

Tachyon inflation in the holographic braneworld

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Received February 13, 2019

Revised July 5, 2019

Accepted July 29, 2019

Published August 27, 2019

Abstract. A model of tachyon inflation is proposed in the framework of holographic cosmology. The model is based on a holographic braneworld scenario with a D3-brane located at the holographic boundary of an asymptotic ADS_5 bulk. The tachyon field that drives inflation is represented by a DBI action on the brane. We solve the evolution equations analytically in the slow-roll regime and solve the exact equations numerically. We calculate the inflation parameters and compare the results with Planck 2018 data.

Keywords: inflation, cosmology with extra dimensions

ArXiv ePrint: [1809.07216](https://arxiv.org/abs/1809.07216)

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1 Introduction

The inflationary universe scenario has been generally accepted as a solution to the horizon problem and some other related problems of the standard Big Bang cosmology. The origin of the field that drives inflation is still unknown and is subject to speculations. Among many models of inflation a popular class comprise tachyon inflation models [1–15]. Tachyon models are of particular interest as in these models inflation is driven by the tachyon field originating in M or string theory. The existence of tachyons in the perturbative spectrum of string theory, both open and closed, indicates that the perturbative vacuum is unstable and that there exists a true vacuum towards which a tachyon field θ tends [16]. The basics of this process are represented by an effective field theory model [17] with a Lagrangian of the Dirac-Born-Infeld (DBI) form

$$\mathcal{L} = -\ell^{-4}V(\theta/\ell)\sqrt{1 - g^{\mu\nu}\theta_{,\mu}\theta_{,\nu}}, \quad (1.1)$$

where ℓ is an appropriate length scale, θ is a scalar field of dimension of length, and

$$X = g^{\mu\nu}\theta_{,\mu}\theta_{,\nu}. \quad (1.2)$$

The dimensionless potential V is a positive function of θ with a unique local maximum at $\theta = 0$ and a global minimum at $|\theta| = \infty$ at which V vanishes.

We plan to study a braneworld inflation model in the framework of a holographic cosmology [18–21]. By *holographic cosmology* we mean a cosmology based on the effective four-dimensional Einstein equations on the holographic boundary in the framework of

anti de Sitter/conformal field theory (AdS/CFT) correspondence. A connection between AdS/CFT correspondence and cosmology has been studied in a different approach based on a holographic renormalization group flows in quantum field theory [22, 23].

As we will argue in the next section, the holographic cosmology has a property that the universe evolution starts from a point at which the energy density and cosmological scale are both finite rather than from the usual Big Bang singularity of the standard cosmology. Then the inflation phase proceeds naturally immediately after $t = 0$. Our model is based on a holographic braneworld scenario with an effective tachyon field on the brane. This paper is a sequel to previous works [24–27] in which we have studied tachyon inflation on a Randall-Sundrum type of braneworld. In the present approach a D3-brane is located at the holographic boundary of an asymptotic ADS_5 bulk. We have improved the analytical calculations of [20] in the slow roll regime up to the second order in the slow roll parameters. We solve the evolution equations numerically and confront our result with the Planck data.

The remainder of the paper is organized in four sections and two appendices. In the next section, section 2, we describe the tachyon cosmology in the framework of a holographic braneworld scenario. The following section, section 3, is devoted to a detailed description of inflation based on the holographic braneworld scenario with tachyon field playing the role of the inflaton. Our numerical results and comparison with observations are presented in section 4. In section 5 we summarize our results and give an outlook for future research.

2 Holographic tachyon cosmology

Our aim is to study tachyon inflation in the framework of holographic cosmology. We assume that the holographic braneworld is a spatially flat FRW universe with line element

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = dt^2 - a^2(t)(dr^2 + r^2d\Omega^2), \quad (2.1)$$

and we employ the holographic Friedmann equations (A.16) and (A.18) derived in appendix A. If we set $k = 0$ and $\mu = 0$, these equations take the form

$$h^2 - \frac{1}{4}h^4 = \frac{\kappa^2}{3}\ell^4\rho, \quad (2.2)$$

$$\dot{h} \left(1 - \frac{1}{2}h^2\right) = -\frac{\kappa^2}{2}\ell^3(p + \rho), \quad (2.3)$$

where, following ref. [25], we have introduced a dimensionless expansion rate $h \equiv \ell H$ and the fundamental dimensionless coupling

$$\kappa^2 = \frac{8\pi G_N}{\ell^2}. \quad (2.4)$$

The holographic cosmology has interesting properties. Solving the first Friedmann equation (2.2) as a quadratic equation for h^2 we find

$$h^2 = 2 \left(1 \pm \sqrt{1 - \frac{\kappa^2}{3}\ell^4\rho}\right). \quad (2.5)$$

Now, because we do not want our modified cosmology to depart too much from the standard cosmology after the inflation era, we demand that eq. (2.5) reduces to the standard Friedmann

equation in the low density limit, i.e., in the limit when $\kappa^2 \ell^4 \rho \ll 1$. Clearly, this demand will be met only by the $(-)$ sign solution in (2.5). Therefore we stick to the $(-)$ sign and discard the $(+)$ sign solution as unphysical. Then, it follows that the physical range of the Hubble expansion rate is between zero and the maximal value $h_{\max} = \sqrt{2}$ corresponding to the maximal energy density $\rho_{\max} = 3/(\kappa^2 \ell^4)$ [19, 28]. Assuming no violation of the weak energy condition $p + \rho \geq 0$, the expansion rate will, according to (2.3), be a monotonously decreasing function of time. The universe evolution starts from $t = 0$ with an initial $h_i \leq h_{\max}$ with energy density and cosmological scale both finite. Hence, as already noted by C. Gao [29], in the modified cosmology described by the Friedmann equations (2.2) and (2.3) the Big Bang singularity is avoided!

2.1 Equations of motion

Tachyon matter in the holographic braneworld is described by the Lagrangian (1.1) in which the scale ℓ can be identified with the AdS curvature radius. The covariant Hamiltonian associated with (1.1) is given by [25]

$$\mathcal{H} = \ell^{-4} V \sqrt{1 + \eta^2}, \quad (2.6)$$

where

$$\eta = \ell^4 V^{-1} \sqrt{g_{\mu\nu} \pi^\mu \pi^\nu} \quad (2.7)$$

and the conjugate momentum π^μ is, as usual, related to $\theta_{,\mu}$ via

$$\pi^\mu = \frac{\partial \mathcal{L}}{\partial \theta_{,\mu}}. \quad (2.8)$$

From the covariant Hamilton equations

$$\theta_{,\mu} = \frac{\partial \mathcal{H}}{\partial \pi^\mu}, \quad \pi^\mu{}_{;\mu} = -\frac{\partial \mathcal{H}}{\partial \theta}, \quad (2.9)$$

we obtain two first order differential equations in comoving frame

$$\dot{\theta} = \frac{\eta}{\sqrt{1 + \eta^2}}, \quad (2.10)$$

$$\dot{\eta} = -\frac{3h\eta}{\ell} - \frac{V_{,\theta}}{V} \sqrt{1 + \eta^2}, \quad (2.11)$$

where the subscript $_{,\theta}$ denotes a derivative with respect to θ . As usual, the Lagrangian and Hamiltonian are identified with the pressure and energy density, respectively i.e.,

$$p \equiv \mathcal{L} = -\ell^{-4} V \sqrt{1 - X} = -\frac{\ell^{-4} V}{\sqrt{1 + \eta^2}}, \quad (2.12)$$

$$\rho \equiv \mathcal{H} = \frac{\ell^{-4} V}{\sqrt{1 - X}} = \ell^{-4} V \sqrt{1 + \eta^2}, \quad (2.13)$$

and $X = \dot{\theta}^2$ in comoving frame.

2.2 Exponential potential

Consider a potential of the form

$$V = V_0 e^{-\omega|\theta|/\ell}, \quad (2.14)$$

which has been studied extensively in the literature related to string theory and tachyons [5, 11, 15, 30–32]. The dimensionless parameters ω and V_0 are positive and basically free. It proves advantageous to redefine the field $\theta \rightarrow \theta - \theta_i$ and integrate equations of motion from the initial θ_i defined as $\theta_i = -\ell\omega^{-1} \ln V_0$ instead of integrating from $\theta = 0$. Then, in the physically relevant domain $\theta_i \leq \theta < \infty$, the potential takes the form

$$V = e^{-\omega(\theta_i + |\theta - \theta_i|)/\ell} = e^{-\omega\theta/\ell}. \quad (2.15)$$

In this way we have traded an arbitrary maximal value $V_0 > 0$ of the potential at the origin for an arbitrary initial value $-\infty < \theta_i < \infty$ of the field. However, as we will discuss next, the initial value θ_i although arbitrary, will be fixed by choosing initial value for h .

The potential (2.15) has a convenient property that one can eliminate dependence on the fundamental parameter κ from the equations of motion. To demonstrate this we introduce a dimensionless time variable $\tilde{t} = t/\ell$ and replace the function θ by a dimensionless function y defined as

$$y = \frac{\kappa^2}{3} e^{-\omega\theta/\ell}. \quad (2.16)$$

Then, from (2.10) and (2.11) with (2.5) and (2.13), we obtain the following equations of motion

$$\frac{dy}{d\tilde{t}} = -\frac{\omega y \eta}{\sqrt{1 + \eta^2}}, \quad (2.17)$$

$$\frac{d\eta}{d\tilde{t}} = -3\eta \left(2 - 2\sqrt{1 - y\sqrt{1 + \eta^2}} \right)^{1/2} + \omega\sqrt{1 + \eta^2}, \quad (2.18)$$

with no κ -dependence.

