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K-essence and kinetic gravity braiding models in two-field measure theory

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Abstract. We show that, in the context of the two-field measure theory, any k-essence model leads to the existence of a fluid made of non-relativistic matter and cosmological constant that would explain the dark matter and dark energy problem at the same time. On the other hand, kinetic gravity braiding models can lead to different behaviors, such as phantom dark energy, stiff matter, and a cosmological constant. For stiff matter, there even exists the case where the scalar field does not have any effect in the dynamics of the Universe.

Keywords: dark energy theory, modified gravity, dark matter theory

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1 Introduction

The dark matter and dark energy problem are fundamental issues to be solved in this century. The discovery of the accelerating expansion of the Universe [1, 2] is one of the most intriguing and puzzling questions in cosmology today [3–8]. Many proposals have been discussed in the literature, ranging from physics beyond the Standard Model of elementary particles, such as supersymmetry, modifications of the gravitational theory (MOND), the cosmological constant, quintessence field [9–11], brane cosmology scenario [12, 13] and k-essence models [14–21]. Among the former proposals the cosmological constant is the simplest choice although it needs a finely tuned value. There are different approaches that try to model the late-time accelerated expansion of the Universe including modified gravity at large distances via extra dimensions or including a dark energy component that does not fulfill the strong energy condition [22] or new proposals, as the case of k-essence theories.

In these kinds of models the scalar field plays an important role in order to describe the dark energy problem. The scalar fields can reproduce the dynamical effects of the cosmological constant at late times and could have a very interesting and adequate behaviour at early times. Scalar field models with non-canonical kinetic terms have been proposed as an alternative to describe the dark energy component of the Universe. The k-essence type Lagrangians were introduced in several contexts, for example, as a possible model for inflation [14, 23]. Later on k-essence models were analyzed as an alternative to describe the characteristics of dark energy and as a possible mechanism of unifying dark energy and dark matter [24]. Purely kinetic k-essence models [25–28] are, in some sense, as simple as quintessence because they depend only on a single function F by means of the expression for the density Lagrangian $\mathcal{L} = F(X)$, where the kinetic term is $X = -g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, $g_{\mu\nu}$ is the metric of the spacetime, and ϕ is the scalar field. Among these fields generalized galileons [29–32] have been studied in detail recently. They have the capacity to induce an accelerated expansion of the Universe when no matter interaction is present an additionally at short distances they possess the Vainshtein screening mechanism. These scalar field models are second-order derivative theories but with the appreciated property that their equations of motions are second order. A nice example of this kind of models is the so-called kinetic gravity braiding models that are constructed by means of a Lagrangian that contain a D'Alambertian operator and an arbitrary function of a non-canonical kinetic terms [33]. The kinetic gravity braiding models have the characteristic that the energy density contains the Hubble parameter and the pressure depends on the second derivatives of the scalar fields which give rise to very interesting effects [34–37].

Recently, the Two Field Measure Theory (TMT) has also been proposed as a possible tool in the search for an explanation to the observations that leads to the dark energy and dark matter hypothesis [38, 39]. In this theory, a new measure is defined to build an action that has extra contributions, additional to the usual measure of general relativity ($\sqrt{-g}$). Different actions have been studied in this context [40–42].

In this work, we study both k-essence and kinetic gravity braiding models that could lead to rich and interesting results in the framework of the TMT. We will show that any k-essence model, in the TMT, will lead to dark matter and dark energy in a natural way. On the other hand, the kinetic gravity braiding models present a richer spectrum of possibilities. Even in the simplest cases for these models can get either a cosmological constant or stiff matter [43, 44].

The paper is organized as follows. In section 2 the TMT is presented where its motivation is enhanced. The next section contains the equations of motion for a general k-essence model in the context of the TMT. Besides it is shown that any arbitrary k-essence model can describe a dark matter and dark energy behavior in an unified way. In section 4, we apply the TMT to kinetic gravity braiding models, starting with a discussion about the imperfect fluid form of these models and then, we investigate what kind of fluid appears when a TMT contribution is added. Finally we give our conclusions.

2 The two field measure theory

In order to discuss the k-essence theories in the context of the TMT, we will describe in this section the main characteristic of the TMT, following closely the discussion given in [45].

We will work in a Friedmann-Robertson-Walker metric

$$ds^2 = g_{tt}dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)\right], \tag{2.1}$$

where $k = -1, 0, 1$, with the metric signature $(+,-,-,-)$[1] and we will set $g_{tt} = 1$ in the following calculations.

First it is important to consider an action that contains two different terms: the usual metric, $g_{\mu\nu}$, and an additional term with a new measure, Φ, in the same space-time:

$$S = \int \mathcal{L}_1\sqrt{-g}d^4x + \int \mathcal{L}_2\Phi d^4x. \tag{2.2}$$

The new measure can be constructed, for instance, with four scalar fields, φ^a $(a = 1, 2, 3, 4)$,

$$\Phi = \epsilon^{\mu\nu\alpha\beta}\epsilon_{ijkl}\partial_\mu\varphi^i\partial_\nu\varphi^j\partial_\alpha\varphi^k\partial_\beta\varphi^l. \tag{2.3}$$

[1]The signature in [45] is $(-,+,+,+)$.

It is assumed that the Lagrangians in these equations are independent from the fields φ^a. It is also assumed that all fields are independent. Therefore, any relation among the fields will be a consequence of the equation of motion.

