Role of Mass Trajectories on Fundamental Fermions

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Assuming a cooperative effect of sea qaurks mass trajectory functions on fundamental fermions are looked for. The u- and d-quark masses in bound states are identified as the corresponding stable equilibrium values on their primary trajectories. The u-d mass difference comparable to p-n one admits us to fix s, c and b quark mass to 130 MeV,1.15 and 4.0 GeV, respectively. The resulting light quark masses are consistent with the current algebra relations. Up to tertiary and quaternary trajectories are derived for d, electron and u, neutrino family, respectively. In general there appear two fixed points on each trajectory but only one of them corresponds to stable equilibrium value, so it is chosen as a new mass unit successively for next trajectory. If the set of predicted trajectories is realized for each family a chaotic behavior is expected from more than two equal mass system. A role of this model on physical world is discussed.

$.1 Introduction

In a previous work [1)] we have proposed the mass formula of common shape on fundamental fermions belonging to the identical charge states, where a single common constant is introduced in order to reproduce the proper masses of relevant fermions. To realize the current quark masses from the proper masses of u and d quarks to acceptable magnitudes of the current algebra, it is suggested to introduce the Cabbibo, Kobayashi and Maskawa-like admixing of light quarks in the same charge states.

Assuming the cooperation of sea quarks in hadron a few u- and d-quark mass associated trajectories will be induced and their stable fixed points on primary trajectories may be identified as the corresponding current quark masses. This will give an another possibility to explain them, starting from their proper masses derived from the mass formula.

Choosing the mass obtained from a stable fixed point on the primary trajectory as a mass unit of next trajectory and so on one can proceed up to quaternary and tertiary trajectory for u,
neutrino and d, electron family, respectively. In general there appear two fixed points on each trajectory. Fortunately only one of them takes the stable equilibrium value for the trajectory function assumed. The set of trajectories in each family may induce

a chaos in general[2)- 5)] on the system provided that their existence is allowed physically. We shall point out several merits of the model in the course of study.

In order to realize the mass trajectories for quarks we propose a function of type ln(x) with a multiplicative parameter plus a one parameter, where the two parameters should be determined case by case. The multiplicative parameter for the primary u quark trajectory is fixed from proper u, theoretical c and observed t masses as inputs. Here it is fixed so as to make the additive parameter of the trajectory is null. Similarly the proper d, theoretical b and somewhat free s ones are used to determine the primary d trajectory. The assumed map function is suggested from our mass formula. The validity of the model may be reduced whether one can find consistent result on the masses for light quarks with the algebra of current discovered first by Leutwyler and extended further. After examining several possibilities under the condition to realize u-d mass difference to p-n one, one may find a possible primary u- and d-trajectories where the theoretical lower bounds of c- and b-quark masses[6)] become preferable, the s-quark one about 130 MeV, u and d quark ones about 4.63 and 6.30 MeV, respectively. The latter three values satisfy consistently the several mass formulas of current algebra relations among bound light quark and meson masses.[6),7)]

The similar work is performed for neutrino and electron trajectories by appropriate substitutions. Recently Duboscq[8)] has reported a new upper bound of tau neutrino mass of CLNS data, where he criticized a current neutrino mixing idea: it is unnatural to use that idea to explain the solar and atmospheric neutrino oscillation observed by SuperKamiokande unless one may observe unstable three types of neutrinos with appropriate masses and life times. If the proposed trajectories induce the chaos by a cooperation of solar neutrinos in sun and atmospheric remnants of Big bang, the missing neutrino behavior becomes independent from the stability of muon and tau neutrino. The full chaos

study is left untouched from the proposed trajectories in this paper. However, one may expect the chaotic behavior if a few trajectories are admitted to a specific object, because it is well known that even a simple doubling map produces a chaos.

It is interesting to seek such a possibility by making use of an electron beam and an electron-positron collision by positronium targets. The crude estimate of lifetimes for para and ortho positronium and their spectra will be found from Bethe and Salpeter's book[9)] and their known decay selection rule is discussed in Weinberg's one.[10)]

There is the similarity between the corresponding set of u-qaurk and neutrino trajectories. The similar applies to d-quark and electron ones.

The stability check of fixed points can be made from the fixed point theorem once the trajectory function is given.

$.2 discusses the u- and d-quark associated trajectories and relevant physics. $.3 and 4 are devoted to neutrino and electron trajectories, respectively. To save the time to read a short original note on quark and lepton mass formula and to make clear the inputs in present work they are repeatedly summarized at the beginning of each section. The last section discusses our result and a future survey in a similar approach.