2.3 Remarks on initial conditions

To solve equations (2.10) and (2.11), or equations (2.17) and (2.18), numerically one has to fix initial values of the functions θ (or y) and η at an initial time. We will assume that the evolution starts at $t = 0$ with a given initial expansion rate $h_i \leq \sqrt{2}$. Then, for a chosen initial η_i , the initial θ_i (or y_i) will be fixed by the first Friedmann equation. We will seek solutions imposing either of the two natural initial conditions: a) $\eta_i = 0$ or b) $\dot{\eta}_i = 0$. As we shall shortly see, the condition a) assures a finite initial \dot{h} whereas b) yields solutions consistent with the slow-roll regime which will be discussed in the next section.

a) $\eta_i = 0$

In this case from (2.12) and (2.13) it follows

$$p_i = -\rho_i \quad (2.19)$$

and, as a consequence of (2.3), \dot{h}_i will be finite even for $h_i = \sqrt{2}$. The initial θ_i is fixed from (2.2) and (2.13)

$$V(\theta_i) = \frac{3}{\kappa^2} \left(h_i^2 - \frac{h_i^4}{4} \right). \quad (2.20)$$

For example, the exponential potential (2.15) yields

$$\theta_i = -\frac{\ell}{\omega} \ln \left[\frac{3}{\kappa^2} \left(h_i^2 - \frac{h_i^4}{4} \right) \right], \quad (2.21)$$

which corresponds to the initial

$$y_i = h_i^2 - \frac{h_i^4}{4}, \quad (2.22)$$

independent of κ .

b) $\dot{\eta}_i = 0$

In this case from (2.11) it follows

$$\eta_i = -\frac{(\ell V_{,\theta}/V)_i}{\sqrt{9h_i^2 - (\ell V_{,\theta}/V)_i^2}} \quad (2.23)$$

and from (2.2) we obtain

$$\left(1 - \frac{h_i^2}{2} \right)^2 = 1 - \frac{\kappa^2}{3} V(\theta_i) \sqrt{1 + \eta_i^2}. \quad (2.24)$$

Given $V(\theta)$ these two equations can, in principle, be solved for η_i and θ_i . In particular, for the potential (2.15) we find

$$\eta_i = \frac{\omega}{\sqrt{9h_i^2 - \omega^2}}, \quad (2.25)$$

$$\theta_i = -\frac{\ell}{\omega} \ln \left[\frac{3}{\kappa^2} \left(h_i^2 - \frac{h_i^4}{4} \right) \sqrt{1 - \frac{\omega^2}{9h_i^2}} \right]. \quad (2.26)$$

The corresponding

$$y_i = \left(h_i^2 - \frac{h_i^4}{4} \right) \sqrt{1 - \frac{\omega^2}{9h_i^2}} \quad (2.27)$$

is again κ -independent. Because of (2.25) the free parameter ω is restricted by $0 < \omega < 3\sqrt{2}$ and the initial h_i by $h_i > \omega/3$. Note that η_i is always positive non-zero and hence, $p_i + \rho_i > 0$. Then, according to (2.3), for the maximal $h_i = \sqrt{2}$ the expansion starts with a negative infinite \dot{h}_i .

3 Inflation on the holographic brane

Tachyon inflation is based upon the slow evolution of θ with the slow-roll conditions [15]

$$\dot{\theta}^2 \ll 1, \quad |\ddot{\theta}| \ll 3H\dot{\theta}. \quad (3.1)$$

In view of (2.10) the conditions (3.1) are equivalent to

$$\eta \ll 1, \quad |\dot{\eta}| \ll \frac{3h}{\ell} \eta, \quad (3.2)$$

so that in the slow-roll regime the factors $(1 - \dot{\theta}^2)^{-1/2} = (1 + \eta^2)^{1/2}$ in (2.12) and (2.13) may be omitted. Then, during inflation we have

$$h^2 \simeq 2(1 - \sqrt{1 - \kappa^2 V/3}). \quad (3.3)$$

Note that the second inequality in (3.2) is consistent with the evolution starting from the initial $\dot{\eta}_i = 0$ as discussed in section 2.3.

Combining (2.10) and (2.11) with (3.1) and (3.2) we find

$$\dot{\theta} \simeq -\frac{\ell V_{,\theta}}{3hV}, \quad (3.4)$$

$$\ddot{\theta} \simeq \frac{\ell V_{,\theta} \dot{h}}{3Vh^2} + \left[\left(\frac{V_{,\theta}}{V} \right)^2 - \frac{V_{,\theta\theta}}{V} \right] \frac{\ell \dot{\theta}}{3h}. \quad (3.5)$$

As mentioned before, the evolution is constrained by the physical range of the expansion rate $0 \leq h^2 \leq 2$.

The most important quantities that characterize inflation are the slow-roll inflation parameters ε_j defined recursively [15, 33]

$$\varepsilon_{j+1} = \frac{\dot{\varepsilon}_j}{H\varepsilon_j}, \quad (3.6)$$

starting from $\varepsilon_0 = H_*/H$, where H_* is the Hubble rate at some chosen time. The next two are then given by

$$\varepsilon_1 \equiv -\frac{\dot{H}}{H^2} \simeq \frac{4 - h^2}{12h^2(2 - h^2)} \left(\frac{\ell V_{,\theta}}{V} \right)^2, \quad (3.7)$$

$$\varepsilon_2 \equiv \frac{\dot{\varepsilon}_1}{H\varepsilon_1} \simeq 2\varepsilon_1 \left(1 - \frac{2h^2}{(2 - h^2)(4 - h^2)} \right) + \frac{2\ell^2}{3h^2} \left[\left(\frac{V_{,\theta}}{V} \right)^2 - \frac{V_{,\theta\theta}}{V} \right]. \quad (3.8)$$

During inflation $\varepsilon_1 < 1$, $\varepsilon_2 < 1$ and inflation ends once either of the two exceeds unity.

In the following we will study the exponential potential

$$V = e^{-\omega\theta/\ell}, \quad (3.9)$$

as in (2.15). As we have shown in section 2.2, this potential has a remarkable property that the evolution does not depend on the fundamental parameter κ . For this potential we have

$$\left(\frac{V_{,\theta}}{V} \right)^2 = \frac{V_{,\theta\theta}}{V} = \frac{\omega^2}{\ell^2}, \quad (3.10)$$

so in this case the last term on the right-hand side of (3.8) vanishes. Note that near and at the end of inflation $h^2 \ll 1$ so that we approximately have $\varepsilon_2 \simeq 2\varepsilon_1$. Hence, the criteria for the end of inflation will be $\varepsilon_{2f} = 1$.

For the purpose of calculating the spectral index we will also need the third slow roll parameter ε_3 given by

$$\varepsilon_3 \equiv \frac{\dot{\varepsilon}_2}{H\varepsilon_2} \simeq \varepsilon_2 + \frac{4h^2(8 - h^4)}{(2 - h^2)(4 - h^2)(8 - 8h^2 + h^4)} \varepsilon_1, \quad (3.11)$$

where the second equality holds for the exponential potential in the slow-roll approximation.

Equation (3.4) with (3.9) may be easily integrated yielding the time as a function of h in the slow-roll regime

$$t = \frac{3\ell}{\omega^2} \left[2(h_i - h) + \ln \frac{(2 - h_i)(2 + h)}{(2 + h_i)(2 - h)} \right], \quad (3.12)$$

where we have chosen the integration constant so that $h = h_i$ at $t = 0$. In our numerical calculations we treat the initial value h_i^2 as a free parameter ranging between 0 and 2.

Another important quantity is the number of e-folds N defined as

$$N \equiv \int_{t_i}^{t_f} H dt \simeq -3 \int_{\theta_i}^{\theta_f} \frac{h^2 V}{\ell^2 V_{,\theta}} d\theta, \quad (3.13)$$

where the subscripts i and f denote the beginning and the end of inflation, respectively. Typically $N \simeq 50$ – 60 is sufficient to solve the flatness and horizon problems. The second equality in (3.13) is obtained by making use of eq. (3.4). For the potential (3.9) the integral can be easily calculated and expressed in terms of elementary functions. With the substitution

$$x \equiv 1 - h^2/2 = \sqrt{1 - \kappa^2 e^{-\omega\theta/\ell}/3} \quad (3.14)$$

we find

$$N = \frac{6}{\omega\ell} (\theta_f - \theta_i) - \frac{12}{\omega^2} \int_{x_i}^{x_f} \frac{x^2 dx}{1 - x^2}, \quad (3.15)$$

where

$$x_{i,f} = 1 - h_{i,f}^2/2 = \sqrt{1 - \kappa^2 e^{-\omega\theta_{i,f}/\ell}/3}. \quad (3.16)$$

The end-value of θ is fixed by the condition $\varepsilon_{2f} = 1$, so that

$$\varepsilon_{2f} \simeq \frac{\omega^2}{\kappa^2 V_f} = \frac{\omega^2}{\kappa^2} e^{\omega\theta_f/\ell} = 1, \quad (3.17)$$

yielding

$$\theta_f = \frac{\ell}{\omega} \ln \frac{\kappa^2}{\omega^2} \quad (3.18)$$

and

$$x_f = \sqrt{1 - \omega^2/3}. \quad (3.19)$$

Using this in (3.15) yields a functional relationship between the parameter ω , the e-fold number N , and the initial value h_i

$$N = \frac{12}{\omega^2} \left[\sqrt{1 - \frac{\omega^2}{3}} - 1 + \frac{h_i^2}{2} + \ln \left(2 - \frac{h_i^2}{2} \right) - \ln \left(1 + \sqrt{1 - \frac{\omega^2}{3}} \right) \right]. \quad (3.20)$$

Expanding the expression in brackets up to the lowest order in ω^2 we find an approximate expression

$$N = \frac{12}{\omega^2} \left[\frac{h_i^2}{2} + \ln \left(1 - \frac{h_i^2}{4} \right) \right] - 1. \quad (3.21)$$

As expected for the potential (3.9), neither N nor the slow-roll parameters depend on the parameter κ .