In particular, it can be shown that the variation of the action will lead to the equation of motion [46]

$$A_i^\mu \partial_\mu \mathcal{L}_2 = 0, \tag{2.4}$$

with

$$A_i^\mu = \epsilon^{\mu\nu\alpha\beta} \epsilon_{ijkl} \partial_\nu \varphi^j \partial_\alpha \varphi^k \partial_\beta \varphi^l, \tag{2.5}$$

and, since we have [46]

$$\det\left(A_i^\mu\right) = \frac{4^{-4}}{4!} \Phi^3 \tag{2.6}$$

an important restriction can be obtained for $\Phi \neq 0$:

$$\mathcal{L}_2 = \text{constant}. \tag{2.7}$$

In this work, we will follow closely the work of Guendelman, et al. [38], where they consider an action of the form

$$S = \int \frac{R}{16\pi G} \sqrt{-g} d^4x + \int \frac{K\left(\phi, \nabla_\mu \phi\right)}{\sqrt{-g}} \Phi d^4x + \int K\left(\phi, \nabla_\mu \phi\right) d^4x,$$

where

$$K\left(\phi, \nabla_\mu \phi\right) = V\left(\phi\right) \sqrt{-g} \sqrt{1 + g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi} \tag{2.8}$$

is a field of the type of DBI theories. In this case, the restriction given by eq. (2.7) turns out to be

$$\frac{K}{\sqrt{-g}} = V\left(\phi\right) \sqrt{1 + \dot{\phi}^2} = M, \tag{2.9}$$

with M a constant and ϕ is a function of time, t. With the help of the energy-momentum tensor

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_m}{\delta g^{\mu\nu}}, \tag{2.10}$$

it is possible to obtain the energy density and pressure of the fluid which are given by

$$\begin{aligned} \varepsilon &= \left(1 + \frac{\Phi}{\sqrt{-g}}\right) \frac{V\left(\phi\right) \dot{\phi}}{\sqrt{1 + \dot{\phi}^2}} - V\left(\phi\right) \sqrt{1 + \dot{\phi}^2} \\ \mathcal{P} &= -V\left(\phi\right) \sqrt{1 + \dot{\phi}^2}, \end{aligned} \tag{2.11}$$

where we have used the definitions for energy density and pressure [34] (given in section 4):

$$\varepsilon = T_{\mu\nu} u^\mu u^\nu, \qquad \mathcal{P} = -\frac{1}{3} T^{\mu\nu} \perp_{\mu\nu} . \tag{2.12}$$

By solving the Lagrange equation for the Lagrangian density in eq. (2.8)

$$-\frac{\partial}{\partial t}\left(\frac{V\left(\phi\right) \Phi \dot{\phi}}{\sqrt{1 + \dot{\phi}^2}} + \frac{V\left(\phi\right) \sqrt{-g} \dot{\phi}}{\sqrt{1 + \dot{\phi}^2}}\right) = \frac{\partial V}{\partial \phi}\left(\Phi \sqrt{1 + \dot{\phi}^2} + \sqrt{-g} \sqrt{1 + \dot{\phi}^2}\right), \tag{2.13}$$

it is possible to find an expression for Φ in terms of ϕ [38]:

$$\Phi = \frac{C\,(r,\theta)}{V^2 - M^2} - \sqrt{-g}, \tag{2.14}$$

where $C(r,\theta)$ is an integration constant given by

$$C\,(r,\theta) = \frac{Dr^2 \sin\theta}{\sqrt{1-kr^2}}, \qquad D = \text{constant}. \tag{2.15}$$

Substituting Φ in the energy density and pressure gives

$$\varepsilon_T = M + \frac{D}{Ma^3}, \tag{2.16}$$

$$\mathcal{P}_T = -M. \tag{2.17}$$

The parameter density is

$$\omega = \frac{\mathcal{P}_T}{\varepsilon_T} = \frac{-1}{1 + \frac{D}{M^2 a^3}}. \tag{2.18}$$

The expression for ε_T has two contributions: a constant term, M, and a dust-like term a^{-3}. It is shown that the inhomogeneous perturbations correspond to a particle fluid. This is achieved by computing the covariant derivative of the energy momentum tensor. The four velocity is defined as

$$u_\mu = \frac{\nabla_\mu \phi}{\sqrt{\nabla_\alpha \phi \nabla^\alpha \phi}}. \tag{2.19}$$

The energy momentum tensor has the perfect fluid form

$$T_{\mu\nu} = \varepsilon_d u_\mu u_\nu + M g_{\mu\nu}, \tag{2.20}$$

where

$$\varepsilon_d = \left(1 + \frac{\Phi}{\sqrt{-g}}\right) \frac{(\partial_\alpha \phi \partial^\alpha \phi)\, V\,(\phi)}{\sqrt{1 + g^{\mu\nu}\partial_\mu \phi \partial_\nu \phi}}. \tag{2.21}$$

The covariant derivative of the second term in eq. (2.20) is zero due to the presence of the metric tensor. The covariant derivative of the first term,

$$\nabla_\mu\,(\varepsilon_d u^\mu u^\nu) = \nabla_\mu\,(\varepsilon_d u^\mu)\, u^\nu + \varepsilon_d u^\mu \nabla_\mu u^\nu, \tag{2.22}$$

is also zero due to the orthogonality property of the vectors u^ν and $u^\mu \nabla_\mu u^\nu$ and the conservation of the current $J^\mu = \varepsilon_d u^\mu$.

