system.

Theoretically, the system can be addressed in two different approaches such as the mean-field Gross–Pitaevskii (GP) equation, which is involved in the solution of time-dependent nonlinear coupled differential equations; another approach is Monte-Carlo integration with some suitable microscopic wave function [25]. GP equations have been solved for isotropic droplets of two species of atoms for different kinds of interactions [26] and anisotropic dipolar systems [27]. Here, we have applied the GP equation technique, as it is most popular and has the advantage of attaining collective excitation using the Bogoliubov approach.

The elementary excitation in a uniform BEC of infinite extent is easy to study using Bogoliubov theory. In this weak

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interacting system of ultracold dilute gas of Bose atoms in the condensate, the low-energy collective excitation is the well-known phonon mode [28]. On the other hand, the excitation in the nonuniform confined state is discrete due to the finite size of the system, which has been studied theoretically [29] and experimentally [30]. The collective excitation of uniform BEC of atoms with SOC has been studied theoretically [31] and experimentally [32], resulting in the phonon–roton mode of excitation.

The liquid droplets are finite in size, but have completely different properties than confined BEC. The density inside the droplet is constant except near the surface. To study the collective excitation inside the liquid droplets, we have considered sufficiently large droplets to avoid the surface effect. To find the Bogoliubov spectra, we have used the constant background density, which is obtained by solving the GP equations. Thus, the excitation is equivalently the collective excitation of bulk liquid of ultracold atoms.

In our study, we have considered the two-dimensional BEC of a mixture of atoms of two internal degrees of freedom of the same element. The short-range delta functions of repulsive potential between the atoms of same species and attractive potential between atoms of different species have been considered. We have considered two types of SOC: a combination of Rashba and Dresselhaus SOC with different strengths, and Raman SOC.

1. Model and method of calculations

The short-range interaction potential between two atoms can be written as $V(\vec{r}_1, \vec{r}_2) = g_{ij}\delta(\vec{r}_1 - \vec{r}_2)$. g_{ij} is the strength of the interaction of atoms of i th and j th species, and can be expressed as $g_{ij} = \frac{4\pi\hbar^2 a_{ij}}{m}$, where m is the mass of each atom and a_{ij} is the scattering length, which can be controlled by magnetic Feshbach resonance. Near the phase transition, the mean-field GP equations are not sufficient to study the nature of the condensate, so we need to consider a self-repulsive beyond-mean-field, Lee–Huang–Yang (LHY) term (quantum fluctuation term) [6]. The LHY interaction in the 2D system takes the logarithmic form [12], and the corresponding nonlinear coupled GP equations of two-component BEC in the presence of Rashba and Dresselhaus SOC can be written as [24]

$$i\frac{\partial\Psi}{\partial t} = -\frac{\nabla^2}{2}\Psi + g(|\psi_1|^2 - |\psi_2|^2)\sigma_z\Psi + \frac{g^2}{4\pi}\rho \ln(\rho)\Psi + \lambda_R\vec{\sigma} \cdot \hat{z} \times \vec{p}\Psi + \lambda_D\vec{\sigma} \cdot \vec{p}\Psi. \quad (1)$$

All the quantities are expressed in suitable natural units of the system [26]. The first term of the right-hand side is the kinetic energy term, the second term is the interaction potential term, the third term is the quantum fluctuation term, the fourth term is the Rashba SOC term with coupling strength λ_R , and the last term is the Dresselhaus term with coupling strength λ_D . $\Psi = (\psi_1\psi_2)^T$ is the two-component wave function and $(\sigma_x, \sigma_y, \sigma_z)$ are the Pauli spin matrices. Here, we have considered

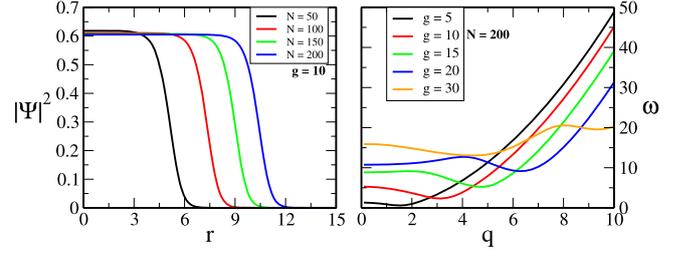


Figure 1. Circular liquid film in absence of SOC: The nature of the ground state as well as excited state in the absence of SOC ($\lambda_R = \lambda_D = 0$). Left panel: density of the condensate as a function of distance from the center for different numbers of particles for a fixed value of $g(=10.0)$. Right panel: Collective excitations of the liquid film for different values of g as indicated inside the figure.