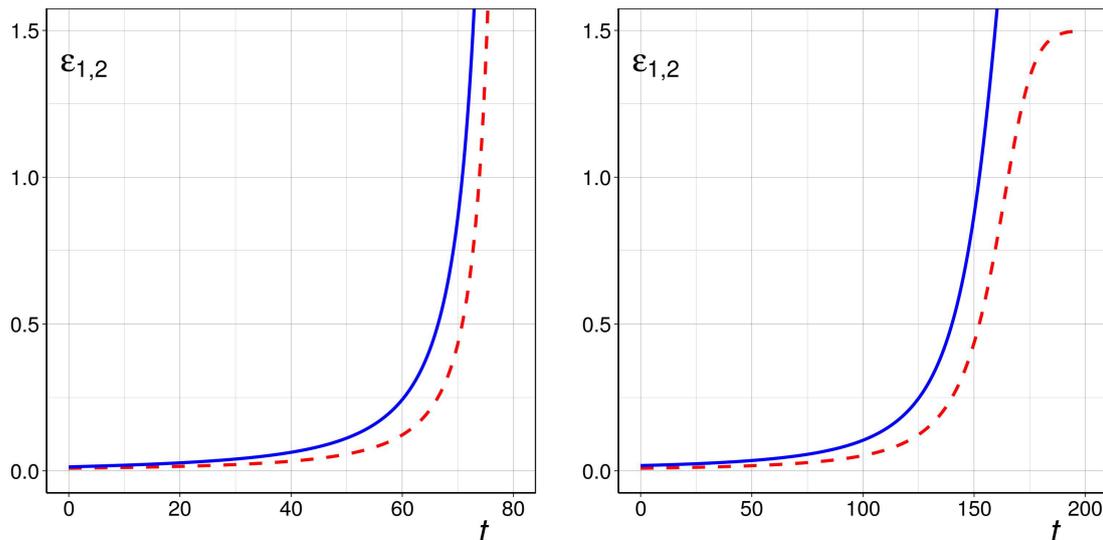


Figure 1. Slow-roll parameters ε_1 (dashed red line) and ε_2 (full blue line) versus time in units of ℓ calculated analytically in the slow-roll approximation (left panel) and numerically (right panel) for $\omega^2 = 0.027$ and the initial value $h_i^2 = 0.6$ corresponding to $N = 60$ according to (3.20).

4 Numerical calculations

In this section we present the results obtained by numerically solving the exact equations of motion (2.17) and (2.18) given the initial conditions at $t = 0$ as described in section 2.3. The numerical procedure is similar to that developed in ref. [24]. For each pair of randomly chosen N and h_i in the intervals $60 \leq N \leq 90$ and $0 < h_i^2 < 2$, respectively, the parameter ω is fixed by (3.21). Then, the set of equations (2.17) and (2.18) supplemented by the equation for N

$$dN = h d\tilde{t} \quad (4.1)$$

is evolved from $t = 0$ up to an end time of the order of a few hundreds of ℓ . In figure 1 we plot the evolution of the slow roll parameters for the initial $h_i^2 = 0.6$ and $\omega^2 = 0.027$ corresponding to $N = 60$ according to (3.20). The inflation actually ends at a time t_f obtained using the function $\varepsilon_2(t)$ and demanding $\varepsilon_2(t_f) = 1$. In comparison with the end time obtained analytically in the slow roll approximation the numerical t_f is substantially larger as is evident by comparing left and right panels of figure 1. As a consequence, the numerically calculated $N(t_f)$ turns out to be larger than the assumed N . Hence, the inflation is assumed to begin at some time $t_i > 0$, rather than at $t = 0$, such that

$$N(t_f) - N(t_i) = N. \quad (4.2)$$

The time t_i is then used to find the initial $\varepsilon_1(t_i)$ and $\varepsilon_2(t_i)$ which in turn are used to calculate the tensor-to-scalar ratio r and spectral index n_s .

4.1 Spectral index and tensor to scalar ratio

A proper calculation of the power spectra by perturbing the Einstein equations (A.8) would go beyond the scope of the present paper. We propose instead a simplified scheme described in appendix B where we derive approximate expressions for the scalar and tensor power spectra \mathcal{P}_S and \mathcal{P}_T , respectively.

For scalar perturbations we calculate the power spectrum in the limit when the modes are well outside the acoustic horizon characterized by the comoving wave number $q = aHc_s^{-1}$. Here c_s is the adiabatic sound speed defined by

$$c_s^2 \equiv \left. \frac{\partial p}{\partial \rho} \right|_{\theta} = \frac{p, X}{\rho, X} = \frac{p + \rho}{2X\rho, X}, \quad (4.3)$$

where the subscript $, X$ denotes a derivative with respect to X and $|_{\theta}$ means that the derivative is taken keeping θ fixed, i.e., ignoring the dependence of \mathcal{L} on θ . This definition coincides with the usual hydrodynamic definition of the sound speed squared as the derivative of pressure with respect to the energy density at fixed entropy per particle. For the tachyon fluid described by the Lagrangian (1.1) the sound speed squared may be expressed as

$$c_s^2 = 1 - X = 1 - \frac{4(2 - h^2)}{3(4 - h^2)}\varepsilon_1, \quad (4.4)$$

where the first equation follows from (2.12) and (2.13) and the second equation is a consequence of the modified Friedmann equations (2.2) and (2.3). This equation shows a substantial deviation from the standard tachyon result [15]

$$c_s^2|_{\text{st}} = 1 - \frac{2}{3}\varepsilon_1. \quad (4.5)$$

The expressions (4.4) and (4.5) agree near the end of inflation, i.e., in the limit $h \rightarrow 0$. This is what one would expect since the holographic cosmology described by equation (2.5) reduces to the standard cosmology in the low density limit.

Next, using the definition (B.37) with (B.22), (B.28), and (B.30) derived in appendix B.1, we evaluate the scalar spectral density at the horizon crossing, i.e., for a wave-number satisfying $q = aH$. Following refs. [15, 34] we make use of the expansion of the Hankel function in the limit $c_s q \tau \rightarrow 0$

$$H_{\nu}^{(1)}(-c_s q \tau) \simeq -\frac{i}{\pi} \Gamma(\nu) \left(\frac{-c_s q \tau}{2} \right)^{-\nu}, \quad (4.6)$$

where q is the comoving wave number and τ denotes the conformal time ($\tau < 0$). Using this we find at the lowest order in ε_1 and ε_2

$$\mathcal{P}_S \simeq \frac{\kappa^2 h^2}{8\pi^2(1 - h^2/2)c_s \varepsilon_1} \left[1 - 2 \left(1 + C + \frac{Ch^2}{2 - h^2} \right) \varepsilon_1 - C\varepsilon_2 \right], \quad (4.7)$$

where $C = -2 + \ln 2 + \gamma \simeq -0.72$ and γ is the Euler constant. In comparison with \mathcal{P}_S obtained in the standard tachyon inflation [15], our result is enhanced by a factor $1 - h^2/2$ in the denominator on the right side of (4.7) and the linear term in ε_1 gets an additional contribution proportional to $h^2/(2 - h^2)$.

Similarly, from the expression (B.46) with (B.44) for tensor perturbations we obtain

$$\mathcal{P}_T \simeq \frac{2\kappa^2 h^2}{\pi^2} [1 - 2(1 + C)\varepsilon_1], \quad (4.8)$$

Hence, the tensor perturbation spectrum is given by the usual expression for \mathcal{P}_T [15].

The scalar spectral index n_S and tensor to scalar ratio r are then given by

$$n_S - 1 = \frac{d \ln \mathcal{P}_S}{d \ln q}, \quad (4.9)$$

$$r = \frac{\mathcal{P}_T}{\mathcal{P}_S}, \quad (4.10)$$

where \mathcal{P}_S and \mathcal{P}_T are evaluated at the horizon crossing.