3 k-essence models and the two field measure theory

Once we have introduced the TMT and the restriction on the new Lagrangian, L_2, we turn our attention to the specific case of k-essence models where we have the function

$$K\,(\phi, \partial_\mu \phi) = G_2\,(\phi, X)\,\sqrt{-g}, \tag{3.1}$$

that represents the more general k-essence models [15], with $X = \nabla_\alpha \phi \nabla^\alpha \phi / 2$ being the kinetic term. In this case the restriction on L_2 will be given as

$$G_2(\phi, X) = M_3, \tag{3.2}$$

with M_3 an arbitrary constant. We can therefore consider the following action in the TMT:

$$S_m = \int \left[G_2(\phi, X)\, \Phi + G_2(\phi, X) \sqrt{-g} \right] d^4x. \tag{3.3}$$

If we vary this action with respect to the metric, $g^{\mu\nu}$, and employ the usual expression for the energy momentum tensor given in eq. (2.10), we obtain

$$T_{\mu\nu} = \left(1 + \frac{\Phi}{\sqrt{-g}}\right) G_{2,X}\, \nabla_\mu \phi \nabla_\nu \phi - g_{\mu\nu} G_2. \tag{3.4}$$

Through this paper we are using the abbreviated notation, $G,_X = dG/dX$ or $G,_X = \partial G/\partial X$.

From this tensor, we can write down the total energy and pressure, that includes the contribution from TMT, as

$$\varepsilon_T = \left(1 + \frac{\Phi}{\sqrt{-g}}\right) 2X G_{2,X} - G_2, \tag{3.5}$$

$$\mathcal{P}_T = -M_3. \tag{3.6}$$

We can find an expression for Φ by using the Lagrange equations. For this purpose we use the following relations

$$\frac{\partial \mathcal{L}_m}{\partial \phi} = G_{2,\phi}\, \psi, \tag{3.7}$$

$$\frac{\partial \mathcal{L}_m}{\partial \dot{\phi}} = -G_{2,X}\, \dot{\phi} \psi, \tag{3.8}$$

where $\psi = \Phi + \sqrt{-g}$ and $\mathcal{L}_m$ is given by

$$\mathcal{L}_m = G_2(\phi, X)\, \Phi + G_2(\phi, X) \sqrt{-g}. \tag{3.9}$$

The Lagrange equations will be then expressed as

$$\sqrt{2X} \frac{d}{d\phi} \left(\psi G_{2,X} \sqrt{2X} \right) = G_{2,\phi}\, \psi. \tag{3.10}$$

If we use now the restriction (3.2) on G_2, we can find a relation between $G_{2,X}$ and $G_{2,\phi}$:

$$dG_2 = G_{2,\phi}\, d\phi + G_{2,X}\, dX, \tag{3.11}$$

$$G_{2,\phi} = -G_{2,X} \frac{dX}{d\phi}. \tag{3.12}$$

If we calculate the differential in the left hand side of equation (3.10)

$$d\left(\psi G_{2,X} \sqrt{2X}\right) = G_{2,X} \sqrt{2X} d\psi + \psi \left(G_{2,XX} \sqrt{2X} + \frac{G_{2,X}}{\sqrt{2X}} \right) dX, \tag{3.13}$$

we can substitute this expression and eq. (3.12) in the eq. (3.10) and find that

$$\frac{d\psi}{\psi} = \left(-\frac{1}{X} - \frac{G_{2,XX}}{G_{2,X}} \right) dX, \tag{3.14}$$

which can be solve to give

$$\psi = \frac{C_3\,(r,\theta)}{XG_{2,X}}. \tag{3.15}$$

From this last expression for ψ we obtain Φ

$$\Phi = \frac{C_3\,(r,\theta)}{XG_{2,X}} - \sqrt{-g}. \tag{3.16}$$

Finally, by adopting a particular expression for the constant C_3 we cancel out the dependence on the energy density in the spatial variables,

$$C_3\,(r,\theta) = \frac{D_3 r^2 \sin\theta}{\sqrt{1-kr^2}}, \tag{3.17}$$

where D_3 is a constant. After this last step, the total energy density and the total pressure will be written as

$$\varepsilon_T = \frac{D_3}{a^3} - M_3, \tag{3.18}$$

$$\mathcal{P}_T = M_3. \tag{3.19}$$

As in the previous section, for this case the inhomogeneous perturbations also correspond to dust particles. The energy momentum tensor, eq. (3.4), can be expressed as

$$T_{\mu\nu} = \varepsilon_d u_\mu u_\nu - M g_{\mu\nu}, \tag{3.20}$$

where u_μ is given by eq. (2.19) and ε_d is now

$$\varepsilon_d = \left(1 + \frac{\Phi}{\sqrt{-g}}\right) \left(\nabla_\alpha \phi \nabla^\alpha \phi\right) G_{2,X}. \tag{3.21}$$

The expression in eq. (3.20) is exactly the same as eq. (2.20). The only change is in the expression for ε_d. Therefore, the covariant derivative of the tensor (3.4) is zero, as it is expected.

