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Compact objects in unimodular gravity

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Abstract. Unimodular gravity provides a theoretical framework that allows for non-conservation of energy-momentum, with possible implications for the cosmological constant problem. It is then important to study the predictions of unimodular gravity in other gravitational regimes. In this work we study stellar dynamics under the assumption of non-conserved energy-momentum. We find that constant density objects can be as compact as Schwarzschild black holes. For polytropic objects, we find modifications due to the non-conservation of energy-momentum that lead to sizeable effects that could be constrained with observational data. Additionally, we revisit and clarify the Reissner-Nordström solution in unimodular gravity. We also study gravitational collapse and discuss possible implications for the growth of structure.

Keywords: dark energy theory, massive stars, modified gravity, neutron stars

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Contents

1 Introduction

Stellar dynamics is an outstanding laboratory to explore the predictions of general relativity (GR) and alternative theories in the strong field regime. A variety of systems, such as dwarf stars, neutron stars, or even collapsed objects like black holes offer observational windows to constrain the theory of gravity. This has led to the study of astrophysical systems in several alternatives to GR, for example, $f(R)$ theories [1, 2], brane-world models [3–8], non-commutative theories [9, 10], scalar and vector-tensor theories [11–19] etc. In particular, it is important to study how the equilibrium equations are modified and what are the consequences for the magnitudes that characterize compact systems, such as the mass, radius, and compactness. For simplicity these studies often begin with constant density objects, but it is also important to use more realistic equations of state and identify degeneracies between modifications due to the theory of gravity and those due to the equation of state. On the other hand, the long-standing cosmological constant problem [20, 21] continues to be a motivation for proposing alternative theories of gravity. One of these alternatives, known as Unimodular Gravity (UG) [22–24], attempts to relax this problem by a mechanism that makes vacuum energy non-gravitating and attributes the observed cosmological constant to an arbitrary integration constant of the theory, helping to evade the gap between the theoretical and observational estimates of the cosmological constant. This form of UG works under the assumption that the energy-momentum tensor is covariantly conserved, and does not offer physical insights on the nature of the cosmological constant: its classical field equations are the same as in GR [25–29]. The study of cosmological perturbations requires some care in the gauge choice, for instance, now the gauge symmetry consists of transverse diffeomorphisms only, but it has been shown that in gauge invariant quantities the equivalence between GR and UG holds [28]. At the quantum level, this equivalence is not yet clear. Differences have been argued to arise, with implications for the hierarchy and radiative stability of the cosmological constant [25, 30–32], but it has also been suggested that the equivalence depends on the details of the classical starting point — whether the determinant of the metric is explicitly constrained or not — as well as on the quantization procedure [33, 34].

As mentioned above, the conservation of the energy-momentum tensor is not automatic in UG but is introduced as an additional assumption. Recently, the possibility to discard this assumption has received some attention in the literature, partially motivated by novel ideas proposed by Perez et al. [35, 36] that do offer an insight on the nature of the cosmological constant by allowing for a non-conserved energy-momentum tensor, a feature that can be incorporated in UG. Further, exhaustive studies on the cosmological implications of this version of UG have been reported in [37] and [38].

Given this scenario, it is important to put UG to test also in extreme gravitational laboratories, such as stellar dynamics, with the assumption that energy-momentum is non-conserved, thus stopping the classical theory from automatically reducing to GR. In this work we study static, spherically symmetric solutions to the UG field equations subject to an additional condition that is required in order to close the system of equations, since in UG the number of independent field equations is reduced by one due to the trace-free property of the field equations. We exploit this additional condition in two ways, first to simplify the system of equations and obtain some analytical results, and then to parameterize the type of non-conservation of energy-momentum in our numerical results and contrast with GR predictions.

This paper is organized as follows: in section 2, we briefly review the theoretical framework of UG. Section 3 is dedicated to the study of static, spherically symmetric solutions. We begin by exploring the relation between solutions for metrics that satisfy explicitly the unimodular condition — constant metric determinant — and solutions that do not satisfy this condition, we justify that the systems that we explore in this work can be analyzed in either coordinate chart. We then study an analytic solution of UG through the imposition of a simplifying ansatz that allows us to obtain the Tolman-Oppenheimer-Volkoff (TOV) equation, which we use to analyze some properties of a star in this conditions. In section 5 we study constant density configurations under an assumption for the non-conservation of energy-momentum; we perform a numerical analysis and explore the behaviour of our solutions near the Buchdahl limit of GR. In section 6 we extend the previous numerical analysis to account for stars described by a polytropic equation of state (EoS), obtaining modifications that could provide constraints on the non-conservation of energy-momentum. Finally, in section 7 we study gravitational collapse, showing that collapse times get modified, and we discuss some consequences for black hole and structure formation in our Universe. Section 8 is devoted to conclusions and perspectives for the study of stellar dynamics in UG. In addition, we include several appendices with details of our calculations and assumptions: we discuss the equivalence between unimodular and FLRW-like metrics; we give a formal justification for the simplifying assumptions used in section 3, we provide details of the equations of motion for constant density objects, we discuss how different assumptions for the non-

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conservation of energy-momentum affect our results, and we revisit the Reissner-Nordström solution, clarifying misleading results reported in previous studies.

2 Unimodular gravity: Lagrangian and equations of motion

Unimodular gravity can be described by the action

$$S = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa}R + \mathcal{L}_{\rm matter}\right), \tag{2.1}$$

where $\kappa = 8\pi G$ with G the Newtonian gravitational constant. Importantly, the metric determinant in (2.1) is restricted to satisfy the *unimodular condition* $\sqrt{-g} = \epsilon_0$, where ϵ_0 is a fixed scalar density usually set to unity, i.e $\sqrt{-g} = 1$. The unimodular action can then be written as

$$S = \int d^4x\epsilon_0\left(\frac{1}{2\kappa}R + \mathcal{L}_{\rm matter}\right), \tag{2.2}$$

where all the tensors in the action are constructed with a metric that satisfies the unimodular condition. After some manipulations — transparent when the unimodular condition is incorporated into the action by means of a Lagrange multiplier — the equations of motion result in

$$\xi_{\mu\nu} := G_{\mu\nu} - \kappa T_{\mu\nu} = -\frac{1}{4}g_{\mu\nu}(R + \kappa T), \tag{2.3}$$

where $T_{\mu\nu}$ is the standard energy-momentum tensor

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_{\rm matter})}{\delta g^{\mu\nu}}. \tag{2.4}$$

Some confusion might arise here since, strictly, the energy-momentum tensor in UG should be defined like in (2.4) but without the metric determinants. However, it can be shown that the combination $T_{\mu\nu} - g_{\mu\nu}T/4$ which appears in the equations of motion is independent of what definition of the energy-momentum tensor is used [25]. In contrast to General Relativity, where the trace of the equations of motion gives $R = -\kappa T$, here the equations of motion are trace-free, and the differences with respect to GR are indeed parameterized by $R + \kappa T$. Furthermore, while in GR the Bianchi identities $\nabla^\mu G_{\mu\nu} = 0$ enforce the covariant conservation of the energy-momentum tensor, i.e. $\nabla^\mu T_{\mu\nu} = 0$, in unimodular gravity there is the possibility to have non-conserved energy-momentum tensors since the conservation that must be satisfied is

$$\nabla^\mu\left(\kappa T_{\mu\nu} - \frac{\kappa}{4}g_{\mu\nu}T - \frac{1}{4}g_{\mu\nu}R\right) = 0. \tag{2.5}$$

If $\nabla^\mu T_{\mu\nu} = 0$ is assumed, then the above equation implies $\partial_\nu(\kappa T + R) = 0$, so that we can write $\kappa T + R = -4\Lambda$ for some integration constant Λ. Plugging this back in the equations of motion of unimodular gravity we get $G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu}$. Then, assuming conservation of the energy-momentum tensor, the equations of motion of GR with a cosmological constant are recovered.

In the formulation of UG described above the unimodular condition is reflected in the trace-free property of the equations of motion. For our purposes this formulation is enough, since what we actually exploit in this work is the fact that this condition reduces the number of independent equations of motion, but it is worth mentioning that there are other

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approaches in the literature, for instance, the field equations of UG can be derived from a fully diffeomorphism invariant action [39].

Rather than assuming $\nabla_\mu T^\mu{}_\nu = 0$, which automatically leads to the usual Einstein field equations, in the next sections we work with (2.3) and look for configurations where the energy-momentum tensor is not conserved. Physical motivations for this possibility have been presented in [40].

3 Static, spherically symmetric solutions

Before we start, it is important to mention that the unimodular condition (hereafter, we consider the unimodular condition as $\sqrt{-g} = 1$ where $\epsilon_0 = 1$) is not the most formal way to define unimodular gravity: what is really relevant is that the equations of motion are obtained by considering an invariant volume form. A volume form is coordinate independent, while $\sqrt{-g} = 1$ is not. The physical consequence of the restricted variation considered in unimodular gravity is the fact that the equations of motion are trace-free. At the level of the equations of motion we can impose any ansatz for the metric; furthermore, at least locally, any metric can be rewritten in a form that satisfies $\sqrt{-g} = 1$.