$.2 Bound u- and d-Quark Masses from their Mass

Trajectories

We assume that the mass formula of charged leptons takes a form

$$m_\tau^{\,k}\ m_e = m_\mu^{\,k+1}\ , \qquad (2.1)$$

where their masses m's are specified by the corresponding subscripts e, μ and τ and the constant k is found to be k=1.88897. Here the numerical figures depend on an accuracy of the observed tau lepton mass.

The similar is assumed to apply remaining fermions in three generations with the common k by making an appropriate substitutions. To find the m_c dependence of m_u from u, c and t input masses we fix t-quark one to the observed m_t = 174.3 GeV throughout this work. It may be suitable to present six figures of estimated numbers for future purpose.

In general the mass dependence of u-trajectory cannot be found unless one knows the corresponding masses with the same quantum number at high energy. Let us assume that c and t
quarks belong to one and the same mass trajectory of u quark. In what follows if we accept this approach the mass values estimated, starting from the proper u qaurk mass found from the mass formula will predict the current u mass as a stable equilibrium value of the fixed point on a primary u trajectory. Choosing the stable equilibrium value of fixed points on the primary trajectory as a unit for a secondary one, it gives a new stable fixed point. The game seems to continue up to the quaternary trajectory. The similar study for d-quark one end up with tertiary one. One may wonder that this way to find trajectories in high degree may be contrary to double counting. We do not worry this criticism unless the trajectories overlap in the range of interest except crossing at a point which will not destroy the feature of the theory.

Admitting this pretext we push our calculation under this hypothesis and to examine the physical consequences by comparing them with the existing experimental data.

The mass dependence of the trajectory function will be approximated by

$$f(x) = a \ln(x) + b, \qquad (2.2)$$

where the parameter a is fixed from the discrete numbers $x_1 = m_u / m_u$, $x_2 = m_c / m_u$ and $x_3 = m_t / m_u$ by choosing b=0 for the primary trajectory. Here m_u is the u-proper mass estimated from the formula analogous to Eq.(2.1). To get a reasonable choice of the corresponding current u and d quark masses and m_s from theoretical values we tried several possibilities so as to adapt u-d mass difference close to the proton-neutron one, where the u quark mass corresponding to its stable fixed point is specified by m_u. It is assumed to correspond the current quark mass. The parameter a of the trajectory function, the corresponding stable fixed point and the first derivative of f(x) at that point is also estimated. Actually there arises only one stable equilibrium value appears on each trajectory for all the examples studied in this paper.

According to the fixed point theorem[3] the stability of fixed point p is guaranteed if the condition

$$|f'(p)| < 1 \qquad (2.3)$$

is satisfied, and it becomes unstable if $|f'(p)| > 1$, while one cannot talk about the stability when $|f'(p)| = 1$. These conditions can
easily be tested once the parameters a and b of the trajectory is determined numerically.

As a start of our program we summarize an estimated result on the primary u-trajectory in Table I for several m_c choices.

Table I. c-qurak mass dependence of u-relevant values on primary u-trajectory for parameter b = 0

m_c (MeV)	1130	1140	1150	1350
m_u (MeV)	0.083100	0.085241	0.087421	0.138928
$m_{u'}$ (MeV)	4.4157	4.52423	4.62912	7.04367
a	13.3867	13.3634	13.3401	12.9441
p	53.1372	53.0757	52.952	50.6997
f'(p)	0.253256	0.252368	0.251928	0.254717

The input c quark mass, the estimated proper and bound u quark one are shown in the first, second and third line of the Table, respectively. The parameter a, fixed point p and f'(p) are listed in the fourth, fifth and sixth line, respectively. They are slowly varying function of m_c. The numerical values in the same column is one-one correspondent. One sees that all fixed points are stable equilibrium values by the fixed point theorem. We did not list unstable equilibrium value of fixed point x = 0, corresponding to b = 0, in this Table, where $f'(0) = \propto$. The bound u quark mass $m_{u'}$ is slowly varying function of m_c. It is related by $m_{u'} = p\, m_u$.