We can see that we have obtained a general result for k-essence theories in the TMT. In this formulation, we will always obtain a dependence a^{-3} for the energy density and a cosmological constant therefore it is possible to model dark energy and dark matter. This is a general result for any k-essence action and, therefore, it applies for particular k-essence action as, for example, linear actions

$$K\,(\phi, \partial_\mu \phi) = V\,(\phi)\,(1 + AX)\,, \tag{3.22}$$

where A is a constant and $X = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$; or logarithmic actions

$$K\,(\phi, \partial\phi) = V\,(\phi)\,\frac{1}{b} \ln\left(\frac{c^\star X^\beta}{1 - \alpha^\star X^\beta}\right), \tag{3.23}$$

where $\alpha^\star$, $c^\star$, b and β are constants.

4 Kinetic gravity braiding models and the TMT

As we have seen, the TMT ensures that a general k-essence action will be able to model both dark matter and dark energy. In this section, we pay attention to a different case, where we consider the kinetic gravity braiding models, and we will show that its behaviour in the TMT can be more complex.

It is well known that the most general scalar field theories in four dimensions that have second order (or less) equations of motion are described by the Lagrangian [47]

$$\mathcal{L} = \sum_{i=2}^{5} \mathcal{L}_i, \tag{4.1}$$

where

$$\mathcal{L}_2 = K(\phi, X), \tag{4.2}$$

$$\mathcal{L}_3 = -G_3(\phi, X)\Box\phi, \tag{4.3}$$

$$\mathcal{L}_4 = G_4(\phi, X) R + G_{4,X}\left[(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi)\right], \tag{4.4}$$

$$\mathcal{L}_5 = G_5(\phi, X) G_{\mu\nu}(\nabla^\mu\nabla^\nu\phi) - \frac{1}{6}G_{5,X}\Big[(\Box\phi)^3 - 3(\Box\phi) \tag{4.5}$$
$$(\nabla_\mu\nabla_\nu\phi)(\nabla^\mu\nabla^\nu\phi) + 2(\nabla^\mu\nabla_\alpha\phi)(\nabla^\alpha\nabla_\beta\phi)\left(\nabla^\beta\nabla_\mu\phi\right)\Big].$$

Here $G_{i,X} \equiv \partial G_i/\partial X$, R is the Ricci scalar, and $G_{\mu\nu}$ is the Einstein tensor.

We will consider now the kinetic gravity braiding action $\mathcal{L}_3$

$$K\left(\phi, X, \ddot{\phi}\right) = -G_3(\phi, X)\Box\phi\sqrt{-g}. \tag{4.6}$$

In order to have a better understanding of the kinetic gravity braiding case, we will discuss first the hydrodynamic fluid picture and show how the kinetic gravity braiding results can be generalized for the TMT case.

4.1 The imperfect fluid in kinetic gravity braiding models and the TMT theory

We can start our discussion by considering first the kinetic gravity braiding action without TMT:

$$S_\phi = \int d^4x\sqrt{-g} \quad \Box\,\phi G(X, \phi), \tag{4.7}$$

where we can write X and $\Box\phi$ in covariant form, that is, $X = \nabla^\mu\phi\nabla_\mu\phi/2$, $\Box\,\phi = \nabla^\mu\nabla_\mu\phi$ and the function G is arbitrary.

From the computations in ref. [33], the equation of motion for the field ϕ is

$$\nabla_\mu J^\mu = \mathcal{P}_\phi, \tag{4.8}$$

where

$$J_\mu = (\mathcal{L}_X - 2G_\phi)\nabla_\mu\phi - G_X\nabla_\mu X, \tag{4.9}$$

$$\mathcal{P}_\phi = -\nabla^\lambda\phi\nabla_\lambda G_\phi. \tag{4.10}$$

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The corresponding energy-momentum tensor, $T_{\mu\nu}$, is given by

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_\phi}{\delta g^{\mu\nu}} = \mathcal{L}_X \nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}\mathcal{P} - \nabla_\mu G\nabla_\nu\phi - \nabla_\nu G\nabla_\mu\phi. \tag{4.11}$$

In ref. [34], that we follow closely in this subsection, the authors rewrite these expressions using the hydrodynamic fluid picture. Let's define some useful quantities used to describe relativistic fluids. We set a local rest frame defining the four-velocity, u_μ,

$$u_\mu \equiv \frac{\nabla_\mu\phi}{\sqrt{2x}}, \tag{4.12}$$

which is normalized, $u_\mu u^\mu = 1$. The four-acceleration is

$$a_\mu \equiv \dot{u_\mu} \equiv u^\nu\nabla_\nu u_\mu, \tag{4.13}$$

where the acceleration is orthogonal to the velocity, $u_\mu a^\mu = 0$, and the dot represents the material derivative along u^λ,

$$\dot{(\,)} = \frac{d}{d\tau}(\,) = u^\lambda\nabla_\lambda(\,)\,. \tag{4.14}$$

The expansion, θ, and the diffusivity, κ, are defined

$$\theta = \nabla_\mu u^\mu, \quad \kappa = 2XG_X. \tag{4.15}$$

Under these scheme, the current in eq. (4.9) is written as

$$J_\mu = \left(-2\dot{\phi}G_\phi + \kappa\theta\right)u_\mu - \kappa a_\mu. \tag{4.16}$$