Nevertheless, for the sake of completeness and clarity, here we review some static, spherically symmetric solutions both in unimodular coordinates (i.e., coordinates where the unimodular condition is satisfied explicitly) and in standard spherically symmetric coordinates. In addition, in appendix A we demonstrate the equivalence between a FLRW metric[1] in its standard form and in a form that fulfills the unimodular condition, showing that it is irrelevant which metric we are using and the physical results are the same in each system. Another important point that it is necessary to remark is that UG coordinates does not have advantages from a numerical point of view[2] and thus, the following analysis has only the goal to underscore the connection between both coordinates system. Let us begin our discussion in unimodular coordinates.

3.1 Static, spherically symmetric solutions of GR in unimodular coordinates

Given a metric of the form

$$ds^2 = -f(r)dt^2 + h(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\,, \tag{3.1}$$

we can perform a coordinate transformation $dr = \sqrt{h(r)/(r^4 f(r))}dy$, $x = \cos\theta$, such that the metric rewrites as.

$$ds^2 = -f(y)dt^2 + \frac{dy^2}{r(y)^4 f(y)} + \frac{r(y)^2 dx^2}{1-x^2} + r(y)^2(1-x^2)d\varphi^2\,, \tag{3.2}$$

as long as the radial coordinate can be expressed in terms of y. For example, for a solution with $f(r) = h(r)$ the coordinate change simplifies to $r = (3y)^{1/3}$. Using this, the Schwarzschild solution in unimodular coordinates reads (see [41] for a more complete study of this solution in UG)

$$\begin{aligned} ds^2 &= -\left(1 - \frac{2M}{(3y)^{1/3}}\right)dt^2 + \left(1 - \frac{2M}{(3y)^{1/3}}\right)^{-1}\frac{dy^2}{(3y)^{4/3}} \\ &\quad + \frac{(3y)^{2/3}dx^2}{1-x^2} + (3y)^{2/3}(1-x^2)d\varphi^2\,. \end{aligned} \tag{3.3}$$

[1] We use for simplicity the FLRW line element in order to illustrate the equivalence between metrics, due to the integrability in this particular case.

[2] Unless it helps to diagonalize the metric, which is not the case with this paper.

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A similar procedure can be applied to (anti)-de Sitter and Reissner-Nordström metrics (the latter is analyzed in appendix D). When $f \neq h$, expressing r in terms of y becomes more complicated, for example, for a constant density TOV solution to the Einstein-Hilbert equations with matter described by $T^0_0 = -\rho_0, T^i{}_i = p(r)$ and all other elements vanishing, the metric can be written in the spherical coordinates (3.1) with

$$\begin{aligned} f(r) &= \frac{\rho_0^2}{(\rho_0 + p(r))^2}\,, \\ h(r) &= 1 - \frac{1}{3} r^2 \kappa \rho_0\,, \\ p(r) &= \rho_0 \frac{\sqrt{R_s^2 \kappa \rho_0 - 3} - \sqrt{r^2 \kappa \rho_0 - 3}}{\sqrt{r^2 \kappa \rho_0 - 3} - 3\sqrt{R_s^2 \kappa \rho_0 - 3}}\,, \end{aligned} \tag{3.4}$$

where the constant R_s is the radius of the compact object, defined by the vanishing of $p(r)$. The change of coordinates requires us to integrate

$$dy = -\frac{\sqrt{3} r^2 \left(\sqrt{3 - r^2 \kappa \rho_0} - 3\sqrt{3 - \kappa {R_s}^2 \rho_0}\right)}{2\sqrt{(3 - r^2 \kappa \rho_0)\,(3 - \kappa {R_s}^2 \rho_0)}} dr\,,$$

and then solve for r as a function of y. In a small ρ_0 approximation (formally defined by introducing a small, dimensionless parameter ϵ such that $\rho_0 \to \epsilon \rho_0$), we have to solve

$$\frac{r^3}{3} + \frac{1}{180} r^3 \kappa \left(9r^2 - 5{R_s}^2\right) \rho_0 + \frac{r^3 \kappa^2 \left(9r^4 - 7{R_s}^4\right) \rho_0^2}{1008} + \frac{5 r^3 \kappa^3 \left(r^6 - {R_s}^6\right) \rho_0^3}{2592} + \mathcal{O}(\rho^4) = y\,. \tag{3.5}$$

Since y is a new coordinate it does not depend on ρ_0, so we can take $r = r_0(y) + \rho_0 r_1(y) + \rho_0 r_2(y) + \ldots$. Proceeding in this way we find a perturbative solution to arbitrary order in ρ_0. The first few terms read

$$\begin{aligned} r_0(y) &= (3y)^{1/3}\,, \\ r_1(y) &= \frac{1}{180}\left(5\; 3^{1/3} y^{1/3} \kappa {R_s}^2 - 27 y \kappa\right)\,, \\ r_2(y) &= \frac{9\; 3^{2/3} y^{5/3} \kappa^2}{2800} - \frac{1}{40} y \kappa^2 {R_s}^2 + \frac{11 y^{1/3} \kappa^2 {R_s}^4}{432\; 3^{2/3}}\,. \end{aligned} \tag{3.6}$$

Notice that $r_0(y)$ coincides with the coordinate transformation of a vacuum solution. We can now write down the unimodular form of the TOV metric for small, constant density objects in GR:

$$\begin{aligned} f(y) &\approx 1 + \frac{1}{6}\left(3^{2/3} y^{2/3} \kappa - \kappa {R_s}^2\right) \rho_0 + \frac{\left(27\; 3^{1/3} y^{4/3} \kappa^2 + 50\; 3^{2/3} y^{2/3} \kappa^2 {R_s}^2 - 75 \kappa^2 {R_s}^4\right) \rho_0^2}{2160} + \ldots \\ p(y) &\approx \frac{1}{12} \kappa \left({R_s}^2 - 3^{2/3} y^{2/3}\right) \rho_0^2 + \frac{\kappa^2 \left(27\; 3^{1/3} y^{4/3} - 35\; 3^{2/3} y^{2/3} {R_s}^2 + 30 {R_s}^4\right) \rho_0^3}{1080} + \ldots\,. \end{aligned} \tag{3.7}$$

The radius of the star is given by $y(R_s)$ and can be obtained from (3.5). The previous results verify that both Schwarzschild and the TOV metric of a constant density object can be expressed in unimodular coordinates. Given that Schwarzschild is a vacuum solution and that the energy-momentum tensor of TOV satisfies $\nabla^\mu T_{\mu\nu} = 0$, they have to be also solutions of unimodular gravity, although not the most general ones since in UG with $\nabla^\mu T_{\mu\nu} = 0$ there is an additional integration constant that embeds these solutions in (anti-)de Sitter spacetime. Now that we are convinced that a change of coordinates from standard to unimodular coordinates exists also for solutions in presence of matter, we continue our study of static, spherically symmetric solutions of unimodular gravity in standard coordinates, allowing for a non-conserved energy-momentum tensor.

3.2 Static, spherically symmetric solutions of UG

In order to study stellar dynamics, we assume that the geometry is described by the spherically symmetric line element given in eq. (3.1), and we consider a matter sector characterized by a perfect fluid whose energy-momentum tensor is expressed in the form

$$T_{\mu\nu} = \rho u_\mu u_\nu + p(g_{\mu\nu} + u_\mu u_\nu), \tag{3.8}$$

where $p = p(r)$ and $\rho = \rho(r)$ are, respectively, the pressure and density of the stellar matter of interest, u_μ is the fluid four-velocity, which satisfies the condition $g_{\mu\nu}u^\mu u^\nu = -1$, and $g_{\mu\nu} + u_\mu u_\nu$ is orthogonal to u_μ. In the following, we derive analytic expressions for the masss and gravitational energy of gravitationally bound objects under some assumptions that allow for analytical progress.

3.2.1 Analytic solution with a particular ansatz

This exercise is aimed to obtain an analytic solution of the UG field equations in order to gain some insight on the physics that happens in this context, and later on extend this knowledge to the numerical solutions. For this task, we assume that either $R_{tt} = 0$, $R_{rr} = 0$ or $R_{\theta\theta} = 0$. This ansatz simplifies the field equations and allows us to obtain analytic results (see appendix B for a formal justification of this ansatz choice).