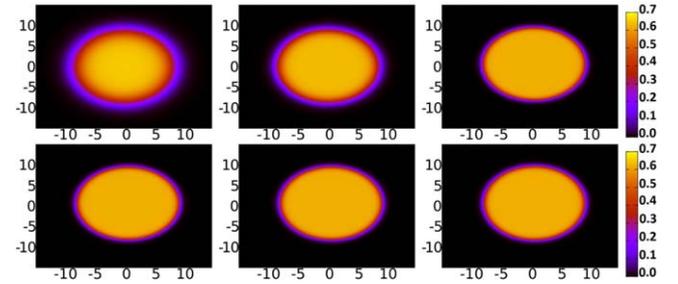


Figure 2. Ground state density for various strengths of Rashba SOC and two-body interaction for 150 atoms. First Row: $g = 3, 5,$ and 10 , respectively, for $\lambda_R = 0.2$. **Second Row:** $\lambda_R = 0.2, 0.3,$ and 0.5 , respectively for $g = 10$.

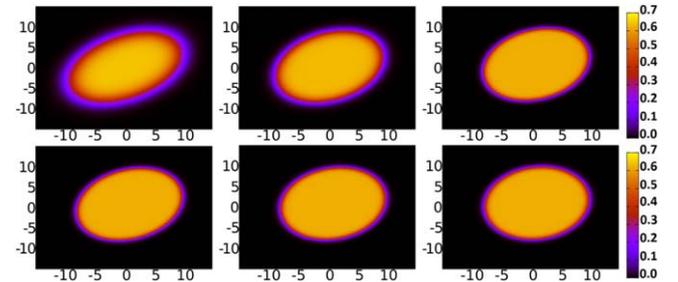


Figure 3. Ground state density for various strengths of Dresselhaus SOC and two-body interaction for 150 atoms at fixed Rashba interaction $\lambda_R = 1.0$. First Row: $g = 3, 5,$ and 10 , respectively, for $\lambda_D = 0.2$. **Second Row:** $\lambda_D = 0.2, 0.3,$ and 0.5 , respectively, for $g = 10$.

$g_{11} = g_{22} = -g_{12} = g$. The normalization of the wave function is given by

$$\int (|\psi_1|^2 + |\psi_2|^2)d^2r = \int \rho(\vec{r})d^2r = N, \quad (2)$$

where N is the total number of particles in the system, and ρ is the density of the condensate. We have used the Crank Nicolson method to solve the nonlinear coupled equations. An alternating direction implicit method [33] has been used to tackle the 2D derivatives.

Li *et al* [24] have established the mixed-mode region in the parameter space. We have calculated the ground state density and excitation in the mixed-mode region, where the two species superpose uniformly on each other [34]. The ground state density has been plotted in the left panel of figure 1 as a function of distance from the center of the condensate for the