Now we turn our attention to the primary d-trajectory. It is assumed that the similar mass formula as Eq.(2.1) with a common k holds in this case by substituting u, c, t by d, s, b in this order. The similar is applied to the d-trajectory function as Eq.(2.2). There are two possibilities on the choice of b-mass in addition to the inherent difficulty associated with the choice of current s-quark mass. We do not discriminate between proper and current s quark in this study. We only list m_b = 4.0 GeV case in Table II and comment on its upper bound 4.4 GeV one for short.

Table II. s-quark mass dependence of d-relevant values on primary d trajectory for parameter b = 0

m_s (MeV)	120	130	140	150
m_d (MeV)	0.159408	0.20088	0.24884	0.303722
$m_{d'}$ (MeV)	5.18149	6.30167	7.56897	8.97016
a	9.31643	9.10377	8.90687	8.72358
p	32.4056	31.3793	30.4170	29.5341
f'(p)	0.286619	0.290203	0.292825	0.295379

The input s-quark mass, proper, bound d-quark mass, parameter a for d-trajectory function f(x), the fixed point p and f'(p) are given in the first, second, third, fourth, fifth and sixth lines of the table, respectively, in this order. The bound quark masses in the third line correspond to the stable fixed points where all P's are the stable equilibrium values.

Comparing these two tables we suggest that columns 3 and 2 in Table I and II, respectively, are suitable combination, since the mass difference of u and d is close to that of proton and neutron.

The $m_{u'} = 4.63$, $m_{d'} = 6.30$ and $m_s = 130$ MeV found from this study are consistent with the known current algebra relations quoted in ref.[6),7)]

We have examined $m_b = 4.4$ GeV case by a similar manner. As shown already the current algebra favors theoretical lowest bound of m_c so we prefer the lowest bound of m_b, because both of them were estimated by making use of similar experimental data and the QCD based theoretical approach.

The primary to quaternary u- and to tertiary d-trajectory associated parameters are tabulated in Table III without avoiding partial overlap of previous statements for the sake of comparison with lepton trajectories studied in next two sections.

Table III. Primary to Quaternary u- and to Tertiary d-trajectory associated parameters for $m_c = 1.15$ GeV, $m_s = 130$ MeV

trajecotry	primary	secondary	tertiary	quaternary
m_u (MeV)	4.62912	218.503	9113.09	324883
m_d (MeV)	6.30167	164.623	423.99	
a for u	13.3401	13.2552	13.2552	13.2552
a for d	9.10377	9.0458	9.0458	

b for u	0	-3.88939	-7.7433	-11.4745
b for d	0	-3.39127	-6.65412	
p for u	52.9520	47.2019	41.7069	36.0408
p for d	31.3703	26.1238	20.799	
f'(p) for u	0.252	0.281	0.318	0.368
f'(p) for d	0.290	0.346	0.435	

The indices are shown in the first line of the table. The first column specifies the meaning of numbers appearing in each line in association with trajectory specification. They are shown by paired lines for u- and d-trajectories in this order. It may be sufficient to give number of three figures on f'(p) for stability test.

Denoting $m_{u'}$ and $m_{d''}$ as the masses of stable equilibrium value of fixed point of secondary u and d trajectory, respectively, they are heavier than that of pion, so this cannot decay strongly. They are much lighter than the kaon so that their quantum numbers are dominated by those of u and d quark, respectively. If one accepts this interpretation the kaon cannot decay strongly from the Okubo, Zweig and Iizuka rule. All excited hadronic states can contain these excited u and d quarks from energetic point of view so it is quite natural to interpret their strong decays through these states. See the end of the last section. The mass corresponding to the stable equilibrium value $m_{u'''}$ of tertiary u trajectory is heavier than that of c but much lighter than that of t quark, while $m_{d'''}$ is close to that of b quark. If our theory turns out to be true it would be interesting to check their contribution to weak t and b quark decays.

It may be convenient to introduce detailed symbolic specification of fixed points and masses in the table form, then it is easy to show the relation between them used already in part at above discussion. For definiteness let us denote the stable fixed points of the primary, secondary tertiary and quaternary u-trajectory by x_1, x_2, x_3 and x_4 and the corresponding u masses by $m_{u'}$, $m_{u''}$, $m_{u'''}$ and $m_{u''''}$, respectively. They are related as

$$m_{u'} = x_1\, m_u\ ,\ m_{u''} = x_2\, m_{u'}\ ,\ m_{u'''} = x_3\, m_{u''}\ ,\ m_{u''''} = x_4\, m_{u'''}\ . \quad (2.4)$$

The similar applies to other families.