The field equations give us the form of $h(r)$ as

$$h(r) = 1 - \frac{2G\mathcal{M}_{UG}(r)}{r}\,, \tag{3.9}$$

where we define

$$\mathcal{M}(r)_{UG} \equiv \mathcal{C}\int_0^r 4\pi r'^2(p+\rho)dr'\,, \tag{3.10}$$

with $\mathcal{C}$ a constant that depends on which component of the Ricci tensor is set to zero: $\mathcal{C} = 3/2$ for $R_{tt} = 0$ and $\mathcal{C} = 1/2$ both for $R_{rr} = 0$ and $R_{\theta\theta} = 0$. After some manipulations that combine the field equations with eq. (2.5), we get

$$\frac{f(r)'}{f(r)} = \pm\frac{p(r)' + \rho(r)'}{p(r) + \rho(r)}\,, \tag{3.11}$$

and with this we can arrive to an equation that contains only the mass function, the pressure, and the density of matter, this is the modified TOV equation in UG under the assumptions mentioned above:

$$-r^2(p' + \rho') = G\mathcal{M}_{UG}\rho\left[1 + \frac{p}{\rho}\right]\left[\pm\mathcal{S}\frac{4\pi r^3(p+\rho)}{\mathcal{M}_{UG}} \mp 2\right]\left[1 - \frac{2G\mathcal{M}_{UG}}{r}\right]^{-1}\,, \tag{3.12}$$

where $\mathcal{S} = 1$ for $R_{tt} = 0$ and $\mathcal{S} = 3$ both for $R_{rr} = 0$ and $R_{\theta\theta} = 0$. One concern is that the TOV equation found in this approach is not continuously connected to the Newtonian equation found in the weak field limit of GR. This is caused by the ansatz imposed to integrate

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and obtain the analytic solution: by choosing these ansatz we are imposing a non-infinitesimal deviation from GR. It is interesting that the system admits solutions under these conditions and this could lead to observable effects useful to constrain the model.

Integrating eq. (3.11) and using the modified TOV equation, we obtain the g_{tt} component of the metric,

$$f(r) = \exp\left\{\pm\int_r^\infty \frac{2G}{r'^2}[\mp\mathcal{M}_{UG} \pm 2\pi\mathcal{S}r'^3(\rho+p)]\left[1-\frac{2G\mathcal{M}_{UG}}{r'}\right]^{-1}\right\}dr', \tag{3.13}$$

where we consider the boundary condition $f(\infty) = 1$ in order to obtain an asymptotic Minkowski space-time. The upper signs corresponds to $R_{tt} = 0$ and the lower signs to the other two cases. Outside the configuration of matter the pressure and density vanish and the Schwarzschild solution is recovered, this is possible since our ansatz is automatically satisfied for Schwarzschild. As a complement, we compute the redshift of spectral lines from the surface of the star as

$$z+1 = \left(1-\frac{2M_{UG}G}{R}\right)^{1/2}, \tag{3.14}$$

where $M_{UG} \equiv \mathcal{M}(R)_{UG}$, given by eq. (3.10), existing a substantial change in comparison with the standard result due to the presence of p and $\mathcal{C}$ in the previous equations. On the other hand, in similarity with GR, we expect that the number of nucleon in the star can be written as [42]

$$N = \int_0^R 4\pi r^2\left[1-\frac{2G\mathcal{M}_{UG}(r)}{r}\right]^{-1/2} n(r)dr, \tag{3.15}$$

being $n(r)$ the proper number density. In addition, the internal energy of the star is given by $E \equiv M - m_N N$, where $m_N = 1.66\times 10^{24}g$ is the rest mass of a nucleon. If we now assume a proper internal material energy density[3] $e(r) \equiv (\rho(r)+p(r)) - m_N n(r)$, we have $E = T + V$, where

$$T = \int_0^R 4\pi r^2\left\{1+\frac{G\mathcal{M}(r)_{UG}}{r}+\dots\right\}e(r)dr, \tag{3.16}$$

$$V = -\int_0^R 4\pi r^2\left\{(1-\mathcal{C})+\frac{G\mathcal{M}(r)_{UG}}{r}+\frac{2}{3}\left(\frac{G\mathcal{M}(r)_{UG}}{r}\right)^2+\dots\right\}(\rho(r)+p(r))dr, \tag{3.17}$$

where T and V are the thermal and gravitational energies in UG, respectively. In order to compare eq. (3.17) with the standard gravitational energy, we propose the following dimensionless variables

$$\tilde{V} = V/M,\ \ \bar{\rho} = \rho/\rho_{\text{eff}},\ \ \bar{p} = p/\rho_{\text{eff}},\ \ \text{and}\ \ x = \sqrt{GM/R}(r/R), \tag{3.18}$$

with $\rho_{\text{eff}} = 3M/4\pi R^3$, M the standard GR mass, and assuming the case where $p \ll \rho$ for a constant ρ. Therefore we finally have

$$\tilde{V} \approx -\bar{\rho}(1-\mathcal{C}) - \frac{3}{5}\bar{\rho}^2 C^m_{\text{comp}}, \tag{3.19}$$

[3]The form of $e(r)$ is inspired by the structure of the UG mass given in eq. (3.10), which depends both on p and ρ, in contrast with the mass and $e(r)$ for GR which depend only on $\rho(r)$ (see [42] for details in the GR case).

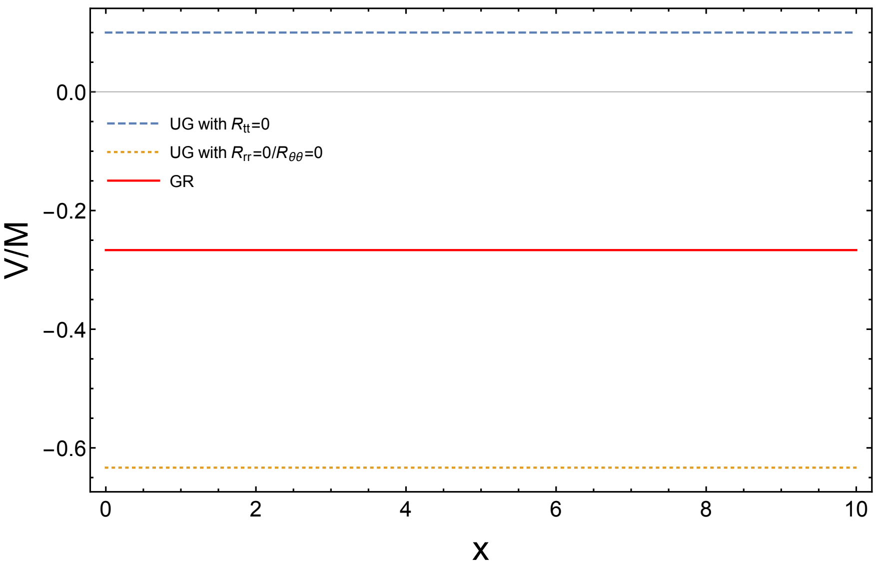

Figure 1. Behavior of gravitational energy described by eq. (3.19), for the two ansatz imposed and a comparison with the standard behavior of GR. As a initial conditions, we propose $\bar{\rho} = 1$ and $p \ll \rho$.

where $C^m_{\rm comp} \equiv GM\mathcal{C}/R$ is the stellar compactness modified by UG together with the ansatz made in this section for the Ricci tensor. Eq. (3.19) is to be compared to the dimensionless gravitational energy in GR, $\tilde{V} = -(3/5)\bar{\rho}^2 C_{\rm comp}$, which is shown in figure 1 assuming $\bar{\rho} = 1$, notice that in this particular case the traditional GR behavior is recovered when $\mathcal{C} = 1$, therefore $C^m_{\rm comp} \to C_{\rm comp} = GM/R$. However, while in GR the constraint $GM/R < 4/9$ holds, it remains to be seen whether UG allows for higher values of the compactness, this is analyzed in the next section. Also, notice that in some cases UG predicts positive values for the gravitational energy, this suggests that the ansatz $R_{tt} = 0$ has no physical interpretations. For the other ansatz we found a lower gravitational energy than the one expected in GR. This result highlights the differences due to the UG modifications to the dynamical equation.

In the next section we present a full numerical study of compact objects in UG without assuming the ansatz of this section, but rather closing the system of equations with an assumption on the type of violation of energy-momentum tensor that allows for a continuous limit to GR.

4 On the choice of non-conservation of $T_{\mu\nu}$

The trace-free property of the equations of motion in UG reduces the number of independent equations of motion. In GR, when a matter Lagrangian and a spherically symmetric ansatz for the metric are considered, the metric equations of motion contain three independent equations — one of them equivalent to the Bianchi identities combined with the conservation of $T_{\mu\nu}$. Together with an EoS, these equations suffice to determine the four free functions of the system (two metric functions and the pressure and density of matter). Under the same considerations, in UG there are only two independent metric equations of motion, which together with the EoS can determine only three of the four free functions. Therefore we need an extra condition to close the system of equations. One possibility is to use this freedom to impose simplifying assumptions on the equations of motion, as in section 3.2.1. Another option is to impose a form of violation of $\nabla_\mu T^\mu_\nu = 0$. This has the advantage that we have