Using (4.7)–(4.10) and keeping the terms up to the second order in ε_i we find

$$r = 8(2 - h^2)\varepsilon_1 \left[1 + C\varepsilon_2 + 2 \left(\frac{Ch^2}{2 - h^2} - \frac{2 - h^2}{12 - 3h^2} \right) \varepsilon_1 \right] \quad (4.11)$$

and

$$\begin{aligned} n_s = 1 - & \left(2 + \frac{2h^2}{2 - h^2} \right) \varepsilon_1 - \varepsilon_2 - \left(2 + \frac{2h^2}{2 - h^2} - \frac{8h^2}{3(4 - h^2)^2} - \frac{8Ch^2}{(2 - h^2)^2} \right) \varepsilon_1^2 \\ & - \left(\frac{8}{3} + \frac{h^2}{3(4 - h^2)} + \frac{4C}{2 - h^2} \right) \varepsilon_1 \varepsilon_2 - C\varepsilon_2 \varepsilon_3. \end{aligned} \quad (4.12)$$

It is understood that the quantities h , ε_1 , and ε_2 in these expressions are to be taken at the beginning of the slow roll inflation. A comparison of (4.11) and (4.12) with the second order predictions of the standard tachyon inflation [15]

$$r|_{\text{st}} = 16\varepsilon_1(1 + C\varepsilon_2 - \varepsilon_1/3), \quad (4.13)$$

$$n_S|_{\text{st}} = 1 - 2\varepsilon_1 - \varepsilon_2 - 2\varepsilon_1^2 - \left(\frac{8}{3} + 2C \right) \varepsilon_1 \varepsilon_2 - C\varepsilon_2 \varepsilon_3 \quad (4.14)$$

shows a substantial deviation. As expected, near the end of inflation, i.e., in the limit $h \rightarrow 0$, expressions (4.11) and (4.12) agree with (4.13) and (4.14), respectively.

In figures 2 and 3 we plot r and $1 - n_S$ as functions of the initial Hubble rate squared h_i^2 for fixed N and varying ω . For each pair (h_i^2, N) the parameter ω , being in a functional relationship with N and h_i , is calculated using the approximate expression (3.21). In figure 4 we present the r versus n_S diagram. The dots represent the numerical data for randomly chosen N ranging between 60 and 90 and h_i^2 between 0 and 2. Variation of N and h_i^2 is represented by color. Each point on the left panel is depicted by a color representing a value of N and similarly on the right panel a value of h_i . Clearly, the numerical data set is bounded by the points corresponding to $N = 60$ from above and $N = 90$ from below. For each point the parameter ω is calculated using (3.21). The results obtained analytically in the slow-roll approximation are depicted by dashed and full black lines corresponding to $N = 60$ and $N = 90$, respectively. In figure 5 the numerical and analytical data are superimposed on the observational constraints taken from the Planck collaboration 2018 [35]. To demonstrate more explicitly the dependence of our theoretical predictions on N we plot r versus n_S in figure 6 for several fixed e-fold numbers N ranging between 60 and 140. Both numerical and analytical results show that agreement with observations is better for larger values of N .

Figures 4 and 5 show that the approximate analytical results are shifted downwards with respect to the numerical data. This shift reflects the departure of the analytical from the numerical curve in figure 3 as a consequence of analytical calculations being subject to the slow-roll approximation. To see this, for definiteness, consider $N = 90$. The corresponding

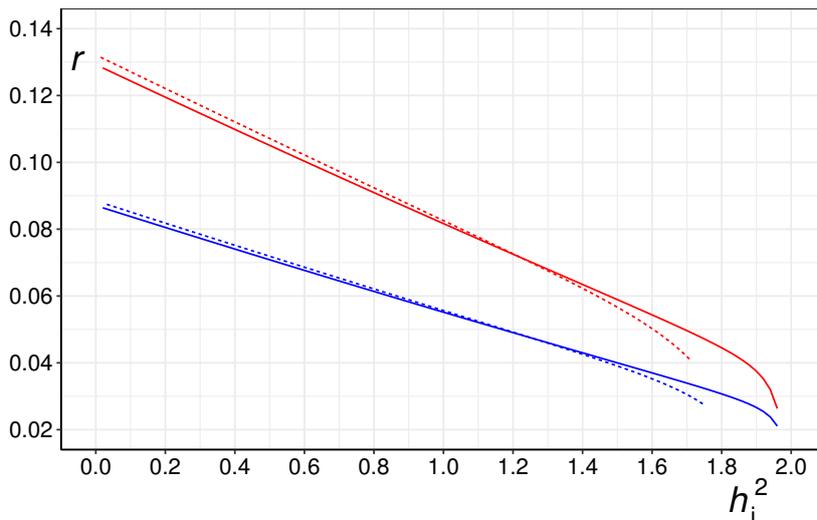


Figure 2. r versus initial h_i^2 calculated analytically (full lines) and numerically (dashed lines) for fixed $N = 60$ (upper red lines) and $N = 90$ (lower blue lines). The parameter ω is varying along the lines in accordance with (3.21).

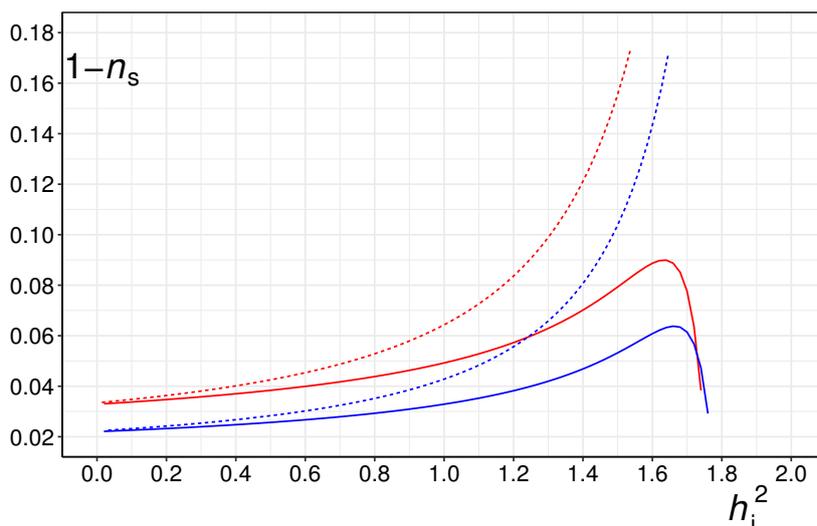


Figure 3. $1 - n_s$ versus initial h_i^2 calculated analytically (full lines) and numerically (dashed lines) for fixed $N = 60$ (upper red lines) and $N = 90$ (lower blue lines). The parameter ω is varying along the lines in accordance with (3.21).

numerical results are represented in figures 4 and 5 by the points at the lower boundary of the numerical data set and the corresponding analytical results by a two-valued function depicted by the full line. The upper (lower) branch of that curve corresponds to the part of the curve in figure 3 left (right) from the maximum. The departure of the analytical from the numerical curve in figure 3 increases with h_i which is consistent with the slow-roll approximation. Clearly, the slow-roll approximation breaks down at the maximum of the curve in figure 3 so the results represented by the lower branch of the curve in figures 4 and 5 are not physically relevant.

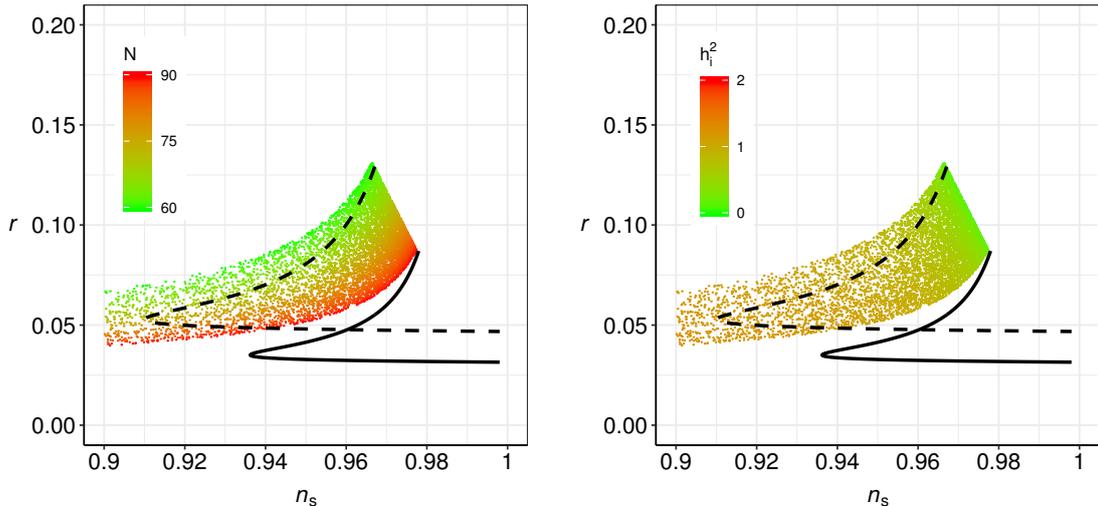


Figure 4. r versus n_s diagram. The dots represent the theoretical predictions obtained by solving the equations of motion numerically for randomly chosen N ranging between 60 and 90 and h_i^2 between 0 and 2. Variation of color represents variation of N (left panel) and variation of h_i (right panel). The parameter ω varies in accordance with (3.21). The analytical results of the slow roll approximation are depicted by the black lines corresponding to $N = 60$ (dashed) and $N = 90$ (full).