Once the trajectory functions are determined it is important to find their associated properties: range of trajectories, the stability check of so far untouched fixed points on secondary, tertiary and quaternary trajectories. There appear two fixed points in general as mentioned already for each trajectory function established up to now. Specifying them by f(x) the fixed point is defined as the point x_i satisfying the condition

$$f(x_i) = x_i\ . \quad (2.5)$$

We shall concentrate our attention to the properties on full trajectories from now on. Let us specify the starting points of trajectory functions and the additional fixed points on u-trajectory for so far untouched on secondary, tertiary and quaternary trajectory by x_{01}, x_{02}, x_{03}, x_{04} and x_{u1}, x_{u2}, x_{u3}, x_{u4}, respectively, and similarly for d-ones by suitable substitutions in mind. The range of these trajectories on abscissa in two-dimensional plot of them will be specified by $[x_{01}, x_{02})$, $[x_{02}, x_{03})$, $[x_{03}, x_{04})$ and $[x_{04},...)$, respectively. The numerical result is tabulated in Table IV.

Table IV. Additional properties of u- and d-Trajectories

traj.	primary	secondary	tertiary	quaternary
u	[0,1.34101)	[1.34101,1.79357)	[1.79357, 2.97445)	[2.97445,...)
d	[0,1.45485)	[1.45485, 2.08674)	[2.08674, ...)	
x_{ui}	0	1.50190	2.10175	2.97445
x_{di}	0	1.76910	2.86398	
u $f'(x_i)$ $\propto$	7.83	5.31	3.46	
d $f'(x_i)$ $\propto$		4.11	2.16	

The numbers are listed in a similar manner to those in Table III. The upper and lower line of pair of numbers in the first, second and third one correspond to the range of trajectories, fixed points and the corresponding first derivative at these points for u- and d-trajectory, respectively. One sees that all fixed points listed here take unstable equilibrium values. If there exists chaos through our proposed trajectory functions it should occur more often at the very beginning of the Big bang Universe, since it is expected that the simultaneous three-quark collisions had been occurred more often at that time. It is interesting to pursue this possibility in connection with the inhomogeneity of the matter distribution observed through the cosmic microwave background[6] in the present-day Universe.

$.3 Electron Neutrino Mass Trajectory and Its Oscillation

The upper bound of muon neutrino mass used in this section is estimated from the observed ones of electron, tau neutrino and muon one, where the last value is estimated from the proposed mass formula. To determine the upper bound of it we have applied the mass formula for two choices of tau neutrino mass: the world average[2] 18.2 MeV and CLNS result[3] 27 MeV . The corresponding muon neutrino mass becomes 81.6945 and 105.729 keV, respectively. They are reduced by some factor from the known upper bound.[6] We shall study the neutrino trajectories by combining these values with the electron neutrino mass 3 eV. To make comparison easier we tabulate the result completely

parallel to Table III and IV in the previous section. The analysis corresponding to tau-neutriono mass choice of PDG and of CLNS is specified by P and C, respectively, in the following. Table V corresponds to neutrino trajectories and stable equilibrium value of fixed points on them. Here the neutrino mass is specified by n in order to avoid complication of suffix.

Table V. Primary to Quaternary Neutrino Trajectories for PDG's and CLNS tau neutrino mass

traj.	primary	secondary	tertiary	quaternary
	(eV)	(keV)	(MeV)	(MeV)
n_e P	175.283	9.21369	0.433118	17.8945
n_e C	181.168	9.87147	0.482809	20.8597
a for P	14.3635	14.272	14.272	14.272
a for C	14.7262	14.6325	14.6325	14.6325
b for P	0	-3.98166	-7.9437	-11.794
b for C	0	-4.01251	-8.01048	-11.9005
p for P	58.4277	52.5646	47.0081	41.3155
p for C	60.3894	58.4879	48.9095	43.2048
f'(p) for P	0.246	0.272	0.304	0.345
f'(p) for C	0.244	0.269	0.299	0.339

The numbers at first two lines gives the estimated neutrino mass corresponding stable equilibrium value for each choice of tau neutrino masses, where we inserted mass units on the top of the first pair of lines owing to the rapid variation of them. The pair of numbers in the second and third line give the corresponding trajectory parameters for both choices. The last two lines give the fixed points and their stability test, correspondingly.

From these numbers one may find the tau-neutrino mass dependence of the trajectory relevant parameters. Comparing them with the corresponding parameters

in Table III, one sees that there is a similarity between the neutrino and u-quark mass trajectories.