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under control the non-conservation of $T^{\mu}{}_{\nu}$ in the model. Also, we can parametrically recover the GR solutions when this non-conservation is small. But, what type of non-conservation should we choose? Most studies on UG actually impose conservation of $T_{\mu\nu}$ and take the "automatic" presence of a cosmological constant as an integration constant as the main characteristic of UG. However, this approach leads to the same dynamics as GR with cosmological constant [29]. On the other hand, a few mechanisms leading to non-conservation of energy momentum have been discussed in the literature. For instance, in [35] non-conservation originating from the interaction between a discrete space-time and matter at a microscopic level is seen as a friction-like force acting on massive particles, which macroscopically generates an accelerated expansion of the universe. Similar energy-momentum diffusion effects have been found in causal set theory [43, 44]. Another scenario where non-conserved energy-momentum arises is found in non-unitary modifications of quantum dynamics, such as the *Continuous Spontaneous Localization* (CSL) collapse model. Recently, NSs have been pointed out as competitive candidates to test this model [45]. Dissipative effects can also be motivated by standard physics of NSs. After they form, NSs undergo a rapid cooling phase driven by the Urca process, in which neutrinos escape from the star carrying energy away [46, 47]. After a few minutes, the Urca process is replaced by a much slower modified version that lasts up to a million years (see, e.g., the *Minimal cooling paradigm* [48]). Along the same lines, with exception of the CSL model, all the interpretations above lead to energy-momentum loss, and would therefore correspond to a negative k in our ansatz for non-conservation. A detailed incorporation of these effects in modified gravity is an interesting line of research that seems natural to pursue in UG, since the framework allows for a non-conserved $T_{\mu\nu}$, which is incompatible with GR. Given the limited number of works in this direction (see [37, 38] for a cosmological study), it seems better to start with the simplest case, $\nabla_\mu T^\mu_\nu = \delta^r_\nu k$, for a constant k.[4] This is the choice we make in section 5 for constant density objects. When studying objects described by a more realistic EoS it might also be important to consider less simple assumptions for the non-conservation of $T^{\mu}{}_{\nu}$. In section 6 we use $\nabla_\mu T^\mu_\nu = \delta^r_\nu k\rho(r)$ for two reasons: first, because it is a straightforward generalization of the constant violation used for constant density objects, and second, because in a preliminary study we found that a constant violation does not allow for objects with compactness higher than the compactness of GR solutions for the same density and equation of state. Thus, if UG with non-conservation of T^μ_ν is taken seriously, observations of highly compact objects could rule out a constant violation of energy-momentum conservation.

5 Constant density objects in UG

We explore solutions for objects with constant density in UG. This simple scenario is always a good starting point in the study of stellar dynamics from where some physical intuition can be drawn, even if we can only access the solutions numerically.

As stressed earlier, the system of equations obtained from variation of the action in UG is under-determined. In order to get a closed system, and at the same time to control the violations of conservation of $T_{\mu\nu}$, we supplement the set of equations with

$$\nabla_\mu T^{\mu}{}_{\nu} = \delta^r_\nu k \,, \tag{5.1}$$

[4]A derivation of the type of non-conservation of energy-momentum from microscopic physics, in the spirit of [36], is left for future work.

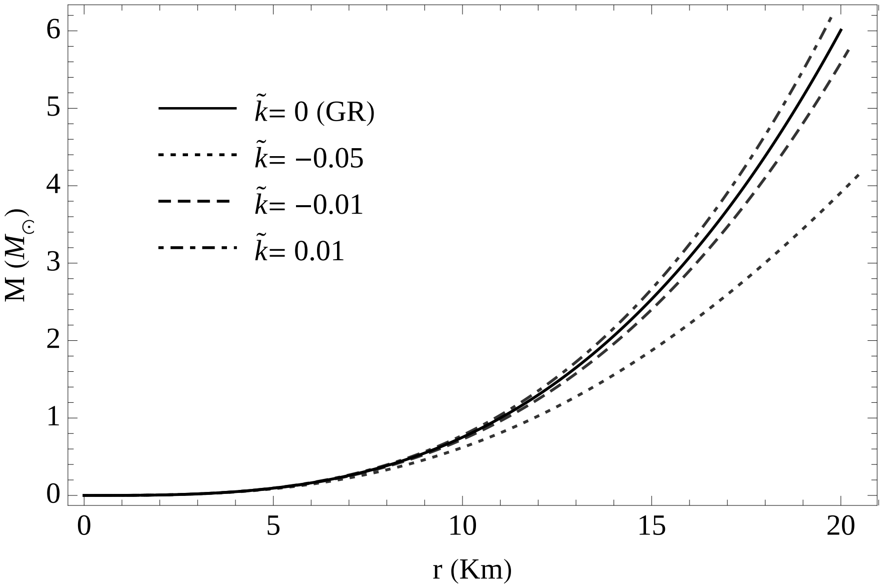

Figure 2. Mass as a function of radius for configurations with different amounts of non-conservation of $T_{\mu\nu}$. The dimensionless constant $\tilde{k}$ is defined by convenience as $k = c^2(1.66 \times 10^{14}\mathrm{gr/cm^3})(10^{-5}\mathrm{cm^{-1}})\,\tilde{k}$. The numerical factors are typical density and length scales of astrophysical compact objects.

for some constant k that parameterizes the non-conservation of the energy-momentum tensor of matter. Given the symmetries of our set-up, the time and angular components of $\nabla_\mu T^\mu{}_\nu$ vanish identically. Then, the fact that the non-conservation of energy-momentum happens only in the radial direction is not an assumption but a consequence of the symmetries of the system, what we are assuming is that this non-conservation is constant. All in all, we solve numerically ξ_{00}, ξ_{11}, and (5.1) for the functions $f(r), h(r)$ and $p(r)$. Initial conditions are set at a small radius r_i by Taylor expanding and solving these equations near $r = 0$. We find that these initial conditions depend on the constant density ρ_0, the central value of the pressure $p(0)$, the parameter k, and the second derivative of the pressure at the origin, $p''(0)$. The presence of $p''(0)$ in the initial conditions is a difference with respect to GR, and can be related to the additional integration constant of UG. In particular, one can check that the initial conditions for $k = 0$ only coincide with those of GR if $p''(0)$ takes the value dictated by the constant density solution of GR: in general, setting the additional integration constant of UG to zero means choosing $p''(0)$ in such a way that it coincides with its value in the GR solution, this is how we fix $p''(0)$. Another thing to note is that the first derivative of the pressure does not vanish at $r = 0$, indeed it is equal to k, this is a consequence of the type of violation of energy-momentum conservation that we impose. More details can be found in appendix C.

Figure 2 shows our results for the masses of constant density configurations with different values of k. The GR solution corresponds to $k = 0$ and it is shown in solid line. Notice that sizable changes in the mass occur without large modifications in the radius of the configuration. This has interesting consequences for the compactness of these objects, defined as the dimensionless ratio $C = G\mathcal{M}_{UG}(R_s)/R_s$, where $\mathcal{M}_{UG}(R_s)$ is the mass function defined in (3.9) evaluated at the radius of the star, R_s. The GR solution displayed in figure 2 has compactness $C \approx 0.4435$, just below the Buchdahl limit [49] $C = 4/9$ that comes from requiring the pressure to be finite at the center of the star. As can be inferred from the same figure, the solution with $k = 0.01$ has a larger compactness, and indeed we find $C_{0.01} = 0.4635$, where we introduced the notation C_k to indicate that C is computed for a configuration with a given value of k.

In view of these results, it is worth exploring the region $k > 0$ in more detail. To do so we parameterize the density in terms of the critical density in GR for uniform distributions of mass:

$$\rho_0 = a\frac{8}{3(R_s^{GR})^2\kappa} \equiv a\rho_{\rm crit}\,, \tag{5.2}$$

for a constant a. The initial conditions are thus determined in terms of a and R_s^{GR}, the latter can be fixed as the radius of the GR configuration by choosing appropriate values for $p(0)$,

$$p(0) = \frac{\rho_0\left(-\sqrt{3}+\sqrt{3-\kappa(R_s^{GR})^2\rho_0}\right)}{\sqrt{3}-3\sqrt{3-\kappa(R_s^{GR})^2\rho_0}}\,. \tag{5.3}$$

For $k \neq 0$ we do not have an explicit relation between the radius of the star and the initial conditions at $r = 0$, so even though we use R_s^{GR} in the initial condition for the pressure, the radius of the star is $R_s \neq R_s^{GR}$. As we explained above, $p''(0)$ is chosen in such a way that for $k = 0$ the GR solution is recovered, i.e., we set to zero the additional integration constant of UG. For given a and R_s^{GR}, the only free parameter in the initial conditions is k, and by the discussion after figure 2 we are interested in $k > 0$.

Figure 3 shows the compactness of configurations with $a = 0.850$ and $a = 0.998$, and values of k between $0.005 \leq k \leq 0.05$. These results confirm that solutions in unimodular gravity can go well beyond the Buchdahl limit $C = 4/9$. Interestingly, they approach asymptotically to the compactness of a Schwarzschild black hole, $C = 1/2$. For $a = 0.850$ and $k \geq 0.04$, instead of a smooth approach $p(r) \to 0$ as r approaches some value that would correspond to the radius of the compact object, we find $dp/dr \to -\infty$ at some finite radius, similar to the radius of configurations with lower k. It is not clear whether this is a numerical problem or a physical limit on the size of k. In any case, this limit is beyond the values of k that give us $C_k \approx 0.5$, so that we do not expect it to be observationally relevant, in the sense that any observed compactness higher than $4/9$ would be sufficiently interesting already.

In the next section we explore how the properties of constant density configurations change when we consider a still simple but more realistic approximation to the equation of state of compact stars.