When dissipation is modeled, the usual condition that diffusion k^μ must satisfy is $u_\mu\kappa^\mu = 0$, that is, the diffusion is only spatial [48]. The last term in eq. (4.16) corresponds to the diffusion, say $\kappa_\mu = -\kappa a_\mu$ (note that $u_\mu\kappa^\mu = 0$ is satisfied through $u_\mu a^\mu = 0$). The equation of motion, eq. (4.8), takes a nice form if one use the density of charge, n, defined as

$$n \equiv u^\mu J_\mu = -2\dot{\phi}G_\phi + \kappa\theta. \tag{4.17}$$

In the case, the aforementioned equation for ϕ, eq. (4.8), becomes

$$\dot{n} + \theta n - \nabla_\mu\left(\kappa a^\mu\right) = \mathcal{P}_\phi. \tag{4.18}$$

Expressing the energy-momentum tensor requires of the energy density, ε, and the total isotropic pressure, $\mathcal{P}$,

$$\varepsilon \equiv T_{\mu\nu}u^\mu u^\nu = -2XG_\phi + \theta m\kappa, \tag{4.19}$$

$$\mathcal{P} \equiv -\frac{1}{3}T^{\mu\nu}\perp_{\mu\nu} = -2XG_\phi - \kappa\dot{m}. \tag{4.20}$$

The energy-momentum is

$$T_{\mu\nu} = \varepsilon u_\mu u_\nu - \perp_{\mu\nu}\mathcal{P} + u_\mu q_\nu + u_\nu q_\mu, \tag{4.21}$$

where $m = \sqrt{2X} = \dot{\phi}$ is the chemical potential and $q_\mu = -m\kappa a_\mu$ is the heat flux that, again, must be purely spatial ($u_\mu q^\mu = 0$). Finally, the conservation of $T_{\mu\nu}$ leads to

$$u_\nu \nabla_\mu T^{\mu\nu} = \dot{\varepsilon} + \theta\left(\varepsilon + \mathcal{P}\right) - \nabla_\lambda\left(m\kappa a^\lambda\right) + m\kappa a_\lambda a^\lambda = 0. \tag{4.22}$$

Up to this point, the action is such that the only contribution comes from the usual measure $\sqrt{-g}\, d^4x$. Now we want to include the contribution from TMT:

$$S = \int d^4x\sqrt{-g}\,\Box\,\phi G\left(X,\phi\right) + \int d^4x\;\Phi\,\Box\,\phi G\left(X,\phi\right). \tag{4.23}$$

Notice the replacement of $\sqrt{-g}$ by Φ in the second integral of this last expression. The new equation of motion becomes:

$$\begin{aligned}
&\nabla_\mu\left[2G_\phi\nabla^\mu\phi - \Box\phi G_X\nabla^\mu\phi + G_X\nabla^\mu X\right] \qquad\qquad (4.24)\\
&+\frac{\Phi}{\sqrt{-g}}\nabla_\mu\left[2G_\phi\nabla^\mu\phi - \Box\phi G_X\nabla^\mu\phi + G_X\nabla^\mu X\right]\\
&\qquad+\frac{1}{\sqrt{-g}}\nabla_\mu\left(G\nabla^\mu\Phi\right) = \nabla^\lambda\phi\nabla_\lambda G_\phi + \frac{\Phi}{\sqrt{-g}}\nabla^\lambda\phi\nabla_\lambda G_\phi\\
&\qquad\qquad+\Box\phi G_X\nabla^\lambda\phi\frac{\nabla_\lambda\Phi}{\sqrt{-g}} - \frac{\nabla^\lambda\Phi}{\sqrt{-g}}\nabla_\lambda G.
\end{aligned}$$

Making use of the expressions in eqs. (4.9) and (4.10), this equation can be reduced to

$$\nabla_\mu\left[\left(\sqrt{-g}+\Phi\right)J^\mu\right] + \nabla_\mu\left(G\nabla^\mu\Phi\right) = \mathcal{P}_\phi\left(\sqrt{-g}+\Phi\right) + G_\phi\nabla^\lambda\nabla_\lambda\Phi. \tag{4.25}$$

The energy-momentum tensor can be expressed similar to equation (4.21):

$$\begin{aligned}
T_{\mu\nu} = &\left[\varepsilon + \varepsilon\frac{\Phi}{\sqrt{-g}} + \sqrt{2X}Gu_\alpha\frac{\nabla^\alpha\Phi}{\sqrt{-g}} + G\Box\phi\frac{\Phi}{\sqrt{-g}} - 2\sqrt{2X}Gu^\lambda\frac{\nabla_\lambda\Phi}{\sqrt{-g}}\right]u_\mu u_\nu\\
&-\perp_{\mu\nu}\left[\mathcal{P} + \mathcal{P}\frac{\Phi}{\sqrt{-g}} - \sqrt{2X}Gu_\alpha\frac{\nabla^\alpha\Phi}{\sqrt{-g}} - G\Box\phi\frac{\Phi}{\sqrt{-g}}\right]\\
&+\left[q_\nu + q_\nu\frac{\Phi}{\sqrt{-g}} - \sqrt{2X}G\perp^\lambda_\nu\frac{\nabla_\lambda\Phi}{\sqrt{-g}}\right]u_\mu\\
&+\left[q_\mu + q_\mu\frac{\Phi}{\sqrt{-g}} - \sqrt{2X}G\perp^\lambda_\mu\frac{\nabla_\lambda\Phi}{\sqrt{-g}}\right]u_\nu.
\end{aligned} \tag{4.26}$$

