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# Erratum: Signatures of extra dimensions in gravitational waves

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A small computational mistake has been noticed in the derivation of equation (2.22a); once corrected, this equation becomes

$$\begin{aligned}\square_D^{(0)} h_{MN=\mu\nu} = {}& e^{-2A}\tilde{\square}_4 h_{\mu\nu} + \Delta_{\mathcal{M}} h_{\mu\nu} - h_{\mu\nu}\Delta_{\mathcal{M}} \ln e^{2A} - \frac{3}{2} e^{-4A} h_{\mu\nu} g^{mn}\partial_m e^{2A}\partial_n e^{2A} \\ & + 2e^{-2A}\tilde{\nabla}_{(\mu} h_{\nu)m} g^{mn}\partial_n e^{2A} + \frac{1}{2} e^{-2A}\tilde{g}_{\mu\nu} h_{mn} g^{mr} g^{np}\partial_r e^{2A}\partial_p e^{2A}\,,\end{aligned} \tag{1}$$

and consequently, equations (2.24a) and (A.3a) both get corrected to

$$\begin{aligned}{}_{\mu\nu}:\ & e^{-2A}\tilde{\square}_4 h_{\mu\nu} + \Delta_{\mathcal{M}} h_{\mu\nu} - h_{\mu\nu}\Delta_{\mathcal{M}} \ln e^{2A} \\ & - 2\tilde{\mathcal{R}}^{\pi}{}_{\mu\nu\sigma} g^{\sigma\rho} h_{\rho\pi} - \frac{1}{2} e^{-2A} g^{pq}\partial_p e^{2A}\partial_q e^{2A}\left(\tilde{g}_{\nu\mu} h_4 + 2h_{\nu\mu} e^{-2A}\right) \\ & + 2e^{-2A}\tilde{\nabla}_{(\mu} h_{\nu)m} g^{mn}\partial_n e^{2A} - \tilde{g}_{\mu\nu} h_{mn} g^{mr} g^{np}\left(\nabla_r\partial_p e^{2A} - e^{-2A}\partial_r e^{2A}\partial_p e^{2A}\right) = 0\,.\end{aligned} \tag{2}$$

This has *no impact on the main analysis and results of the paper*. However, as we now detail, consequences for Appendix A are important, since the correction simplifies equations studied there, starting with equation (A.7).

That equation gets corrected to

$$e^{-2A}\tilde{\square}_4 h_{\mu\nu} + \Delta_{\mathcal{M}} h_{\mu\nu} - h_{\mu\nu}\left(\Delta_{\mathcal{M}} \ln e^{2A} + e^{-4A}(\partial e^{2A})^2 + \frac{2}{3} e^{-2A}\Lambda_4\right) = 0\,, \tag{3}$$

leading to the following correction of (A.8):

$$\tilde{\square}_4 \tilde{h}_{\mu\nu} + e^{2A}\Delta_{\mathcal{M}}\tilde{h}_{\mu\nu} + 2g^{pq}\partial_p\tilde{h}_{\mu\nu}\partial_q e^{2A} - \frac{2}{3}\Lambda_4\tilde{h}_{\mu\nu} = 0\,. \tag{4}$$



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This equation now reproduces precisely an equation obtained in [1], where the RS1 model [2] was taken as the background. This matching is an important computational cross-check, and indicates possible generalisations beyond the RS1 model. Further rewritings can be obtained by considering two different rescalings of the internal metric. The first one, $g_{mn} = e^{-2A}\tilde{g}_{mn}$, was considered in the paper. It required working-out the relation between two Laplacians, given above (A.9). We also corrected a minor mistake in this relation, which is now given by

$$\Delta_{\mathcal{M}}\varphi = e^{2A}\tilde{\Delta}_{\mathcal{M}}\varphi + \frac{1}{2}(6-D)\tilde{g}^{pq}\partial_p e^{2A}\partial_q\varphi\,. \tag{5}$$