6 Polytropic stars in UG

In this section we build upon the previous results in order to study compact objects described by a polytropic equation of state, chosen in such a way that GR configurations with masses and radii in the range of neutron stars are obtained. As before, we use a diagonal energy-momentum tensor $T_0^0 = -\rho(r), T^i{}_i = p(r)$, with

$$\rho(r) = \rho_0\left(\chi(r) + \frac{K}{\Gamma-1}\chi(r)^\Gamma\right)\,, \tag{6.1}$$

$$p(r) = K\rho_0\chi(r)^\Gamma\,, \tag{6.2}$$

where $\chi(r)$ is a dimensionless function, while K and γ are the free parameters of the EoS. These parameters are determined by requiring that the properties of the resulting configuration in GR match a realistic equation of state, in particular, $K = 0.0225$ and $\Gamma = 2.34$ are

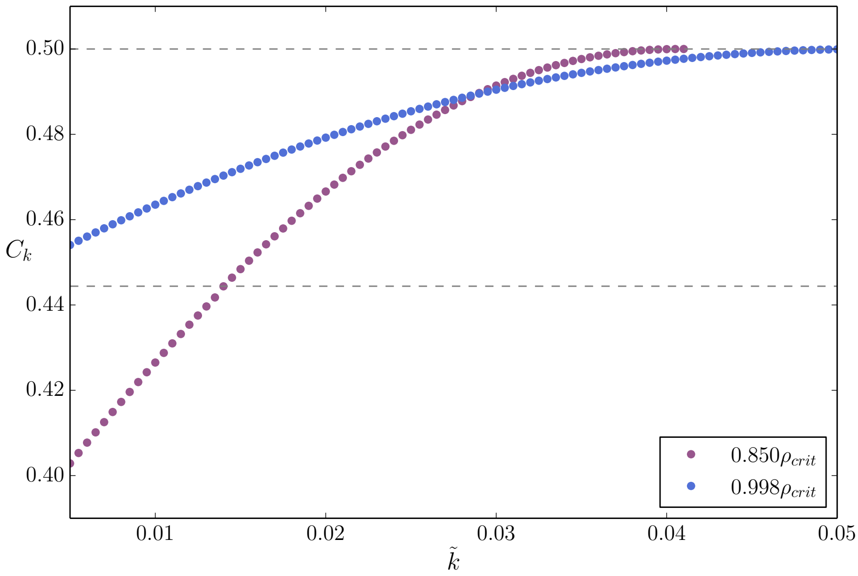

Figure 3. Compactness as a function of the non-conservation of $T_{\mu\nu}$ for objects with two fixed densities near the critical density of GR solutions. The dashed line at $C = 4/9$ represents the GR limit for the compactness of constant density objects, achieved only for objects with $\rho = \rho_{\rm crit}$. In UG, objects with $\rho < \rho_{\rm crit}$ can surpass this limit if the non-conservation of $T_{\mu\nu}$ is large enough.

compatible with the masses of PSR J1614-2230 ($1.97 \pm 0.04 M_\odot$ [50]) and PSR J0348+0432 ($2.01 \pm 0.04 M_\odot$ [51]), two of the most massive neutron stars (NS) observationally confirmed to date.[5]

In addition to the EoS, in order to close the system of equations we decide to assume a type of violation of conservation of $T_{\mu\nu}$. Following the discussion in section 4, here we focus on

$$\nabla_\mu T^\mu{}_\nu = \delta^r_\nu k \rho(r)\,. \tag{6.3}$$

This is a generalization of the constant violation assumed in the case of constant density.

We look for solutions numerically, setting initial conditions at a small radius r_i by solving the equations of motion in a Taylor expansion around $r = 0$. In contrast to GR, where the initial conditions depend only on $\chi(0)$, here they depend on $\chi(0)$, $\chi''(0)$ and k. The value of $\chi''(0)$ is related to the additional integration constant of UG — the one associated with a cosmological constant; to set this contribution equal to zero we fix $\chi''(0)$ to be the same as in the GR solution. Thus, the only free parameters of our solution are $\chi(0)$ and k.

Figure 4 shows our results for polytropic configurations with $k = 0$ — which recovers GR, $k = -0.002$ and $k = 0.002$. The left panel shows the mass-radius curves for equilibrium configurations with central densities in the range 10^{14}–10^{19} kg/m^3. Assuming the same values of K and Γ for every k, we find that the mass-radius curves for negative (positive) k lie below (above) the GR curve. Similar results are found for the compactness of these configurations, shown in the right panel of figure 4. These results show that the GR solution is continuously recovered as $k \to 0$, but relatively large deviations in the compactness of low density objects appear even for small breaking of conservation of $T_{\mu\nu}$; therefore, this

[5]Recently, the mass of PSR J2215+5135 has been estimated to be around $2.27 \pm 0.17 M_\odot$ [52], but this result depends on the orbital inclination, which has not been independently confirmed.

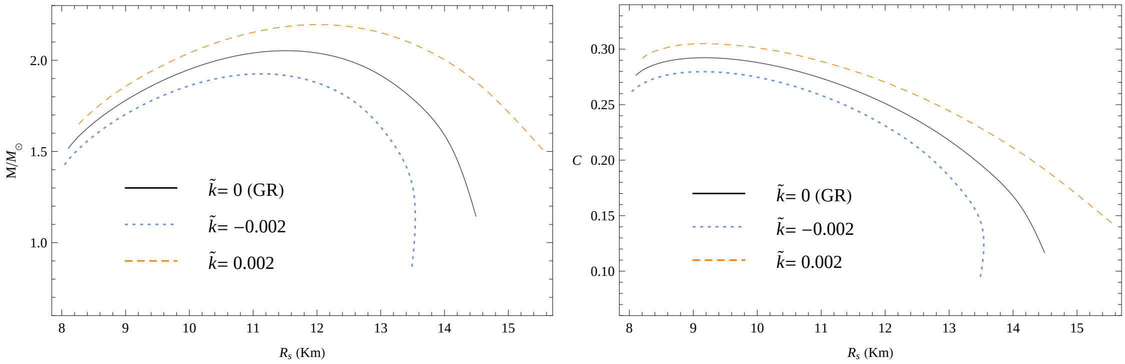

Figure 4. Mass-radius curves and compactness for compact polytropic stars, with non-conservation of energy-momentum parameterized by the dimensionless constant $\tilde{k} = (1.2 \times 10^5 \text{cm})k$. Notice that the changes in the properties of the star due to $\tilde{k}$ get smaller as the matter density increases.

conservation breaking could be constrained, e.g., by observations of the compactness and tidal deformabilities of NS (see, e.g. [53]). Furthermore, it is important to highlight that we generically find solutions with masses higher than the ones allowed in GR by the static equilibrium criterion $dM/d\rho_c > 0$ [54]: stars with such masses, if observed, could hint towards modifications of the theory of gravity. Similar effects are found in scalar-tensor and vector-tensor gravity [12, 13, 16, 18, 55]. Also in connection to other modified gravity models, notice that the deviations due to UG shown in figure 4 get smaller as the central density of the stars increases, this behaviour is reminiscent of screening mechanism (see, e.g. [11]), we speculate that it can be a consequence of the metric non-linear relation and the non-conservation of $T_{\mu\nu}$ implied by eq. (2.5).

Other constraints could be imposed by studying the sound speed $c_s^2 = dp/d\rho$ of our solutions. Causality requires $dp/d\rho \leq 1$, and also we should have $dp/d\rho \geq 0$. However; we find that the maximum c_s^2 in every solution is attained at the center of the star and it depends very weakly on the value of k. This is shown in the left panel of figure 5, where the shaded area is the region excluded by causality. The solutions in that area are already excluded by the static equilibrium criterion, so that no new constraints arise from the sound speed. The right panel of figure 5 shows that the changes to the sound speed inside the star induced by k are also small.

By construction, for $k = 0$ the maximum mass of the configurations shown in figure 4 is about the observed $2M_\odot$ limit. For negative k, the maximum mass is reduced. Although the parameters K and Γ of the EoS can be readjusted in such a way that the observed maximum mass limit is recovered, these changes to the EoS also affect other quantities. For instance, let $\mathcal{M}$ be the observed maximum mass and $\mathcal{R}$ its radius computed in GR with a polytropic equation of state. In UG with $\tilde{k} = -0.01$ we can obtain a mass-radius curve with maximum at $(\mathcal{R}, \mathcal{M})$ (i.e. we require that the radius of the maximum mass configuration is the same in GR and in UG) by setting the polytropic parameters $K \approx 0.0175$ and $\Gamma \approx 2.88$. However, when we study the sound speed of these objects we find that the limit $c_s^2 \leq 1$ is violated before reaching the maximum mass. Thus, we learn that by combining conditions on the mass, radius, and causality of the solutions we can constrain the size of $\tilde{k}$ even if some freedom is allowed in the EoS: neutron stars in UG, compatible with $\mathcal{M} \approx 2M_\odot$ and $\mathcal{R} \approx 12$ Km and described by a polytropic equation of state, require $\tilde{k} > -0.01$. Tighter constraints are not yet possible due to the observational uncertainty of the mass-radius curves (see, e.g. [56]). For