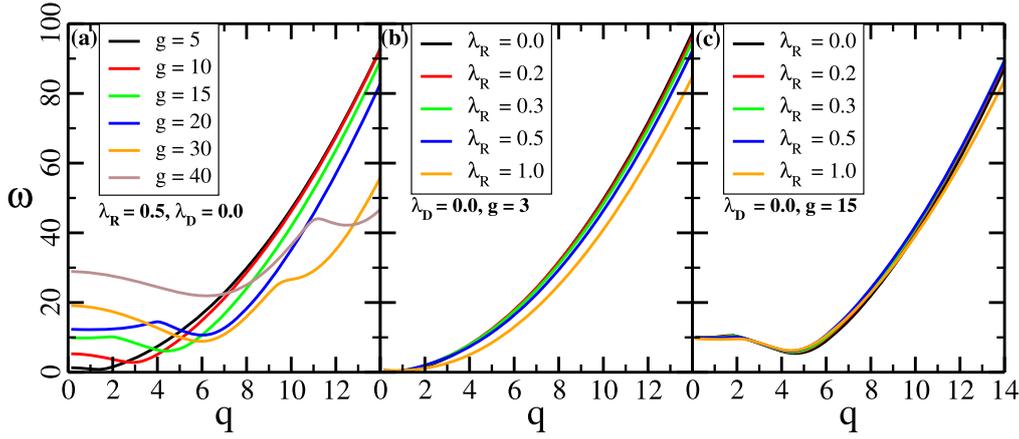


Figure 4. Rashba SOC: Collective excitation of the condensate under Rashba SOC. (a) Variation of excitations with g for fixed Rashba SOC. (b), (c) Variation under Rashba SOC for fixed g .

system without SOC. This shows that the density does not depend on the number of particles. If we increase the number of particles, the size of the film will increase. The ground state density values for different combinations of Rashba and Dresselhaus SOC have been shown in figures 2 and 3.

1.1. Collective excitation

After getting the ground state wave function, we applied Bogoliubov theory of excitation over the ground state. We chose the excited state wave function as a perturbation over the ground state, $\psi_j^{\text{exc}} = \psi_j + \delta\psi_j$. The perturbation part of the wave function can be written as $\delta\psi_j = U_j \exp(i\vec{q} \cdot \vec{r} - i\omega t) - V_j^* \exp(-i\vec{q} \cdot \vec{r} + i\omega t)$, where $\{U_j, V_j\}$ are amplitude of the excitation, \vec{q} is quasi-momentum of the excitation, and ω is the excitation energy. If we put these in our GP equations, we will find the equations for $\{U_j, V_j\}$, in the first-order approximation of $\{\delta\psi_i\}$

$$\begin{pmatrix} H_1 & X + A\psi_1\psi_2^* & B\psi_1^2 & A\psi_1\psi_2 \\ X^* + A\psi_1^*\psi_2 & H_2 & A\psi_1\psi_2 & B\psi_2^2 \\ -B\psi_1^{*2} & -A\psi_1^*\psi_2^* & -H_1 & -X^* - A\psi_1^*\psi_2 \\ -A\psi_1^*\psi_2^* & -B\psi_2^{*2} & -X - A\psi_1\psi_2^* & -H_2 \end{pmatrix} \times \begin{pmatrix} U_1 \\ U_2 \\ V_1 \\ V_2 \end{pmatrix} = \omega \begin{pmatrix} U_1 \\ U_2 \\ V_1 \\ V_2 \end{pmatrix}, \quad (3)$$

where

$$\begin{aligned} H_1 &= \frac{q^2}{2} + 2g|\psi_1|^2 - g|\psi_2|^2 + \frac{g^2}{4\pi}(2|\psi_1|^2 + |\psi_2|^2)\ln(\rho) \\ H_2 &= \frac{q^2}{2} + 2g|\psi_2|^2 - g|\psi_1|^2 + \frac{g^2}{4\pi}(2|\psi_2|^2 + |\psi_1|^2)\ln(\rho) \\ A &= -g + \frac{g^2}{4\pi}\ln(\rho) \\ B &= g + \frac{g^2}{4\pi}\ln(\rho) \\ X &= \lambda_R(q_y + iq_x) + \lambda_D(q_x + iq_y). \end{aligned}$$

The energy eigen value is obtained by diagonalizing the matrix. The results are shown in the right panel of figure 1 for the system without SOC and in figures 4 and 5 for the SOC. The results are discussed in Section 2.

1.2. Raman SOC

After the discovery of the SOC-BEC in 2011 [20], another form of spin-orbit coupling, the Raman SOC, was developed. The coupled GP equations for the system with $g_{1,1} = g_{2,2} = -g_{1,2} = g$ can be written as

$$\begin{aligned} i\frac{\partial\psi_1}{\partial t} &= \left[-\frac{\nabla^2}{2} + ik_0\partial_x + \frac{\delta}{2} + V_{\text{INT}} \right] \psi_1 + \frac{\Omega}{2}\psi_2 + L_{\text{HY}}\psi_1 \\ i\frac{\partial\psi_2}{\partial t} &= \left[-\frac{\nabla^2}{2} - ik_0\partial_x - \frac{\delta}{2} - V_{\text{INT}} \right] \psi_2 + \frac{\Omega}{2}\psi_1 + L_{\text{HY}}\psi_2 \end{aligned} \quad (4)$$