4.2 Comment on primordial non-Gaussianity

The prime diagnostic of non-Gaussianity of inflationary fluctuations is described by the three-point correlation function [36]

$$\langle \hat{\zeta}_{q_1} \hat{\zeta}_{q_2} \hat{\zeta}_{q_3} \rangle = (2\pi)^3 \delta(\mathbf{q}_1 + \mathbf{q}_2 + \mathbf{q}_3) f_{\text{NL}} F(q_1, q_2, q_3), \quad (4.15)$$

where $\hat{\zeta}_q$ is the operator associated with the curvature perturbation ζ introduced in appendix B.1. The quantity f_{NL} is a dimensionless parameter defining the amplitude of non-Gaussianity and the function F captures the momentum dependence.

Our model belongs to the class of k -essence inflation models. In these models the Lagrangian $\mathcal{L}(\theta, X)$ has a non-canonical dependence on the kinetic term X defined in (1.2). As a consequence, the adiabatic sound speed c_s defined by (4.3) in these models may significantly deviate from 1. In the k -essence models, the largest non-Gaussianity is peaked at the so called *equilateral* configuration with $q_1 \sim q_2 \sim q_3$. The non-Gaussianity amplitude of equilateral triangle $f_{\text{NL}}^{\text{equil}}$ in a general k -essence is given by [36, 37]

$$f_{\text{NL}}^{\text{equil}} = -\frac{35}{108} \left(\frac{1 - c_s^2}{c_s^2} \right) + \frac{20}{81} \Lambda, \quad (4.16)$$

where

$$\Lambda \equiv X^2 \frac{\mathcal{L}_{,XX}^2 - (1/3)\mathcal{L}_{,X}\mathcal{L}_{,XXX}}{\mathcal{L}_{,X}^2 + 2X\mathcal{L}_{,X}\mathcal{L}_{,XX}}. \quad (4.17)$$

Precisely as in the string theory motivated DBI model [38], the quantity Λ in the tachyon model turns out to be identically zero and the amplitude is directly proportional to $1 - c_s^2$. In the tachyon model with standard cosmology one has

$$f_{\text{NL}}^{\text{equil}} \Big|_{\text{st}} = -\frac{35}{108} \left(\frac{1 - c_s^2}{c_s^2} \right), \quad (4.18)$$

where c_s is given by (4.5).

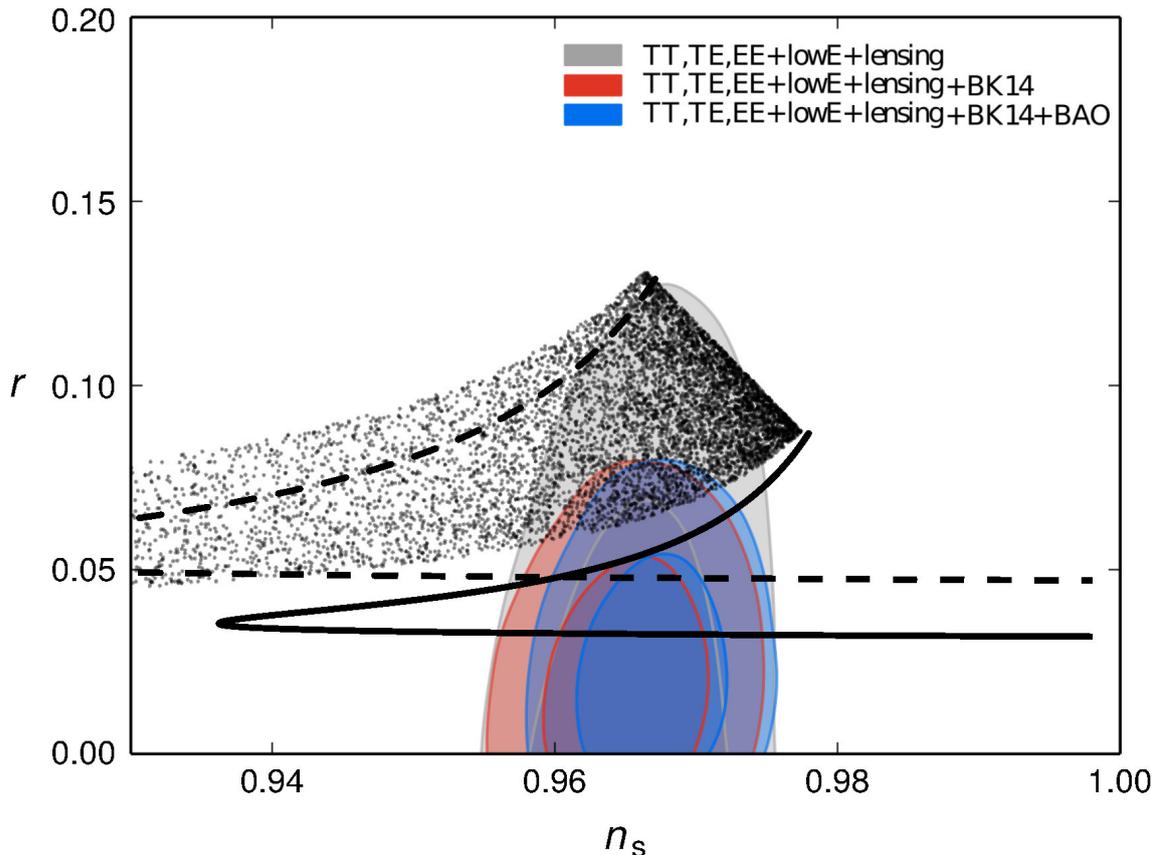


Figure 5. r versus n_s diagram with observational constraints from ref. [35]. As in figure 4 the dots represent the theoretical predictions obtained by solving the equations of motion numerically for randomly chosen N and h_i^2 and the analytical results in the slow roll approximation are depicted by the dashed ($N = 60$) and full ($N = 90$) lines.

Now, we estimate the non-Gaussianity amplitude in our tachyon model. As the sound speed deviates from unity most at the end of the slow roll regime, we estimate the equilateral amplitude at the end of inflation neglecting possible post-inflationary effects. From (4.18) and the calculation of the two point function presented in appendix B.1, where the main difference between our model and the standard tachyon inflation is the factor $1 - h^2/2$ which appears in the denominator of (4.7), we expect $f_{\text{NL}}^{\text{equil}}$ in our model to be of the form (4.18) possibly multiplied by a factor $(1 - h^2/2)$ raised to some power. For the purpose of an estimate this factor can be neglected since $h^2 \ll 1$ at the end of the slow roll regime. An estimate based on (3.14), (3.19), and $\omega^2 = 0.027$ yields $h^2 \lesssim 0.01$ at the end of inflation. Hence, we can use the result (4.18) with c_s given by (4.4) and $\varepsilon_1 = 1$ yielding

$$f_{\text{NL}}^{\text{equil}} \approx -\frac{70(1 - h^2/2)}{108(1 + h^2/4)} \approx f_{\text{NL}}^{\text{equil}} \Big|_{\text{st}} = -\frac{70}{108}. \quad (4.19)$$

This value is well within the observational constraints provided by the Planck 2015 collaboration [39]: $f_{\text{NL}}^{\text{equil}} = 2.6 \pm 61.6$ from temperature data and $f_{\text{NL}}^{\text{equil}} = 15.6 \pm 37.3$ from temperature and polarization data.

In conclusion, the estimated non-Gaussianity in our model at the end of inflationary period cannot be distinguished from that in the standard tachyon inflation. Possible post-

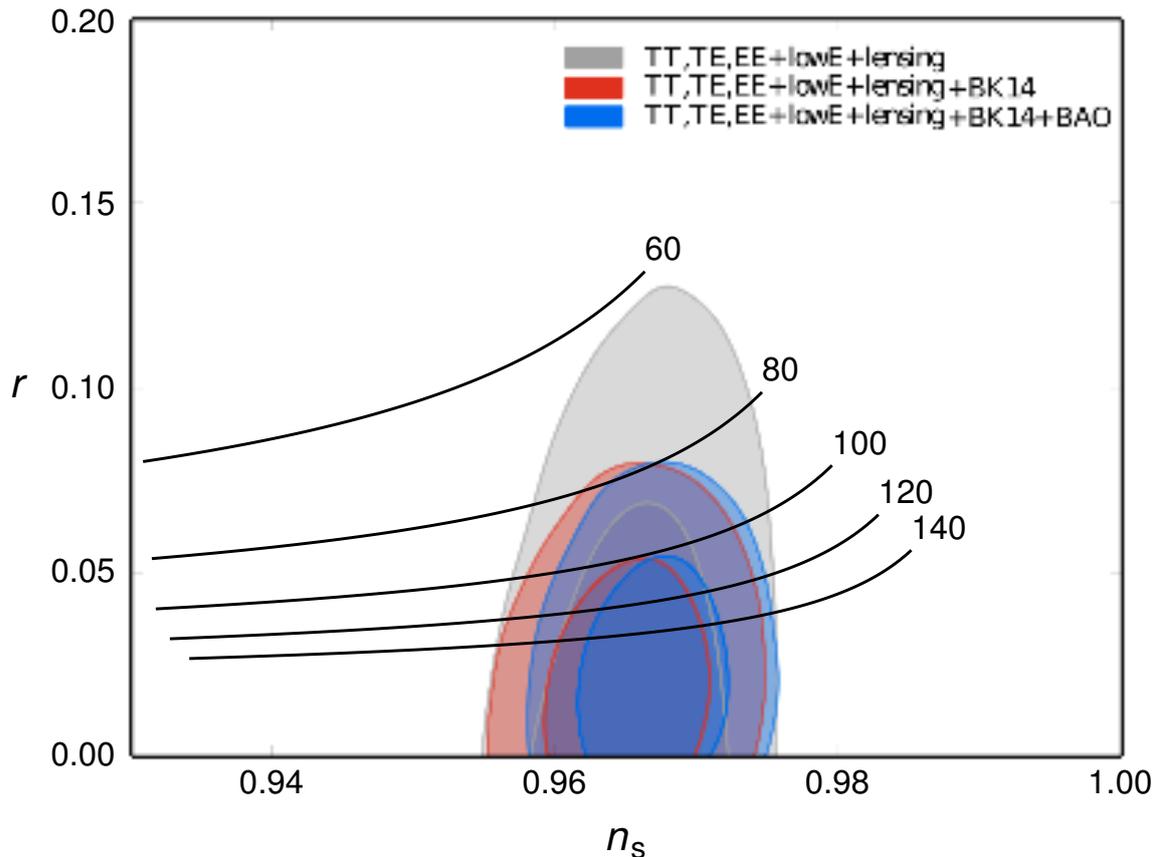


Figure 6. r versus n_s diagram with observational constraints from ref. [35] for several fixed N (indicated at the end of each line) and varying h_i^2 in the interval 0.01–2.