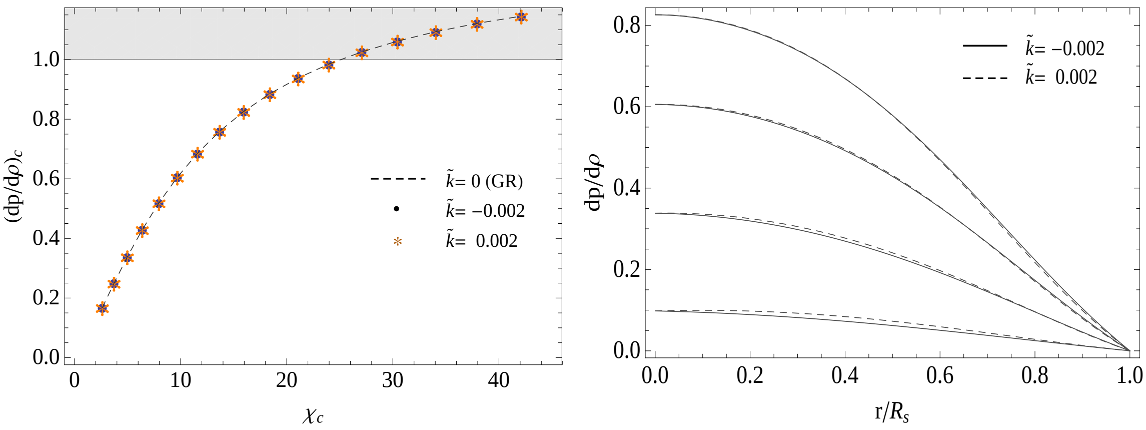

Figure 5. Left panel: sound speed at $r = 0$ for polytropic stars with different amounts of non-conservation of $T_{\mu\nu}$. In all cases, the results agree with GR. Right panel: sound speed inside the star for four different central densities. The GR curves (not displayed) lie between the curves for $\tilde{k} = -0.002$ and $\tilde{k} = 0.002$. Small deviations from GR happen inside the star.

more realistic EoS, whose parameters are determined from effective theories and assumptions on the behaviour of nuclear matter, there are two possibilities. First, the parameters of the EoS get modified once the microscopic non-conservation of energy-momentum is taken into account, in this case the situation would be similar to our discussion above for the polytropic equation of state. Second, the parameters of the EoS remain unchanged. An interesting consequence of this scenario would be that some equations of state previously discarded for not reproducing $\mathcal{M}$ could be revived by assuming a minimum, positive value for $\tilde{k}$. This could be the case, for instance, for the BSk19 EoS [57] (see also [58]), which predicts a maximum mass of $1.86M_\odot$. Comparing with figure 4 we speculate that $\tilde{k}$ of order 10^{-3} can lift this value to approximately $2M_\odot$, thus making this EoS viable. In order to analyze these possibilities, a detailed study of the different equations of state proposed in the literature and their underlying assumptions is required. Such a study is beyond the scope of this work.

Another quantity of interest is the surface redshift, i.e., the gravitational redshift of emission lines originating near the surface of the star. As stated in eq. (3.14), this redshift is closely related to the compactness. For the objects shown in figure 4 we find redshifts modified by about 10% with respect to their GR value. For instance, a polytropic object with a radius of 12Km has $z \approx 0.42$ in GR, $z \approx 0.36$ in UG with $\tilde{\kappa} = -0.002$ and $z \approx 0.47$ in UG with $\tilde{\kappa} = 0.002$. These changes are within the lower and upper bounds on the redshifts expected in GR [59], and therefore do not offer an observational test of UG with $-0.002 \lesssim z \lesssim 0.002$.

To conclude our discussion of the structure of the star, let us comment on the properties of the pressure at $r = 0$. In contrast to GR, where the equilibrium equations demand $p'(0) = 0$, the solutions displayed in this section have $p'(0) \neq 0$ — indeed $p'(0) = k\rho(0)$, thus, for positive k the maximum pressure is not necessarily the pressure at $r = 0$. Also, the second derivative of the pressure at $r = 0$ is not necessarily negative as is the case in GR but can become positive if k is sufficiently large, this can be seen by exploring the perturbative solutions near $r = 0$. Nevertheless, for the stable solutions shown here we always have $p''(0) < 0$, indicating that the profiles of $p(r)$ near $r = 0$ are concave downwards, and even if $k > 0$, $p'(r)$ becomes negative at a very small distance away from the origin and from there the pressure decays monotonically to zero.

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Intuitively one would expect solutions with non-monotonic $p(r)$ to be perturbatively unstable. Thus, a valid question to ask is whether there is a simple way to remove them from the model. Indeed this can be done for polytropic objects by considering a non-conservation of the form

$$\nabla_\mu T^\mu_\nu = \delta^r_\nu k(\rho(r) - \rho(0)) \,. \tag{6.4}$$

By expanding the equations of motion near $r = 0$ it is easy to verify that this leads to solutions with $p'(0) = 0$. Furthermore, $p''(0)$ — or equivalently $\chi''(0)$ is a free parameter that can always be chosen negative so to guarantee that the maximum pressure is at $r = 0$. Nevertheless, this choice is not free of problems: once again the properties of the solutions are such that for configurations with $dM/d\rho > 0$ the compactness is smaller than the compactness of the corresponding GR objects. Also, the constant term $k\rho_0$ implies that the non-conservation of T^μ_ν is larger near the surface of the star, which is counter-intuitive in particular for a polytropic model where $\rho(R_s) = 0$ and we expect a smooth transition to the vacuum solution. Summing up, we found that configurations with compactness equal or smaller than GR bounds are generic in UG with non-conserved energy-momentum tensor, while higher compactness is possible if we allow for the maximum density of the star to be shifted away from the origin. It would be interesting to study the theoretical viability of these solutions, as well as their existence in models where the non-conservation of $T_{\mu\nu}$ is not directly sourced by matter but by the curvature. We leave this for future work.

7 Gravitational collapse in UG

In this section we study the Snyder-Oppenheimer (SO) model, which is the simplest case of gravitational collapse, in the UG scenario without energy-momentum conservation, assuming a spherically symmetric collapse of dust with negligible pressure.

The metric related to this type of collapse is the well known homogeneous and isotropic line element [60] written in the form:

$$ds^2 = -dt^2 + R(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2\right], \tag{7.1}$$

where $R(t)$ is the comoving radius of the star, k is the curvature of the star which always must be imposed positive and $d\Omega^2 \equiv d\theta^2 + \sin^2\theta d\varphi^2$ is the solid angle. Using (2.3) with the addition of a dust energy-momentum tensor ($p = 0$) we have

$$\ddot{R}R - \dot{R} - k = -4\pi G R^2 \rho_{\rm dust}. \tag{7.2}$$

Additionally, eq. (2.5) generates

$$\dot{\rho}_{\rm dust} + 3\mathcal{H}\rho_{\rm dust} = \frac{\mathcal{H}^3}{4\pi G}(1 - j), \tag{7.3}$$

where $\mathcal{H} \equiv \dot{R}/R$ and $j \equiv \dddot{R}/R\mathcal{H}^3$, the last parameter is defined in order to encode the non-conservation of the energy-momentum tensor and help us to elucidate if we are not facing with spurious solutions due to the derivatives acting on the Ricci scalar, which translate into third order derivatives of $R(t)$ (in principle this should not be a problem since (2.5) is contained in the second order equations of motion, but one has to be careful when including (2.5) in the system of equations as we do in this section). Therefore, using eqs. (7.2), (7.3) and integrating we obtain

$$\mathcal{H}^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho_{\rm dust} - \frac{k}{R^2} + \frac{2}{3}\int_{t_0}^{t} \mathcal{H}^3\,(j-1)\,dt' + \Lambda, \tag{7.4}$$

where Λ is an integration constant. Notice that for $j = 1$ and $\Lambda = 0$ the traditional behavior for stellar collapse is recovered, therefore the GR limit is approached as $j \to 1$, so deviations in gravitational collapse can be parameterized by $j \neq 1$.

In figure 6 we present the results of the numerical solution of eqs. (7.3) and (7.4) with dimensionless variables $\tau \to H_0 t$, $\bar{\rho} \to 4\pi G H_0^2 \rho/3$ and $\bar{k} \to H_0^2 k$, where H_0 is an appropriate constant that has units of s^{-1}. In all cases we assume that the collapse initiates at a normalized radius $R(\tau) = 1$ for $\tau = 0$. As expected, our results show that UG differs from GR when we use different values of the parameter j. Here, we explore small and constant violations to energy-momentum conservation in order to observe the differences at large values of τ in a simple model. We find that the collapse time is notably modified when we increase the presence of unimodular gravity. More exotic forms of j could even stop the collapse of the star, therefore it would be interesting to study in detail how this modifies black hole formation and population. Assuming that this process has to be very similar in UG and in GR, we should expect $j \approx 1$, allowing the collapse of the star and only producing small violations to the energy-momentum tensor.

It would also be interesting to study collapse in the context of structure formation. From the results above, it would be possible for subtle differences in the presence of violations to the conservation of energy-momentum to modify collapse times. In this vein we suggest that the reionization [61, 62] epoch could be an excellent laboratory to validate or refute some aspects of UG, in particular we propose a future study, through a comparison with the empirical star-formation rate proposed by [63] and with the strength of the 21-cm signal through the so-called differential brightness temperature T_{21}, in star formation eras [64].