Then, (4) becomes

$$\tilde{\Box}_4\tilde{h}_{\mu\nu} + e^{4A}\tilde{\Delta}_{\mathcal{M}}\tilde{h}_{\mu\nu} + \frac{10-D}{2}e^{2A}\tilde{g}^{pq}\partial_p\tilde{h}_{\mu\nu}\partial_q e^{2A} - \frac{2}{3}\Lambda_4\tilde{h}_{\mu\nu} = 0\,. \tag{6}$$

The second rescaling worth being considered is to have the same warp factor along all dimensions, leading us to introduce $g_{mn} = e^{2A}\bar{g}_{mn}$. Using the above Laplacian formula reversewise, one rewrites (4) to

$$\tilde{\Box}_4\tilde{h}_{\mu\nu} + \bar{\Delta}_{\mathcal{M}}\tilde{h}_{\mu\nu} + \frac{D-2}{2}e^{-2A}\bar{g}^{pq}\partial_p\tilde{h}_{\mu\nu}\partial_q e^{2A} - \frac{2}{3}\Lambda_4\tilde{h}_{\mu\nu} = 0\,. \tag{7}$$

Interestingly, this reproduces precisely the equation obtained on general grounds in [3] with $h_{mn} = h_{\mu n} = 0$ in 4d Transverse-Traceless gauge. That work also generalises [4] where the same analysis was carried-out with only one extra dimension. The comparison to these references, not known to us while initially completing the paper, is again an important cross-check, and indicates possible generalisations beyond these works.

The next step of the analysis, and the rest of Appendix A, was devoted to the decomposition of $\tilde{h}_{\mu\nu}(x,y)$ into a tower of Kaluza–Klein modes, once one restricts to $\mathcal{M}$ being compact. With these corrected equations, this is done more easily. One should first build a differential operator acting on $\tilde{h}_{\mu\nu}(x,y)$ by combining the two terms with internal first and second derivatives in (4), (6) or (7). The appropriate basis is then that of eigenmodes of this operator; we refer e.g. to [3] for more detail. While this method was successfully used in phenomenological papers working with the RS1 background, it is in general more difficult to apply, as one needs some knowledge of the warp factor. Typically, this warp factor is not explicitly known in string compactifications. We still hope to come back to this case of a non-constant warp factor, which could provide interesting new effects.

Finally, let us emphasize that the correction of the general wave equations, namely of (2.24a) to (2), does not alter the following important result discussed below (2.25): from these equations, one can see that $D$-dimensional General Relativity does not allow for an amplitude damping of the four-dimensional gravitational wave, even with extra dimensions, contrary to the “extra-dimensional leakage” observed in [5] that we attribute to modified gravity. The results of the paper, summarized in Section 4, are not altered either.

## References


[1] H. Davoudiasl, J.L. Hewett and T.G. Rizzo, *Phenomenology of the Randall-Sundrum Gauge Hierarchy Model*, *Phys. Rev. Lett.* **84** (2000) 2080 [hep-ph/9909255] [INSPIRE].

[2] L. Randall and R. Sundrum, *A large mass hierarchy from a small extra dimension*, *Phys. Rev. Lett.* **83** (1999) 3370 [hep-ph/9905221] [INSPIRE].

[3] C. Bachas and J. Estes, *Spin-2 spectrum of defect theories*, *JHEP* **06** (2011) 005 [arXiv:1103.2800] [INSPIRE].

[4] C. Csáki, J. Erlich, T.J. Hollowood and Y. Shirman, *Universal aspects of gravity localized on thick branes*, *Nucl. Phys.* **B 581** (2000) 309 [hep-th/0001033] [INSPIRE].

[5] C. Deffayet and K. Menou, *Probing Gravity with Spacetime Sirens*, *Astrophys. J.* **668** (2007) L143 [arXiv:0709.0003] [INSPIRE].

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