From this tensor, the energy density, ε_T , the pressure, $\mathcal{P}_T$ and the energy flow, Q_ν, are:

$$\varepsilon_T = \varepsilon + \varepsilon\frac{\Phi}{\sqrt{-g}} + \sqrt{2X}Gu_\alpha\frac{\nabla^\alpha\Phi}{\sqrt{-g}} + G\Box\phi\frac{\Phi}{\sqrt{-g}} - 2\sqrt{2X}Gu^\lambda\frac{\nabla_\lambda\Phi}{\sqrt{-g}}, \tag{4.27}$$

$$\mathcal{P}_T = \mathcal{P} + \mathcal{P}\frac{\Phi}{\sqrt{-g}} - \sqrt{2X}Gu_\alpha\frac{\nabla^\alpha\Phi}{\sqrt{-g}} - G\Box\phi\frac{\Phi}{\sqrt{-g}}, \tag{4.28}$$

$$Q_\nu = q_\nu + q_\nu\frac{\Phi}{\sqrt{-g}} - \sqrt{2X}G\perp^\lambda_\nu\frac{\nabla_\lambda\Phi}{\sqrt{-g}}. \tag{4.29}$$

These expressions contain the energy density, ε, and pressure, $\mathcal{P}$, of the kinetic gravity braiding models (Lagrangian 4.7) given by eqs. (4.19), (4.20), respectively. From the definition of

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Q_ν, the condition $u_\nu Q^\nu = 0$ is satisfied. The expression for the conservation of the energy-momentum tensor can be written as eq. (4.22):

$$u_\nu \nabla_\mu T^{\mu\nu} = \dot{\varepsilon_T} + \theta\left(\varepsilon_T + \mathcal{P}_T\right) + \nabla_\mu Q^\mu - Q^\nu a_\nu + u_\nu Q^\mu \nabla_\mu u^\nu = 0. \tag{4.30}$$

5 Some solutions to the equation of motion in cosmology

In order to describe our Universe, we must take into account the cosmological principle. This tells us that the field ϕ is only time-depend. An equivalent form of the equation of motion, eq. (4.25), is obtained by means of Lagrange equations:

$$\frac{d^2}{dt^2}\left[\frac{\partial \mathcal{L}}{\partial \ddot{\phi}}\right] - \frac{d}{dt}\left[\frac{\partial \mathcal{L}}{\partial \dot{\phi}}\right] + \frac{\partial \mathcal{L}}{\partial \phi} = 0. \tag{5.1}$$

From this point of view, the equation for ϕ, not written in a covariant way, is:

$$\begin{aligned} G_\phi\left(\ddot{\phi} + 3H\dot{\phi}\right)\left[\sqrt{-g} + \Phi\right] - \frac{d}{dt}\left[\left(3H\dot{\phi}^2 G_X - 2G_\phi \dot{\phi} + 3HG + G_\phi \dot{\phi}\right)\left[\sqrt{-g} + \Phi\right]\right] \\ + \frac{d}{dt}\left(G\frac{d}{dt}\left[\sqrt{-g} + \Phi\right]\right) = 0. \end{aligned} \tag{5.2}$$

Now that we have the equation of motion for the field ϕ, eq. (4.25), and the expressions for the energy density and pressure, eqs. (4.27), (4.28), respectively, we want to calculate an expression for the new measure field, Φ, so that we can find the behaviour of the fluid represented by the above equations. The approach is to consider special cases for the new measure Φ and the function G in the kinetic gravity braiding action eq. (4.23) and try to solve the differential equation (5.1). We will consider the following cases $G = cte$ and $G = G(X)$. From the definition of four velocity, eq. (2.19), the only non-zero component is:

$$u_0 = \dot{\phi}. \tag{5.3}$$

The expansion coefficient, θ, becomes

$$\theta = \nabla_0 u^0 = \partial_0 u^0 + \Gamma^0_{\mu 0} u^0 = 3H. \tag{5.4}$$

An isotropic and homogeneous Universe is modeled via a perfect fluid, meaning that the energy flow Q_ν vanish for all ν, as can be verified from eq. (4.29). The energy density and pressure, eqs. (4.27), (4.28), take the structure:

$$\varepsilon_T = \varepsilon\left(1 + \frac{\Phi}{\sqrt{-g}}\right) - \frac{\dot{\phi}G}{\sqrt{-g}} a^3 \frac{d}{dt}\left[\frac{\Psi}{a^3}\right] + G\Box\phi\frac{\Phi}{\sqrt{-g}}, \tag{5.5}$$

$$\mathcal{P}_T = \mathcal{P}\left(1 + \frac{\Phi}{\sqrt{-g}}\right) - \frac{\dot{\phi}G}{\sqrt{-g}} a^3 \frac{d}{dt}\left[\frac{\Psi}{a^3}\right] - G\Box\phi\frac{\Phi}{\sqrt{-g}}, \tag{5.6}$$

where we have computed the covariant derivatives using the fact that $\Psi = \sqrt{-g} + \Phi$ is a scalar density of weight one. If we take the equation of motion, eq. (5.2), together with the restriction (2.7) in the form

$$G\Box\phi = B, \tag{5.7}$$

where B is a constant, we can check the validity of the continuity equation,

$$\dot{\varepsilon_T} + 3H\left(\varepsilon_T + \mathcal{P}_T\right) = 0. \tag{5.8}$$

The next step is to find the form of the new measure, Φ, to give the complete description of the fluid. We have to solve the system of two equations, the equation of motion and the restriction, with three unknowns: the new measure contained in Ψ , the scale factor a and the field ϕ. So we need an extra equation. When we just have a k-essence action, there is no appearance of the scale factor, a, or of the Hubble factor, $H = \dot{a}/a$, but in the kinetic gravity braiding action, the presence of H is behind the D'Alembertian operator acting on ϕ. We know that the dynamics of the Universe is governed by Friedmann equations so, the fluid represented by ε_T and $\mathcal{P}_T$ must obey:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\varepsilon_T - \frac{k}{a^2}, \tag{5.9}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\varepsilon_T + \mathcal{P}_T\right). \tag{5.10}$$

Here, G is the Newton's constant and k is the spatial curvature.