inflationary persistence of isocurvature perturbations, as discussed recently by C. van de Bruck, T. Koivisto, and C. Longden [40], may alter this conclusion. However, a study of such effects is beyond the scope of the present paper.

5 Conclusions

We have investigated a model of tachyon inflation based on a holographic braneworld scenario with a D3-brane located at the boundary of the ADS_5 bulk. The slow-roll equations in this model turn out to differ substantially from those of the standard tachyon inflation with the same potential. We have studied in particular a simple exponentially attenuating potential. For a given number of e-folds our results depend only on the initial value of the Hubble rate and do not depend on the fundamental coupling κ . A comparison of our results with the most recent observational data [35, 41] shows reasonable agreement as demonstrated in figure 5. Apparently, the agreement with observations is better for larger values of the numbers of e-folds N .

It would be of considerable interest to perform precise calculations for other types of tachyon potentials that are currently on the market. To this end, one would need to estimate the phenomenologically acceptable range of the fundamental coupling parameter κ and solve

the exact equations numerically for various potentials. Based on our experience from the previous work [24, 27] we do not expect a substantial deviation from the present results.

Acknowledgments

This work has been supported by the ICTP — SEENET-MTP project NT-03 Cosmology — Classical and Quantum Challenges. N. Bilić has been supported by the European Union through the European Regional Development Fund — the Competitiveness and Cohesion Operational Programme (KK.01.1.1.06) and by the H2020 CSA Twinning project No. 692194, “RBI-T-WINNING”. N. Bilić, G.S. Djordjevic and M. Milosevic are partially supported by the STSM CANTATA-COST grant. D. Dimitrijevic, G.S. Djordjevic, M. Milosevic, and M. Stojanovic acknowledge support by the Serbian Ministry of Education, Science and Technological Development under the projects No. 176021 (G.S.Dj., M.M.), No. 174020 (D.D.D.) and No. 176003 (M.S.). G.S. Djordjevic is grateful to the CERN-TH Department for hospitality and support.

A Cosmology on the holographic brane

In this appendix we present a brief review of the holographic cosmology elaborated in [19]. The basic idea is to use the action of the second Randall-Sundrum (RSII) model [42] as a regulator of the bulk action and derive the Friedmann equations on the AdS₅ boundary using the AdS/CFT prescription and holographic renormalization [43].

A general asymptotically AdS₅ metric in Fefferman-Graham coordinates [44] is of the form

$$ds^2 = G_{ab}dx^a dx^b = \frac{\ell^2}{z^2} (g_{\mu\nu}dx^\mu dx^\nu - dz^2), \quad (\text{A.1})$$

where the length scale ℓ is the AdS curvature radius and we use the Greek alphabet for 3+1 spacetime indices. Near $z = 0$ the metric $g_{\mu\nu}$ can be expanded as

$$g_{\mu\nu}(z, x) = g_{\mu\nu}^{(0)}(x) + z^2 g_{\mu\nu}^{(2)}(x) + z^4 g_{\mu\nu}^{(4)}(x) + \dots \quad (\text{A.2})$$

Explicit expressions for $g_{\mu\nu}^{(2n)}$ in terms of arbitrary $g_{\mu\nu}^{(0)}$ can be found in ref. [43]. The pure gravitational on-shell bulk action is infra-red divergent and can be regularized by placing the RSII brane near the boundary, i.e., at $z = \epsilon\ell$, $\epsilon \ll 1$, so that the induced metric on the brane is

$$\gamma_{\mu\nu} = \frac{1}{\epsilon^2} g_{\mu\nu}(\epsilon\ell, x) = \frac{1}{\epsilon^2} \left(g_{\mu\nu}^{(0)} + \epsilon^2 \ell^2 g_{\mu\nu}^{(2)} + \dots \right). \quad (\text{A.3})$$

The bulk splits in two regions: $0 \leq z < \epsilon\ell$ and $\epsilon\ell \leq z < \infty$. We can either discard the $0 \leq z < \epsilon\ell$ region (one-sided regularization) or invoke the Z_2 symmetry and identify two regions (two-sided regularization). For simplicity we shall use the one-sided regularization. The regularized on shell bulk action is [45]

$$S^{\text{reg}}[\gamma] = \frac{1}{8\pi G_5} \int_{z \geq \epsilon\ell} d^5x \sqrt{G} \left[-\frac{R^{(5)}}{2} - \Lambda_5 \right] + S_{\text{GH}}[\gamma] + S_{\text{br}}[\gamma], \quad (\text{A.4})$$

where S_{GH} is the Gibbons-Hawking boundary term which is required to make a variational procedure well defined. The brane action is given by

$$S_{\text{br}}[\gamma] = \int d^4x \sqrt{-\gamma} (-\sigma + \mathcal{L}[\gamma]), \quad (\text{A.5})$$

where σ is the brane tension and the Lagrangian \mathcal{L} describes matter on the brane. The renormalized action is obtained by adding necessary counter-terms and taking the limit $\epsilon \rightarrow 0$

$$S^{\text{ren}}[\gamma] = S^{\text{reg}}[\gamma] + S_1[\gamma] + S_2[\gamma] + S_3[\gamma], \quad (\text{A.6})$$

where the expressions for the counter-terms S_1 , S_2 and S_3 may be found in refs. [43, 46]. Next, the variation with respect to the induced metric $\gamma_{\mu\nu}$ of the regularized on shell bulk action (RSII action) should vanish, i.e., we demand

$$\delta S^{\text{reg}}[\gamma] = 0. \quad (\text{A.7})$$

The variation of the action yields effective four-dimensional Einstein's equations on the boundary

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}^{(0)} = 8\pi G_N (\langle T_{\mu\nu}^{\text{CFT}} \rangle + T_{\mu\nu}), \quad (\text{A.8})$$

where $R_{\mu\nu}$ is the Ricci tensor associated with the metric $g_{\mu\nu}^{(0)}$ and the energy-momentum tensor

$$T_{\nu}^{\mu} = \text{diag}(\rho, -p, -p, -p) \quad (\text{A.9})$$

corresponds to the matter Lagrangian \mathcal{L} on the brane. According to the AdS/CFT prescription, the expectation value of the energy-momentum tensor of the dual conformal theory is given by

$$\langle T_{\mu\nu}^{\text{CFT}} \rangle = \frac{2}{\sqrt{-g^{(0)}}} \frac{\partial S^{\text{ren}}}{\partial g^{(0)\mu\nu}} = \lim_{\epsilon \rightarrow 0} \frac{2}{\sqrt{-g}} \frac{\partial S^{\text{ren}}}{\partial g^{\mu\nu}}. \quad (\text{A.10})$$

This expectation value has been derived explicitly in terms of $g_{\mu\nu}^{(2n)}$, $n = 0, 1, 2$, in ref. [43] for an arbitrary metric $g_{\mu\nu}^{(0)}$ at the $z = 0$ boundary.