where $V_{\text{INT}} = g(|\psi_1|^2 - |\psi_2|^2)$ is the interaction part of the equations, δ is the detuning parameter (we have chosen $\delta = 0$ in our study), $L_{\text{HY}} = \frac{g^2}{4\pi}\rho \ln(\rho)$ is the quantum fluctuation part of the equations, Ω is the Rabi frequency, and k_0 is the strength of SOC. In the ultracold atomic system, one can vary all parameters in a controlled manner. Solving the above-mentioned equation, we get the density of the condensate, shown in the figure 6.

The excitation spectra can be obtained by diagonalizing the matrix (similarly as earlier)

$$\begin{pmatrix} H_1 - k_0q_x & \frac{\Omega}{2} + A\psi_1\psi_2^* & B\psi_1^2 & A\psi_1\psi_2 \\ \frac{\Omega}{2} + A\psi_1^*\psi_2 & H_2 + k_0q_x & A\psi_1\psi_2 & B\psi_2^2 \\ -B\psi_1^{*2} & -A\psi_1^*\psi_2^* & -H_1 - k_0q_x & -(\frac{\Omega}{2} + A\psi_1^*\psi_2) \\ -A\psi_1^*\psi_2^* & -B\psi_2^{*2} & -(\frac{\Omega}{2} + A\psi_1\psi_2^*) & -H_2 + k_0q_x \end{pmatrix}. \quad (5)$$

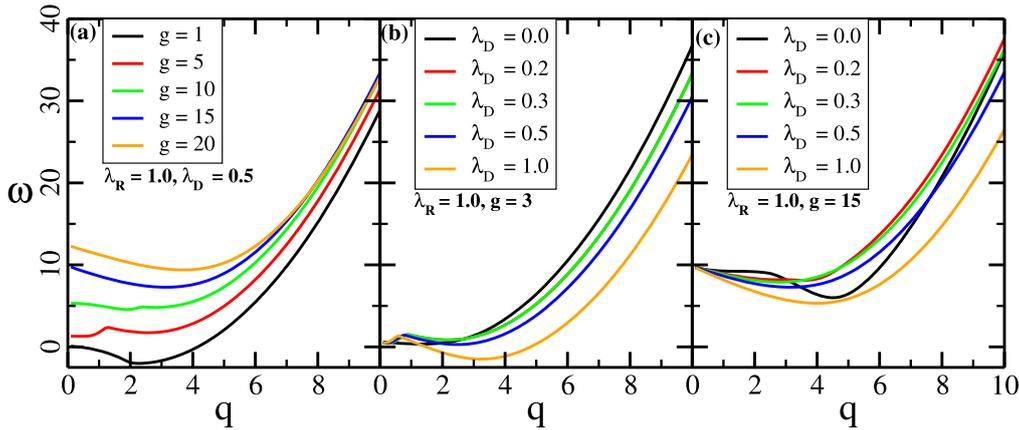


Figure 5. Rashba and Dresselhaus SOC combination: Collective excitation of film of atoms with combined Rashba and Dresselhaus SOC.

The excitation spectra are shown in figure 7 for various strengths of SOC and Rabi frequencies.

2. Results and discussion

a) **Circular film in absence of SOC:** We have solved the GP equation in 2D BEC for the system of SOC atoms in the liquid phase. In this study, we have considered the repulsive interaction between atoms of same species and attractive interaction between atoms of different species. The quantum fluctuation is responsible for preventing the collapse of the condensate due to attractive interaction. We get the circular liquid film for the system without SOC. It is clear from our calculation (left panel of figure 1) that if we increase the number of particles, the size of the film increases, keeping the density fixed. The collective excitation for such a system is a phonon-like mode for weak two-body short-range interaction, and if we increase the strength of the interaction, the mode becomes roton like and the position of the roton minimum shifts to the larger momentum as we increase the strength of interaction; see the right panel of figure 1. If we increase the strength of interaction, we will have two roton minima instead of one.