In addition, we notice that the solution of eq. (7.3) for values of $j \to 1$ (as we expect) can be approximately written as $\rho_{\rm dust} \approx \rho(0)_{\rm dust} R^{-3} + \text{Corr}$, where the corrections comes from the violations to the energy-momentum tensor, being $\rho(0)$ the central stellar density. This corrections contribute to the effective mass of the star, allowing the possibility of a larger population of NS or black holes in the Universe. Moreover, the final fate of the star strongly depends on the violations to non-conservation in UG, and also on the Chandrasekhar and Oppenheimer-Volkoff limits associated to the UG mass. Therefore, the study of the SO collapse is not enough to give a verdict on the destiny of the compact object.

8 Discussion and conclusions

This paper presents a systematic study of static, spherically symmetric solutions in unimodular gravity with non-conserved energy-momentum tensor. This non-conservation has relevant consequences for the cosmological constant problem. However, to our knowledge, it has not been exhaustively studied in the strong gravity regime. Here we report some progress in this direction: we address issues regarding the choice of coordinate system, showing that a coordinate transformation from unimodular to standard spherically symmetric coordinates is possible in presence of matter, we study compact and polytropic configurations of matter in detail, and we analyze a simple model for gravitational collapse. In addition, in appendix D

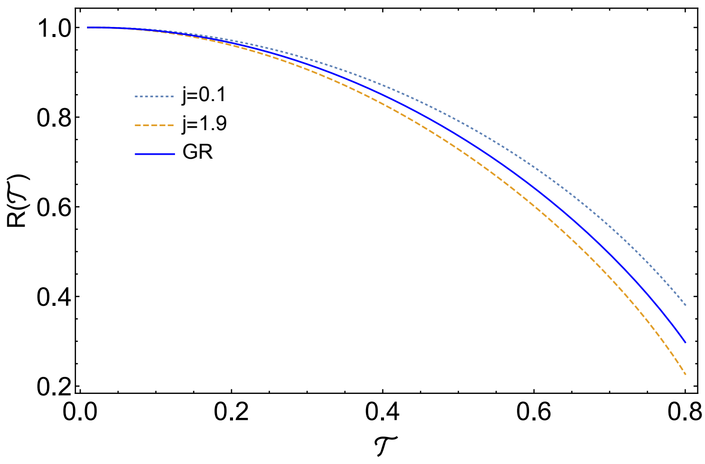

Figure 6. Numerical solutions for gravitational collapse in unimodular gravity. We choose as initial conditions: $R(0) = 1$, $dR(0)/d\tau = 0$, $\bar{\rho}(0) = 1$ and $\bar{k} > 0$.

we revisit the Reissner-Nördstrom solution, which had been claimed to incorporate effects not accounted for in GR, even under the assumption that the energy-momentum tensor is conserved [65]. This would contradict the classical equivalence between GR and UG. We clarify this by pointing out that the solution reported in [65] makes use of results that only hold in a different version of UG, known as density-metric unimodular gravity [66], and we rederive the RN solution, finding full compatibility with GR.

Let us summarize our main results. We start by finding the TOV equation in UG under a specific ansatz that allows for an analytic treatment of the field equations. We show a comparison between the gravitational energy V/M in GR and in UG, obtaining that the ansatz $R_{tt} = 0$ leads to positive values that indicate non-physical results in the case of constant energy density. Furthermore, different stellar compactness are obtained due to the presence of a constant related to the ansatz chosen to close the UG system of equations.

For constant density objects we parameterize continuous deviations from GR by choosing an appropriate ansatz for the type of non-conservation of $T_{\mu\nu}$. We find that their compactness goes well beyond the Buchdahl limit as this non-conservation increases, and approaches asymptotically to the compactness of a black hole.

We have also studied neutron stars described by a polytropic EoS. We find that the type of violation of $T_{\mu\nu}$ becomes relevant. We focus on a choice that allows for objects with higher compactness than their GR counterparts, but this comes with the peculiarity that the maximum pressure of the star is shifted away from the origin by a distance related to the size of the violation to energy-momentum conservation. This is in stark contrast with GR, where a monotonically decreasing pressure is guaranteed by the equilibrium TOV equations. Similarly, in a generic class of modified gravity models the equations do admit solutions where the pressure increases with r near the origin, but it was shown that a complete solution does not exist, i.e., that the pressure never turns from increasing to decreasing as a function of r [67]. The existence of this type of solutions in UG is thus a novel prediction. A detailed study of the stability of these solutions, left for future work, is a promising tool to constrain this model. Furthermore, by combining causality and conditions on the mass and radius of the maximum mass configuration we are able to set a bound on the parameter that controls the non-conservation of energy-momentum.

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Finally, we revisit homogeneous and isotropic gravitational collapse in UG. The non-conservation of energy-momentum is encoded in the fluid equation through the j parameter. We show that even if we take this parameter close to its GR value, the solutions exhibit modifications to the collapse time that could impact the process of black hole formation, providing another way to constrain the violations to energy-momentum conservation. We discuss that the growth of structure in our Universe could be also affected, specifically in the reionization epoch [61, 62], modifying the population of collapsed objects (white dwarfs, NS or black holes). In this vein, we propose that in future works, the results must be extended and compared with the empirical star-formation rate, resulting in important consequences for the observed peak of cosmic star formation history at $z \approx 2$ [63]; not less important could be a profound study of 21-cm signal through the T_{21} temperature which is not only sensitive to the Hubble expansion but also to the temperatures that are intimately related with the stellar formation and population [64].

In summary, the results presented in this work suggest several scenarios where the non-conservation of energy-momentum allowed in UG could be constrained. We have chosen to analyze simple forms of non-conservation, showing effects that we expect to be generic for other choices, like the modifications to the compactness of neutron stars and the change in gravitational collapse times. In addition, we remark that UG theory can be considered as a particular case of $f(R,T)$ theories as reported in [68]. In this study, the authors adopted a Lagrangian, which contains a linear combination of Ricci scalar and a trace of the energy-momentum tensor, developing a thermodynamic and cosmological analysis, where at the end, it is possible to demonstrate that the results degenerate to the UG theory, stressing a subtle relation between $f(R,T)$ and unimodular gravity models. Finally, it would be interesting to derive a form of non-conservation motivated by the possible discretization of space-time, as some authors have done in the study of the cosmological constant problem [35, 36]. This is ongoing research that will be presented elsewhere.

Acknowledgments

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A Equivalence between metrics

A point that tends to cause confusion in UG is whether it is necessary to use a metric that fulfills the unimodular condition or a metric in traditional spherical coordinates — or any coordinates — can be used. The requirement that the metric determinant equals a constant is a coordinate-dependent statement, and one should prefer a statement about coordinate-independent objects, like the volume form. We also remark that the goal of the unimodular condition is to restrict the variations of the metric and not the metric per se. Nevertheless, it is good to show explicitly the equivalence between metrics in unimodular and in other system of coordinates. Here we do so for a FLRW metric.

Let us compare physical results derived for a FLRW line element written in the form $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$ and for a metric that fulfills the unimodular condition, used in Alvarez

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et al. [31] to study a cosmological scenario in UG. This metric reads

$$ds^2 = -b(\tau)^{-2/3}d\tau^2 + b(\tau)^{1/2}d\vec{x}^2. \tag{A.1}$$

Notice that eq. (A.1) can be constructed from the standard FLRW line element via a change of variables $a \to b^{1/4}$ and $dt \to b^{-3/4}d\tau$.

Assuming a perfect fluid energy-momentum tensor and using the field equations of UG (see eq. (2.3)) we have

$$\frac{b''}{b} - \frac{1}{4}\left(\frac{b'}{b}\right)^2 = -16\pi G b^{-3/2}(\rho + p), \tag{A.2}$$

where primes indicate derivatives with respect to τ. This is the Friedmann equation under the unimodular condition, but the physical interpretation in this form is not straightforward. However, if we return to the $a(t)$ and t variables to recover the traditional FLRW line element, we have: $\dot{H} = -4\pi G(\rho+p)$, which is the same equation used previously by several authors [26, 27]. Therefore, the result is independent of using the FLRW metric or the metric of eq. (A.1). The essential point is, which metric provides the best insight into the physical interpretation of the results.