In the following we will specify the boundary geometry to be of a general FRW form

$$ds_{(0)}^2 = g_{\mu\nu}^{(0)} dx^{\mu} dx^{\nu} = dt^2 - a^2(t) d\Omega_k^2, \quad (\text{A.11})$$

where

$$d\Omega_k^2 = d\chi^2 + \frac{\sin^2(\sqrt{k}\chi)}{k} (d\vartheta^2 + \sin^2\vartheta d\varphi^2) \quad (\text{A.12})$$

is the spatial line element for a closed ($k = 1$), open hyperbolic ($k = -1$), or open flat ($k = 0$) space. Assuming an AdS Schwarzschild geometry in the bulk one obtains [18, 19, 47]

$$\langle T_{\mu\nu}^{\text{CFT}} \rangle = t_{\mu\nu} + \frac{1}{4} \langle T^{\text{CFT}\alpha}_{\alpha} \rangle g_{\mu\nu}^{(0)}. \quad (\text{A.13})$$

The second term on the right-hand side corresponds to the conformal anomaly

$$\langle T^{\text{CFT}\alpha}_{\alpha} \rangle = \frac{3\ell^3}{16\pi G_5} \frac{\ddot{a}}{a} \left(H^2 + \frac{k}{a^2} \right), \quad (\text{A.14})$$

where $H = \dot{a}/a$ is the Hubble expansion rate on the boundary. The first term on the right-hand side of (A.13) is a traceless tensor, the nonvanishing components of which are

$$t_{00} = -3t_i^i = \frac{3\ell^3}{64\pi G_5} \left[\left(H^2 + \frac{k}{a^2} \right)^2 + \frac{4\mu}{a_0^4} - \frac{\ddot{a}_0}{\dot{a}_0} \left(H^2 + \frac{k}{a^2} \right) \right], \quad (\text{A.15})$$

where the dimensionless parameter μ is related to the black hole mass [48, 49]. Then, from (A.8) we obtain the holographic Friedmann equations [18, 19]

$$H^2 + \frac{k}{a^2} - \frac{\ell^2}{4} \left(H^2 + \frac{k}{a^2} \right)^2 = \frac{8\pi G_N}{3} \rho + \frac{\ell^2 \mu}{a^4}. \quad (\text{A.16})$$

From this, by making use of the energy conservation equation

$$\dot{\rho} + 3H(p + \rho) = 0, \quad (\text{A.17})$$

we obtain the second Friedmann equation

$$H^2 + \frac{k}{a^2} + \frac{\ddot{a}}{a} \left[1 - \frac{\ell^2}{2} \left(H^2 + \frac{k}{a^2} \right)^2 \right] = \frac{4\pi G_N}{3} (\rho - 3p), \quad (\text{A.18})$$

where the pressure p and energy density ρ are the components of the energy-momentum tensor as defined in (A.9).

B Cosmological perturbations

Here we derive the spectra of the cosmological perturbations for the holographic cosmology with tachyon k -essence. Calculation of the spectra proceeds by identifying the proper canonical field and imposing quantization of the quadratic action for the near free field. The procedure for a general k -inflation is described in [50] and applied to the tachyon fluid in refs. [3, 15, 34].

We shall closely follow J. Garriga and V.F. Mukhanov [50] and adjust their formalism to account for the modified Friedmann equations. In the following we consider a spatially flat background with Friedman equations of the form (2.2) and (2.3) in which the pressure and energy density p and ρ corresponding to the tachyon Lagrangian (1.1) are defined in (2.12) and (2.13). Equation (2.2) can be written in the usual Friedmann form

$$H^2 = \frac{8\pi G_N}{3} \tilde{\rho}, \quad (\text{B.1})$$

where

$$\tilde{\rho} = \frac{6}{\kappa^2 \ell^4} \left(1 \pm \sqrt{1 - \frac{\kappa^2}{3} \ell^4 \rho} \right). \quad (\text{B.2})$$

Equation (B.1) with (B.2) suggests considering another k -essence Lagrangian $\tilde{\mathcal{L}} = \tilde{\mathcal{L}}(X, \theta)$, such that the effective energy density $\tilde{\rho}$ defined in (B.2) is obtained from $\tilde{\mathcal{L}}$ by the usual prescription

$$\tilde{\rho} = 2X\tilde{\mathcal{L}}_{,X} - \tilde{\mathcal{L}}. \quad (\text{B.3})$$

Then, varying the action

$$\tilde{S} = \int d^4x \sqrt{-g} \left[-\frac{R}{16\pi G_N} + \tilde{\mathcal{L}}(X, \theta) \right], \quad (\text{B.4})$$

one obtains Einstein's equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G_N \tilde{T}_{\mu\nu}, \quad (\text{B.5})$$

where

$$\tilde{T}_{\mu\nu} = (\tilde{p} + \tilde{\rho})u_\mu u_\nu - \tilde{p}g_{\mu\nu} \quad (\text{B.6})$$

and

$$\tilde{p} \equiv \tilde{\mathcal{L}}, \quad u_\mu = \frac{\theta_{,\mu}}{\sqrt{X}}. \quad (\text{B.7})$$

In principle, the Lagrangian $\tilde{\mathcal{L}}$ can be expressed as an explicit function of X and θ by integrating eq. (B.3) but in the following we will not need an explicit expression for $\tilde{\mathcal{L}}$.

Assuming isotropy and homogeneity, equations (B.5) yield the Friedmann equation (B.1) (or equivalently eq. (2.2)) and energy conservation equation

$$\dot{\tilde{\rho}} + 3H(\tilde{p} + \tilde{\rho}) = 0. \quad (\text{B.8})$$

In this way we have obtained the holographic Friedman equation (A.16) (with $k = 0$ and $\mu = 0$) from a standard k -essence action (B.4). Now, we can apply the procedure of ref. [50] directly to the modified k -essence described by (B.4) keeping in mind that the background evolution is governed by our original equations (2.10) and (2.11) with (2.2) and (2.3).

B.1 Scalar perturbations

Assuming a spatially flat background with line element (2.1), we introduce the perturbed line element in the longitudinal gauge

$$ds^2 = (1 + 2\Phi)dt^2 - (1 - 2\Phi)a^2(t)(dr^2 + r^2d\Omega^2). \quad (\text{B.9})$$

Next, we apply directly the procedure of ref. [50] to our modified k -essence. The relevant Einstein equations at linear order are given by

$$a^{-2}\Delta\Phi - 3H\dot{\Phi} + 3H^2\Phi = 4\pi G_N\delta\tilde{T}_0^0, \quad (\text{B.10})$$

$$(\dot{\Phi} + H\Phi)_{,i} = 4\pi G_N\delta\tilde{T}_i^0, \quad (\text{B.11})$$

where the perturbations of the stress tensor components $\delta\tilde{T}_\nu^\mu$ are induced by the perturbations of the scalar field $\theta(t, x) = \theta(t) + \delta\theta(t, x)$, Using the energy conservation (A.17) and the definition (1.2) of X one finds

$$\delta\tilde{T}_0^0 = \frac{\tilde{p} + \tilde{\rho}}{\tilde{c}_s^2} \left[\left(\frac{\delta\theta}{\dot{\theta}} \right)^\cdot - \Phi \right] - 3H(\tilde{p} + \tilde{\rho}) \frac{\delta\theta}{\dot{\theta}}, \quad (\text{B.12})$$

$$\delta\tilde{T}_i^0 = (\tilde{p} + \tilde{\rho}) \left(\frac{\delta\theta}{\dot{\theta}} \right)_{,i}, \quad (\text{B.13})$$

where the quantity \tilde{c}_s is the adiabatic speed of sound defined by (4.3). Using (B.12) and (B.13) equations (B.10) and (B.11) take the form

$$\left(\frac{\delta\theta}{\dot{\theta}} \right)^\cdot = \Phi + \frac{\tilde{c}_s^2}{4\pi G_N a^2 (\tilde{p} + \tilde{\rho})} \Delta\Phi, \quad (\text{B.14})$$

$$(a\Phi)^\cdot = 4\pi G_N a (\tilde{p} + \tilde{\rho}) \frac{\delta\theta}{\dot{\theta}}. \quad (\text{B.15})$$

So far we have merely applied the formalism of [50] in which we have only used the energy conservation with no need to use the modified Friedmann cosmology so equations (B.14)

and (B.15) coincide with those derived in [50]. However, from now on we invoke the modified Friedmann dynamics encoded in equations (2.2) and (2.3). That means, in particular, that all background variables, such as a , H , and \dot{H} are obtained by solving equations (2.10) and (2.11) with (2.2), and, as may be easily shown, $\tilde{c}_s = c_s$. As in ref. [50], we introduce new functions

$$\xi = \frac{a\Phi}{4\pi GH}, \quad \zeta = \Phi + H \frac{\delta\theta}{\dot{\theta}}. \quad (\text{B.16})$$

The quantity ζ is gauge invariant and measures the spatial curvature of comoving (or constant- θ) hyper-surfaces. During slow-roll inflation ζ is equal to the curvature perturbation on uniform-density hyper-surfaces [36]. Substituting the definitions (B.16) into (B.14) and (B.15) and using (2.3) we find

$$\dot{\xi} = a \frac{p + \rho}{H^2} \zeta - \frac{h\dot{h}}{2} \xi, \quad (\text{B.17})$$

$$\dot{\zeta} = \frac{c_s^2 H^2}{a^3(p + \rho)} \Delta \xi + \frac{\dot{h}}{2} \left(\zeta - \frac{4\pi G_N}{a} H \xi \right). \quad (\text{B.18})$$

where $h = H\ell$. Compared with the standard equations of Garriga and Mukhanov [50], equations (B.17) and (B.18) have additional terms on the righthand sides proportional to h as a consequence of the modified Friedmann equations of the holographic cosmology.

In principle, one can find solutions to these equations numerically. However, for the sake of comparison with previous calculations in other models, we prefer to look for approximate solutions in the slow roll regime. We now show that in this regime the additional terms in (B.17) and (B.18) are suppressed with respect to the standard terms by a factor $\varepsilon_1 \equiv -\dot{H}/H^2$. With hindsight, we approximate time derivatives by $|\dot{\xi}| \approx H|\xi|$ and $|\dot{\zeta}| \approx H|\zeta|$ and check the validity of this approximation *a posteriori*. With this we immediately see that the magnitude of the last term on the right-hand side of (B.17) is smaller than the magnitude of the left-hand side by a factor $\varepsilon_1 h^2/2$. Then, neglecting the last term we find an approximate relation

$$H|\xi| \approx \frac{a(p + \rho)}{H^2} |\zeta|. \quad (\text{B.19})$$

Using this we find that the magnitude of the second term on the right-hand side of (B.18) is of the order $\varepsilon_1 h^2 H |\zeta|/2$ and is suppressed with respect to the left-hand side the magnitude of which is of the order $H|\zeta|$. For a consistency check we can use another relation $|\Delta \xi| \approx q^2 |\xi| \approx H^2 a^2 c_s^{-2} |\xi|$ approximately valid at the sound horizon crossing. Using this we find that the magnitude of the first term on the right-hand side is of the order $H|\zeta|$ and it dominates the second term and is comparable with the left hand side of (B.18).