5.1 Stiff matter

The equation of motion, eq. (5.2) can be written in the following way

$$\Psi\frac{G_\phi}{G}B + \frac{d}{dt}\left[a^3\frac{d}{dt}\left(\frac{G\Psi}{a^3}\right) - \Psi\frac{G_X}{G}\dot{\phi}B\right] = 0 \tag{5.11}$$

with the help of the restriction (5.7). If we set $B = 0$ the solution of the equation of motion can be obtained from the simplified relation

$$\frac{d}{dt}\left(\frac{G\Psi}{a^3}\right) = \frac{A}{a^3}, \tag{5.12}$$

where A is an integration constant. Considering the following form of the restriction

$$G\Box\phi = G\left(\ddot{\phi} + 3H\dot{\phi}\right) = G\frac{1}{a^3}\frac{d}{dt}\left(a^3\dot{\phi}\right) = 0,$$

then its solution can be cast as $\dot{\phi} = C/a^3$ where C is a constant. Taking into account this solution the expressions for the energy density can also be simplified:

$$\varepsilon_T = \mathcal{P}_T = A\frac{\dot{\phi}}{\sqrt{-g}}, \tag{5.13}$$

which turns out to be the equation of state for stiff matter [43, 44].

For a complete description of the model, we need to solve the equation for the new measure Φ, eq. (5.12). However, if we use Friedmann equation, eq. (5.9), we can obtain the time dependence of the energy density and pressure, eq. (5.13). We consider a flat Universe, $k = 0$,

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{FA_{\mathrm{stiff}}}{a^6}, \tag{5.14}$$

where we set $\sqrt{-g} = \alpha a^3$, $A_{\rm stiff} = AC/\alpha$, $F = 8\pi G/3$ and α is a constant. The solution of the Friedmann equation is

$$\frac{a^3}{3} = \sqrt{-FA_{\rm stiff}}\,(t-t_0) + \frac{a_0^3}{3}. \tag{5.15}$$

The energy density and pressure as a functions of time are:

$$\varepsilon_T = \mathcal{P}_T = -\frac{A_{\rm stiff}}{9}\left[\sqrt{-FA_{\rm stiff}}\,(t-t_0) + \frac{a_0^3}{3}\right]^{-2}. \tag{5.16}$$

It is very interesting to note that when $A = 0$ and, therefore, $\Psi = D\frac{a^3}{G}$ (where D is a constant) there is not any effect of the braided fields in the dynamics of the Universe because $\varepsilon_T = 0$.

5.2 Function G is constant

The equation of motion, eq. (5.2), forms a total derivative and can be simplified:

$$\frac{d}{dt}\left[G\frac{d}{dt}\Psi - 3HG\Psi\right] = 0,$$

$$Ga^3\frac{d}{dt}\frac{\Psi}{a^3} = A. \tag{5.17}$$

where A is a real number. Another equation that must fulfill the fields comes from the restriction of TMT models

$$G\Box\phi = G\left(\ddot{\phi} + 3H\dot{\phi}\right) = G\frac{1}{a^3}\frac{d}{dt}\left(a^3\dot{\phi}\right) = B,$$

where B is a constant. The energy density and pressure can also be simplified:

$$\varepsilon_T = -A\frac{\dot{\phi}}{\sqrt{-g}} + B\frac{\Phi}{\sqrt{-g}}, \tag{5.18}$$

$$\mathcal{P}_T = -A\frac{\dot{\phi}}{\sqrt{-g}} - B\frac{\Phi}{\sqrt{-g}}. \tag{5.19}$$

The terms ε and $\mathcal{P}$ coming from the contribution of the kinetic gravity braiding model are zero, eqs. (4.19), (4.20), because their dependence on the derivatives of G.

Cosmological constant. If we select $A = 0$ and $B \neq 0$, we get

$$\varepsilon_T = -\mathcal{P}_T = B\frac{\Phi}{\sqrt{-g}}. \tag{5.20}$$

From the equation of motion, eq. (5.17), the new measure $\Phi = Da^3$ (D is a constant coming from the integration). In this case, the energy density and pressure are constants $\varepsilon_T = -\mathcal{P}_T = BD/\alpha$. As in the stiff matter case, we can use Friedmann equation for a flat Universe to obtain the expected exponential expansion of Universe

$$\frac{a}{a_0} = \exp\left[\sqrt{FA_{CC}}\,(t-t_0)\right], \tag{5.21}$$

where $A_{CC} = BD/\alpha$.

In the general case when both, A and B, are non zero we could not obtain analytical expressions for $a, \varepsilon_T, \mathcal{P}_T$.

It is interesting to note that a general cosmological constant behavior of the Universe can be obtained if it is chosen the new measure as $\Phi = -\sqrt{-g}$ and therefore $\Psi = 0$. The eq. (5.2) for the field ϕ is trivially satisfied and the expressions for the energy density and the pressure are the same as in eq. (5.20).