b) **The film of atoms with Rashba and Dresselhaus SOC:** The film is nearly circular but not precisely circular due to the Rashba SOC, as shown in figure 2. The density inside the film is constant over the entire region. There is very small asymmetry in collective excitation along the X- and Y-directions in momentum space. We have plotted the energy spectra for different Rashba SOC strengths for different atom–atom interaction strengths in figure 4(a). The variation of energy spectra with Rashba SOC has been shown in figures 4(b) and (c) for $g = 3$ and 15 , respectively. The nature of the excitation is almost same as that of the system without SOC. Phonon mode disappears in the presence of both the Rashba and Dresselhaus SOC (see figure 5), and we have roton mode of excitation even at small g .

c) **The film of atoms with Raman SOC:** The ground state of the BEC of atoms with Raman SOC separate into two centers (see figure 6) due to the properties of single-particle

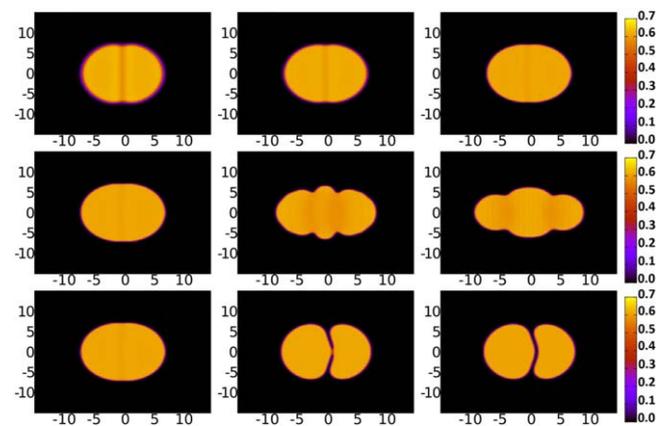


Figure 6. Density of the condensate in the presence of Raman SOC interaction for 100 atoms with different strength parameters. **First Row:** Variation of the shape of the droplet with respect to interaction strength $g = 20, 30$, and 50 , respectively. The other parameters are $k_0 = 1.0$ and $\Omega = 1.0$. **Second Row:** Variation with respect to SOC strength $k_0 = 1.0, 3.0$, and 5.0 , respectively. The other parameters are $g = 50.0$ and $\Omega = 1.0$. **Third Row:** Variation with respect to Rabi frequency $\Omega = 1.0, 10.0$, and 50.0 , respectively. The other parameters are $g = 50.0$ and $k_0 = 1.0$.

quantum state. The effective interaction gives a negative contribution to the total energy, whereas the kinetic energy (comes from the surface of the droplet, the surface tension) gives a positive contribution. The kinetic energy tries to evaporate the droplet whereas the interaction tries to bound the particles to form the droplet. If we increase the anisotropy by changing the interaction parameters, there is a great possibility of evaporation for anisotropic droplets due to the large surface area with high gradient. We have considered a large number of particles and kept the parameters within the safe region of stable droplets. We have used any part of the condensate to get the collective excitation as the density and phase of the ground state is identical in the two regions. The excitation spectra are shown in figure 7. The energy spectra are symmetric along the Y-direction and asymmetric along the X-direction of momentum space.

The energy spectrum has two roton minima for some parameters. One important thing we have observed is that the

