

Addendum: GravitoMagnetic Field in Tensor-Vector-Scalar Theory

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Abstract. We observe that requiring consistency between the near horizon geometry of a black hole in TeVeS and that of the Einstein theory leads to a relation between the coupling constants of the TeVeS theory. We notice that the consistency of the TeVeS gravitomagnetic field and that measured by the Gravity Probe B gives another relations between the parameters. We report that these relations uniquely identify the value of the scalar and vector coupling constants of the theory.

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The Event Horizon Telescope [1] has recently announced that the radius of the event horizon of the M87 black hole is compatible with that of the Schwarzschild metric. This suggests that in any theory of gravity, the near horizon geometry of a stationary black hole of a given mass must coincide to that of the Schwarzschild metric with the same mass. Einstein-Hilbert theory meets this criterion since the Schwarzschild metric is its exact solution. It, however, provides a non-trivial constraint on the parameters of a general modified theory of gravity. J. W. Moffat and V. T. Toth have shown that the consistency of the near horizon geometry of a black hole in MOG and the Schwarzschild metric specifies the value of the α parameter of the MOG theory [2]. We would like to consider the TeVeS theory [3] and study the compatibility of its near horizon geometry with the Schwarzschild metric.

We observe that the non-rotating static solution of the TeVeS in its strong gravity regime, for the branch of the solution that leads to the observed values of the β and γ PPN coefficients, can be made identical to that of the Einstein gravity, as reported in the section V of [4]. We first should impose regularity on the physical event horizon:

$$\frac{r_c}{r_g} = \frac{1}{4} + \frac{\kappa}{8\pi} \frac{G m_s}{r_g}, \quad (\text{eq. (78) of [4]}) \quad (1)$$



where r_c is the Schwarzschild radius

$$16 \left(\frac{r_c}{r_g} \right)^2 = 1 + \frac{\kappa}{\pi} \left(\frac{Gm_s}{r_g} \right)^2 - \frac{K}{2}, \quad (\text{eq. (71) of [4]}), \quad (2)$$

where Gm_s encodes the fall off of the scalar field. r_g encodes the mass of the black hole in the TeVeS theory. We next require it to be the same as the Schwarzschild radius:

$$r_g = r_c. \quad (3)$$

In so doing the near horizon geometry of a black hole for a given mass in TeVeS theory coincides to that of the Einstein theory. Solving (1), (2) and (3) gives:

$$K = -30 + \frac{72\pi}{\kappa}. \quad (4)$$

We notice that the consistency between the TeVeS gravitomagnetic field and that predicted by the Einstein-Hilbert theory and measured by the Gravity Probe B [5] demands [6]:

$$K = \frac{\kappa}{2\pi}. \quad (5)$$

Eq. (5) and (4) then enable us to find the values of the coupling constants of the theory:

$$K = 3 \left(\pm\sqrt{29} - 5 \right), \quad (6)$$

$$\kappa = 6\pi \left(\pm\sqrt{29} - 5 \right), \quad (7)$$

inserting which into (1) gives

$$\frac{Gm_s}{r_g} = \frac{1}{4} \left(5 \pm \sqrt{29} \right). \quad (8)$$

One may wish not to consider a negative value for m_s and this uniquely identifies the coupling constants. This is the first time that the coupling constants of the TeVeS theory are fixed. We observe that the values of K, κ are large while the literature most often assumed that they are small. We further notice they satisfy

$$K < \frac{2\kappa}{\pi} \left(\frac{Gm_s}{r_g} \right)^2 \quad (\text{eq. (73) of [4]}), \quad (9)$$

so the total energy density of vacuum contributed by the scalar and vector fields remains positive in the whole of space-time. This is very nontrivial and points that TeVeS can be both consistent with the data and be free of quantum instability.

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