5.3 Function G depends only on X

The energy density and pressure are represented by eqs. (5.5), (5.6). The contributions of the kinetic gravity braiding model, eqs. (4.19), (4.20), becomes:

$$\varepsilon = 6H\dot{\phi}XG_X, \tag{5.22}$$
$$\mathcal{P} = -2XG_X\ddot{\phi}. \tag{5.23}$$

The equation of motion, eq. (5.2), is a total derivative:

$$\frac{d}{dt}\left[G\frac{d}{dt}\Psi - \left(3H\dot{\phi}^2G_X + 3HG\right)\Psi\right] = 0,$$
$$G\frac{d}{dt}\Psi - \left(3H\dot{\phi}^2G_X + 3HG\right)\Psi = A, \tag{5.24}$$

where A is a constant. The restriction of the model is still necessary:

$$G\frac{1}{a^3}\frac{d}{dt}\left(a^3\dot{\phi}\right) = B, \tag{5.25}$$

with B a constant. The next step is to try to solve the system together with the Friedmann equation. A simple case is when $A = 0$ and $G = X$ in eq. (5.24). The equation of motion give us a solution for Φ in terms of the scale factor a:

$$\frac{d\Psi}{dt} = 9H\Psi, \quad \rightarrow \quad \Psi = Ca^9,$$
$$\Phi = Ca^9 - \alpha a^3. \tag{5.26}$$

We have used that $\Psi = \Phi + \sqrt{-g}$ and $\sqrt{-g} = \alpha a$. The energy density and pressure coming from the kinetic gravity braiding model are:

$$\varepsilon = 6HX\sqrt{2X}, \quad \mathcal{P} = -2X\ddot{\phi}. \tag{5.27}$$

If one uses these expressions, together with the restriction (5.25), in eqs. (5.5), (5.6) we obtain the total energy density and pressure:

$$\varepsilon_T = B\frac{C}{\alpha}a^6 - B, \tag{5.28}$$
$$\mathcal{P}_T = -3B\frac{C}{\alpha}a^6 + B. \tag{5.29}$$

The equation of state parameter is defined by $\omega = \mathcal{P}_T/\varepsilon_T$. The figure 1 shows the parameter ω as function of the scale factor a for some particular values of the free parameters. It starts as a cosmological constant when $a = 0$ and tends to $\omega = -3$ for large values of the scale factor. For this case the model has the behavior of phantom dark energy [49–52].

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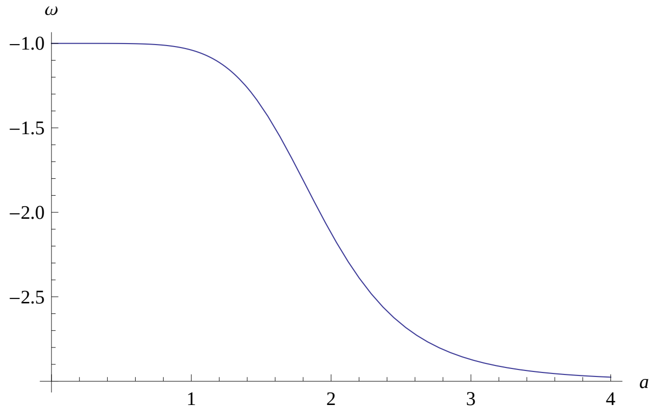

Figure 1. Equation of state parameter for the energy density and pressure given by eqs. (5.28), (5.29). We take $C = \alpha = 1$ and $B = -50$

6 Conclusions

The dark energy problem is one of the most intriguing challenges in modern cosmology. The scalar fields are one of the most usual fields employed to model dark energy. The TMT model introduces an additional measure which has a very interesting property that unified in a same description the dark matter and energy for the DBI action. In this paper we have investigated, in the context of TMT, a general form of purely kinetic k-essence field and we have shown the same unification.

Furthermore, we studied the properties of the kinetic braiding models in the TMT framework and found the general expressions for the equations of motion. We shown that the kinetic gravity braiding models, in the general case, can be described as an imperfect fluid with corresponding modified expressions for the energy density, the pressure and the heat flux that reduce to the original kinetic gravity braiding relations when the new measure is zero.

We have shown that, independent of the kinetic gravity braiding model, by an adequate selection of the new measure it is possible to obtain the cosmological scenarios that include stiff matter or a cosmological constant. For the stiff matter case, and by means of a particular value of the integration constant, there exists a cosmological scenario where the scalar field does not have any effect in the dynamics of the Universe.

In trying to find the behavior of the fluid represented by kinetic gravity braiding model in the TMT framework, we split the task in two simple cases depending on the form of G. The first one, when G is constant, lead us to a cosmological constant scenario even when in the original kinetic gravity braiding model the energy density and the pressure is zero. It is very interesting to note that this behavior is the opposite situation with respect to the stiff matter special case where the effects of the scalar field disappear on the dynamics of the Universe. The second case, taking G as a linear function of X, gave us expressions of the energy density and pressure, that for a particular choice of the free parameters, reproduce a phantom dark energy behavior, a case that can be of interest for neutrino physics [53].

Finally, we have shown that the kinetic gravity braiding models give rise to a variety of interesting cosmological effects. A general form of the function G opens a window to study its cosmological consequences in more detail.

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