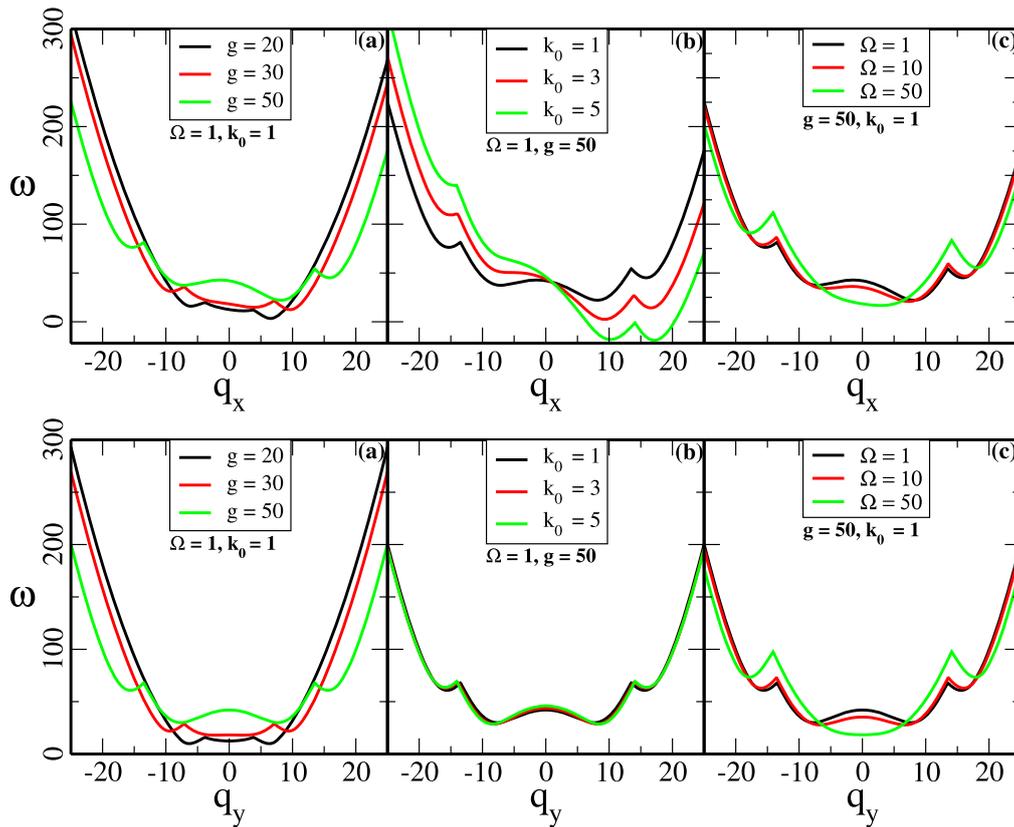


Figure 7. Energy spectra of two components of BEC in the presence of Raman SOC. (a) variation with respect to the two-particle interaction strength. (b) Variation with respect to strength of SOC. (c) Variation with respect to Rabi frequency.

excitation becomes zero at a particular value of k_0 ($k_0 \simeq 4.5$), and it becomes negative if we further increase the value of k_0 , so the BEC collapses for large SOC. The long wavelength excitation and position of roton minima do not depend on the variation of k_0 , but the roton energy depends strongly on k_0 .

Variation of excitation with the Rabi frequency Ω for fixed g ($g = 50$) and SOC ($k_0 = 1$) is shown in the right part of figure 7. Long wavelength mode of the excitation changes abruptly with the Rabi frequency. For low value of Ω , we have maximum of spectra at $q \rightarrow 0$. If we increase the value of Ω , the magnitude of curvature reduces and becomes zero at $\Omega = 40$, after which we have positive curvature and minimum for higher Rabi frequency value.

3. Conclusion

We have studied here the bulk properties corresponding to the interiors of large droplets (in the form of film) for two different spin-orbit coupling systems. For simplicity, we take the mixture of two BECs of atoms of same isotope of an element with different internal degrees of freedom. The droplet forms due to the particle-particle interaction, and SOC makes the droplet asymmetric by deforming it. At small SOC, the qualitative nature of the excitation remains identical to that without SOC. If we increase the strength of the SOC, the droplet becomes unstable (which is obvious from the collective excitation) and evaporates due to the anisotropy. We

have seen that the excitation contains phonon mode for very small interacting systems, and otherwise it contains roton minima, whereas for gas phase we have both phonon and roton modes [31]. Some spectra contain a single roton minimum and some contain double-roton minima depending upon the strength of the interaction. In the presence of Rashba-Dresselhaus SOC, we lose the phonon mode of excitation even at small interactions. Roton energy strongly depends on k_0 , the strength of Raman SOC, whereas the long-wavelength mode of excitation strongly depends on the Rabi frequency. We have very slow qualitative change in the nature of excitations inside the droplet due to the addition of SOC. We hope there will be abrupt qualitative changes of surface modes of excitation due to the change of shape of the droplet in presence of SOC; this remains for us to study.

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