B Ansatz election

Lemma B.1 *Considering the metric (3.1), there is a chart such that $R_{tt} = 0$ and $R_{\theta\theta} \neq 0$.*

Proof. Let p a point in the space-time with metric (3.1), then there is a chart $\mathcal{U}'$ containing p, where the first partial derivative of the metric tensor vanishes at the point. Also, since $h(r) = 1 - 2M_{UG}G/\hat{r}$ and $\partial_{\hat{r}}h|_p = -2G(\frac{M'_{UG}\hat{r} - M_{UG}}{\hat{r}^2})\Big|_p = 0$, we have

$$\begin{aligned}
\partial^2_{\hat{r}}h &= \frac{-2G}{\hat{r}^4}\Big[(M''_{UG}\hat{r} + M'_{UG} - M'_{UG})\hat{r}^2 - 2(M'_{UG}\hat{r} - M_{UG})\hat{r}\Big] \\
&= -2G\Big[\frac{M''_{UG}}{\hat{r}} - 2\frac{(M'_{UG}\hat{r} - M_{UG})}{\hat{r}^3}\Big] \\
&= \frac{-2G}{\hat{r}}\Big(M''_{UG} - 2\partial_{\hat{r}}h\Big)
\end{aligned} \tag{B.1}$$

and at the point p

$$M'_{UG} = \frac{M_{UG}}{\hat{r}}, \qquad M''_{UG}\Big|_p = -\frac{1}{2G}\partial_{\hat{r}}h\Big|_p = 0, \tag{B.2}$$

resulting

$$\partial^2_{\hat{r}}h\Big|_p = \frac{4G}{\hat{r}}\partial_{\hat{r}}h\Big|_p = 0. \tag{B.3}$$

Finally, we can infer

$$R_{tt}(p) = 0, \qquad R_{\theta\theta}(p) = -1 + h(r) \neq 0, \tag{B.4}$$

which is the ansatz that we used.

C Equations of motion for constant density objects in UG

In section 5 we obtained numerical solutions for constant density objects. Here we elaborate further on the analytic treatment of the system of equations. The angular part of the Einstein equations satisfies ${\xi^\theta}_\theta = {\xi^\varphi}_\varphi$ because of our spherically symmetric set-up, and ${\xi^t}_t + {\xi^r}_r + {\xi^t}_t + {\xi^\theta}_\theta = 0$ because of the trace-free condition. Thus, there are only two independent equations, say ${\xi^t}_t$ and ${\xi^r}_r$. Notice that the trace-free condition plays the role usually played by the Bianchi Identities and the conservation of $T_{\mu\nu}$ in eliminating one of the metric equations of motion. Explicitly, under ansatz (3.1) the independent equations read

$$f^2\left(4rh' + 4h + 6\kappa p r^2 + 6\kappa\rho r^2 - 4\right) + hr^2 f'^2 - fr\left(rf'h' + 2h\left(rf'' + 2f'\right)\right) = 0\,, \quad \text{(C.1)}$$

$$-2f^2\left(2rh' - 2h + \kappa p r^2 + \kappa\rho r^2 + 2\right) + hr^2\left(f'\right)^2 - fr\left(rf'h' + h\left(2rf'' - 4f'\right)\right) = 0\,, \quad \text{(C.2)}$$

where f, h and p are functions of r and $\rho = \rho_0$ is constant. In order to close the system of equations we choose to assume a type of non-conservation of $T_{\mu\nu}$: $\nabla_\mu {T^\mu}_\nu = k_\nu$ for some constant vector k_ν with units of density over distance. Only the radial component of these equations is not trivial

$$pf' + \rho_0 f' + 2fp' = 2fk\,, \quad \text{(C.3)}$$

where $k_\nu = \delta^r_\nu k$. After a few manipulations we obtain that the metric satisfies

$$\nu'(r) = \frac{h' + \kappa p r + \kappa\rho r}{h}\,, \quad \text{(C.4)}$$

$$h(r) = 1 + c_1 r^2 - \frac{1}{4}\kappa k r^3\,, \quad \text{(C.5)}$$

with $f(r) = e^{\nu(r)}$. The GR form of the radial component is recovered when $k = 0$ and $c_1 = -\kappa\rho_0/3$. The arbitrariness of c_1 comes as a consequence of the additional integration constant contained in UG. The pressure is determined by the equation

$$\begin{aligned}&\kappa r\left[8c_1 r\left(\rho_0 - kr\right) + 2\kappa k^2 r^3 - k\left(3\kappa\rho_0 r^2 + 8\right) + 4\kappa\rho_0^2 r\right] \\ &+ \kappa r^2\left(8c_1 + 8\kappa\rho_0 - 3\kappa k r\right)p(r) + 4\kappa^2 r^2 p(r)^2 + \kappa r\left(8c_1 r^2 - 2\kappa k r^3 + 8\right)p'(r) = 0.\end{aligned} \quad \text{(C.6)}$$

As in GR, this is a Ricatti equation, but with more complicated coefficients. Writing $c_1 = -\kappa\rho_0/3 - \gamma$ we can verify that the GR solution for a constant density object in presence of cosmological constant $\Lambda = 3\gamma$ is recovered when $k = 0$. It is interesting to note that this effective cosmological constant is not related to the non-conservation of $T_{\mu\nu}$. For arbitrary k we could not find exact solutions. As a complement to the numerical analysis performed in section 5, let us study the near-origin solutions. Expanding eq. (C.6) near $r = 0$ and setting $c_1 = -\kappa\rho_0/3$ we find

$$p'(0) = k\,, \quad \text{(C.7)}$$

$$p''(0) = -\frac{1}{6}\kappa(\rho_0 + p(0))(\rho_0 + 3p(0))\,, \quad \text{(C.8)}$$

$$p'''(0) = -\frac{1}{12}k\kappa(7\rho_0 + 15p(0))\,. \quad \text{(C.9)}$$

Odd orders are turned on by k, and even orders in general do receive modifications due to k ($p''(0)$ is independent of k by construction). If we choose a different value of c_1, modifications due to the extra integration constant in UG appear at every order except $p'(0)$. As a

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consequence, the maximum pressure is shifted away from the origin if $k > 0$. As shown in section 6, this feature persists when a polytropic EoS is considered. It would be interesting to explore its implications in more detail, for example, as another way to constrain k by stability conditions or as a new effect non-degenerated with the EoS. Results in these directions will be reported elsewhere.

D Reissner-Nordström solution in UG

For completeness we briefly review and clarify some aspects of the Reissner-Nordström solution in UG, partially reported in [65]. As discussed after eq. (3.1), a spherically symmetric space-time can be described in unimodular coordinates by the line element

$$ds^2 = -f(y)dt^2 + \frac{dy^2}{r(y)^4 f(y)} + \frac{r(y)^2 dx^2}{1-x^2} + r(y)^2(1-x^2)d\varphi^2 \,, \tag{D.1}$$

where $dr = \sqrt{h(r)/(r^4 f(r))}dy$, with $f(r)$ and $h(r)$ the g_{tt} and g^{rr} components of the metric in spherical coordinates. In particular, for a Reissner-Nordström black hole we have

$$f(r) = h(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} \,, \tag{D.2}$$

and $r = (3y)^{1/3}$. Here we want to verify that this solution, supplemented with a cosmological constant, is the only solution in UG for an electrically charged black hole. To this end, we insert (D.1) without any assumptions on the form of $f(y)$ and $r(y)$ in the equations of motion

$$R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R = \frac{1}{4}\left(T_{\mu\nu} - \frac{1}{4}g_{\mu\nu}T\right) \,, \tag{D.3}$$

$$\nabla_\mu F^\mu{}_\nu = 0 \,, \tag{D.4}$$

where

$$T_{\mu\nu} = -\frac{1}{2}g_{\mu\nu}F^{\alpha\beta}F_{\alpha\beta} + 2F^\alpha{}_\mu F_{\alpha\nu} \,, \tag{D.5}$$

and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. Notice that — as we do throughout this work — we are using the GR energy-momentum tensor

$$T_{\mu\nu} = \frac{1}{\sqrt{-g}}\frac{\delta(\sqrt{-g}F^{\alpha\beta}F_{\alpha\beta})}{\delta g^{\mu\nu}}$$

instead of the UG version $E_{\mu\nu} = \frac{\delta(F^{\alpha\beta}F_{\alpha\beta})}{\delta g^{\mu\nu}}$ used in [65]. This is justified since, as shown in [25],

$$T_{\mu\nu} - \frac{1}{4}g_{\mu\nu}T = E_{\mu\nu} - \frac{1}{4}g_{\mu\nu}E \,.$$

Neglecting the integration constant in $r(y)$ since it is an arbitrary constant in a change of coordinates, fixing the integration constant in $f(y)$ in such a way that $f(y \to \infty) = 1$, and using gauge invariance of A_μ to set to zero another integration constant, we arrive to the general solution

$$r(y) = (3y)^{1/3} \,, \tag{D.6}$$

$$f(y) = 1 + \frac{Q^2}{(3y)^{2/3}} - \frac{2M}{(3y)^{1/3}} + by^{2/3} \,, \tag{D.7}$$

$$A(y) = \frac{2Q}{(3r)^{1/3}} \,. \tag{D.8}$$

This is nothing more than the RN solution in presence of a cosmological constant expressed in unimodular coordinates. This differs from the results in [65], where additional corrections to the RN solution are reported. To our understanding, the difference arises because in [65] the equations of motion are computed by using the results of [66] to evaluate the curvatures appearing in (D.4), these results hold for a theory that is closely related but different to unimodular gravity, in which the metric is considered as a tensor density of weight $-1/2$ instead of a tensor density of weight 0, for this reason this theory is dubbed *density-metric unimodular gravity*, and as pointed out in [66], the solutions to this theory differ from the solutions to GR and to standard UG. This subtlety is not mentioned in [65], and we think it is important that we have clarified it here.

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