By neglecting the sub-dominant terms and keeping the leading order in ε_1 , equations (B.17) and (B.18) can be conveniently expressed as

$$\dot{\xi} = z^2 c_s^2 \zeta, \quad (\text{B.20})$$

$$\dot{\zeta} = z^{-2} \Delta \xi, \quad (\text{B.21})$$

where

$$z = \frac{a(p + \rho)^{1/2}}{c_s H} = \frac{a}{c_s} \sqrt{\frac{\varepsilon_1}{4\pi G_N} \left(1 - \frac{h^2}{2} \right)}. \quad (\text{B.22})$$

Hence, in our approximation we basically neglect the contribution of the conformal fluid in the perturbations and the modified Friedman dynamics is reflected in a modified definition of the quantity z .

By introducing the conformal time $\tau = \int dt/a$ and a new variable $v = z\zeta$, it is straightforward to show from equations (B.20) and (B.21) that v satisfies a second order differential equation

$$v'' - c_s^2 \Delta v - \frac{z''}{z} v = 0. \quad (\text{B.23})$$

By making use of the Fourier transformation

$$v(\tau, \mathbf{x}) = \frac{1}{(2\pi)^3} \int d^3 q e^{i\mathbf{q}\mathbf{x}} v_q(\tau) \quad (\text{B.24})$$

we also obtain the mode-function equation

$$v_q'' + \left(c_s^2 q^2 - \frac{z''}{z} \right) v_q = 0. \quad (\text{B.25})$$

As we are looking for a solution to this equation in the slow-roll regime, it is useful to express the quantity z''/z in terms of the slow-roll parameters ε_i . In the slow-roll regime one can use the relation

$$\tau = -\frac{1 + \varepsilon_1}{aH} + \mathcal{O}(\varepsilon_i), \quad (\text{B.26})$$

which follows from the definition of ε_1 (3.7) expressed in terms of the conformal time. Using this and (B.22) we obtain at linear order in ε_i

$$\frac{z''}{z} = \frac{\nu^2 - 1/4}{\tau^2}, \quad (\text{B.27})$$

where

$$\nu^2 = \frac{9}{4} + \frac{3}{2} \left(2 + \frac{h^2}{2 - h^2} \right) \varepsilon_1 + \frac{3}{2} \varepsilon_2. \quad (\text{B.28})$$

We look for a solution to (B.25) which satisfies the positive frequency asymptotic limit

$$\lim_{\tau \rightarrow -\infty} v_q = \frac{e^{-ic_s q \tau}}{\sqrt{2c_s q}}. \quad (\text{B.29})$$

Then the solution which up to a phase agrees with (B.29) is

$$v_q = \frac{\sqrt{\pi}}{2} (-\tau)^{1/2} H_\nu^{(1)}(-c_s q \tau), \quad (\text{B.30})$$

where $H_\nu^{(1)}$ is the Hankel function of the first kind of rank ν .

In the limit of the de Sitter background all ε_i vanish so $\nu = 3/2$ in which case the solution to (B.25) with (B.27) is given by

$$v_q = \frac{e^{-ic_s q \tau}}{\sqrt{2c_s q}} \left(1 - \frac{i}{c_s q \tau} \right). \quad (\text{B.31})$$

Now, we can use this as an approximate solution in the slow-roll regime to check the validity of our estimate which led to eqs. (B.20) and (B.21). We set $\zeta \approx v_q/z$ at the sound horizon crossing, i.e., we take $c_s q = aH$. Using eq. (B.26) we find

$$\dot{\zeta} \approx \left(\frac{v_q}{z}\right) \cdot \approx \left(\alpha H - \frac{\dot{z}}{z}\right) \zeta, \quad (\text{B.32})$$

where α is a complex constant with magnitude of order 1. From (B.22) it follows $\dot{z}/z = H + \mathcal{O}(\varepsilon_i^2)$ so $|\dot{\zeta}| \approx H|\zeta|$ in accord with our previously assumed relation. Then, the relation $|\dot{\xi}| \approx H|\xi|$ also follows by virtue of eqs. (B.19) and (B.20).

Next, consider the action for a scalar field v

$$S[v] = \frac{1}{2} \int d\tau d^3x \left(v'^2 - c_s^2 (\nabla v)^2 + \frac{z''}{z} v^2 \right). \quad (\text{B.33})$$

The variation of this action obviously yields (B.23) as the equation of motion for v . Applying the standard canonical quantization [51] the field v_q is promoted to an operator

$$\hat{v}_q = v_q \hat{a}_{\mathbf{q}} + v_{-q}^* \hat{a}_{-\mathbf{q}}^\dagger, \quad (\text{B.34})$$

where the operators $\hat{a}_{\mathbf{q}}$ and $\hat{a}_{\mathbf{q}}^\dagger$ satisfy the canonical commutation relation

$$[\hat{a}_{\mathbf{q}}, \hat{a}_{\mathbf{q}'}^\dagger] = (2\pi)^3 \delta(\mathbf{q} - \mathbf{q}'). \quad (\text{B.35})$$

Then, the power spectrum of the field $\zeta_q = v_q/z$ is obtained from the two-point correlation function

$$\langle \hat{\zeta}_q \hat{\zeta}_{q'} \rangle = \langle \hat{v}_q \hat{v}_{q'} \rangle / z^2 = (2\pi)^3 \delta(\mathbf{q} + \mathbf{q}') |\zeta_q|^2. \quad (\text{B.36})$$

The dimensionless spectral density

$$\mathcal{P}_S(q) = \frac{q^3}{2\pi^2} |\zeta_q|^2 = \frac{q^3}{2\pi^2 z^2} |v_q|^2, \quad (\text{B.37})$$

with v_q given by (B.30), characterizes the primordial scalar fluctuations. The difference with respect to the standard expression is basically in a modified definition of z and in a modification of v_q owing to a new expression (B.28) for the rank ν of the Hankel function.

B.2 Tensor perturbations

The tensor perturbations are related to the production of gravitational waves during inflation. The metric perturbations are defined as

$$ds^2 = dt^2 - a^2(t) (\delta_{ij} + h_{ij}) dx^i dx^j, \quad (\text{B.38})$$

where h_{ij} is traceless and transverse. In the absence of anisotropic stress the gravitational waves are decoupled from matter and the relevant Einstein equations at linear order are

$$h''_{ij} + 2aHh'_{ij} - \Delta h_{ij} = 0. \quad (\text{B.39})$$

To solve this one uses the standard Fourier decomposition

$$h_{ij}(\tau, \mathbf{x}) = \frac{1}{(2\pi)^3} \int d^3q e^{i\mathbf{q}\mathbf{x}} \sum_s h_q^s(\tau) e_{ij}^s(q), \quad (\text{B.40})$$

where the polarization tensor $e_{ij}^s(q)$ satisfies $q^i e_{ij}^s = 0$, and $e_{ij}^s e_{ij}^{s'} = 2\delta_{ss'}$ with comoving wave number q and two polarizations $s = +, \times$. The amplitude $h_q^s(t)$ then satisfies

$$h_q'' + 2aHh_q' + q^2 h_q = 0, \quad (\text{B.41})$$

where we have suppressed the dependence on s for simplicity and bear in mind that we have to sum over two polarizations in the final expression. As before, we introduce a new, canonically normalized variable

$$v_q = \frac{a}{16\pi G_N} h_q \quad (\text{B.42})$$

which satisfies the equation

$$v_q'' + \left(q^2 - \frac{a''}{a} \right) v_q = 0. \quad (\text{B.43})$$

This equation is of the same form as (B.25) with $c_s = 1$ and z replaced by a . Then, the properly normalized solution v_q is given by

$$v_q = \frac{\sqrt{\pi}}{2} (-\tau)^{1/2} H_\nu^{(1)}(-q\tau), \quad (\text{B.44})$$

with $\nu^2 = 9/4 + 3\varepsilon_1$. The quantization proceeds in a similar way as in the scalar case and the power spectrum of the field $h_q = (16\pi G_N/a)v_q$ is obtained from the two-point correlation function

$$\langle \hat{h}_q \hat{h}_{q'} \rangle = \langle \hat{v}_q \hat{v}_{q'} \rangle \frac{(16\pi G_N)^2}{a^2} = (2\pi)^3 \delta(\mathbf{q} + \mathbf{q}') |h_q|^2. \quad (\text{B.45})$$

The dimensionless spectral density which characterizes the primordial tensor fluctuations is given by

$$\mathcal{P}_T(q) = \frac{q^3}{\pi^2} |h_q|^2 = \frac{q^3}{\pi^2} \left| \frac{16\pi G_N}{a} v_q \right|^2, \quad (\text{B.46})$$

with v_q given by (B.44).

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