The electron neutrino mass is close to their stable equilibrium values on the secondary and tertiary trajectories. This may explain naturally the phenomena of neutrino oscillation without worrying about their quantum number as well as large mass difference between the electron neutrino and remaining ones provided that the model is pointing the right direction. If chaos occurs through the assumed set of neutrino trajectories the missing mass[6] in the Universe can be reduced to the neutrinos. They become the dark matter candidates by assuming that they are all stable or have lifetimes in excess of the current age of the Universe.

This requires to study the properties of neutrino trajectories left untouched in Table V. They are summarized in Table VI.

Table VI. Additional properties of Neutrino Trajectories

traj.	primary	secondary	tertiary	quaternary
P	[0,1.32179)	[1.32179, 1.74472)	[1.77472, 2.85017)	[2.85017,...)
C	[0,1.31550)	[1.31550, 1.72883)	[1.72883, 2.25532)	[2.25532,...)
P x_{ti}	0	1.46464	2.00835	2.77555
C x_{ti}	0	1.45262	1.97223	2.71515
P f'(x_i) $\propto$		8.74	6.11	4.14
C f'(x_i) $\propto$		9.07	6.39	4.39

The parameters x_{ti} (i=1,...,4) represent the unstable fixed points on the corresponding trajectories. When an additional study on this line of idea is required the tables presented in this section will become useful.

$.4 Electron Mass Trajectory and Search of Its Oscillation

The model discussed above two sections has of interests in confronted with available information. The mass formula associated with electron, muon and tau gave us the precious knowledge on the constant k common to the remaining fundamental fermion family as stated already. If the present model is tested directly from an experiment associated with electron system a la Poincaré one gets solid basis on it. In the

course of stability test up to tertiary trajectory it is required to give the numbers with more figures so we give six significant figures for all of them in the next tabulation. The stable fixed points and e-trajectory parameters are shown in Table VII.

Table VII. Primary to Tertiary e-Trajectory associated parameters

trajectory	primary	secondary	tertiary
m_e (MeV)	12.1389	226.961	2982.58
a	7.49896	7.45121	7.45121
b	0	-3.12283	-6.0512
p	23.7552	18.6970	13.1414
f'(p)	0.315677	0.398524	0.567003

The meaning of the index and symbols is given in the first line and column, respectively, in the table. They are nothing but a repetition of previous pages and not necessarily be expalined in detail. One may find similarity between e-trajectories and d-ones in Table III. The range and properties of unstable fixed point of e-trajectories are given in Table VIII.

Table VIII. Additional properties of electron Trajectories

traj.	primary	secondary	tertiary	range
[0,1.52860)		[1.52860, 2.25266)	[2.25266, ...)	
p	0	1.98469	3.70250	
f'(p)	∝	2.75435	1.01248	

The last number for f'(p) for tertiary trajectory is close to 1 but slightly bigger than it so p is the unstable fixed point on that trajectory. We hope that the experiments by positronium target for both electron beam and electron-positron collider will be performed in future as a direct test of the model presented in this paper.

$.5 Discussion

We have proposed the fundamental fermion associated trajectories in this paper. In the hadronic world the masses of equilibrium values on each trajectory may be counted as the cause of its resonance behavior from the fundamental point of view as we shall discuss shortly later. One may wonder that he can reduce current u- and d-quark mass by reducing the c- and s-ones simultaneously since the certain reduction of s-qaurk one from 130 MeV were suggested by many physicists.[11] This requires the simultaneous reduction of b-quark mass as seen from $.2.

We did not prove the presence of chaos and its detail from the presence of quark mass trajectories shown. If it turns out to be true it may give us more precise information on the inhomogeneous matter distribution in the Universe. The similar possibility arises to explain the solar neutrino oscillation and the content of dark matter in the Universe without introducing extra unknown matter.

To push these ideas furthermore it is desirable to study the simultaneous collision more than two electrons as a direct test of the model. An electron beam or electron-positron collider experiment by positronium target was proposed for this purpose.

We have done a preliminary study[12] of meson and baryon mass trajectories by making use of an analogous map function as quark one and the least squares fit whenever a few of the resonant states carry one and the same quantum numbers. None of the trajectory parameter a exceeds e (a base of natural logarithm), so no fixed point of the primary trajectory exists and the higher trajectory is forbidden in the hadronic world. This may be counted as a reason why the state corresponding to the stable fixed points of secondary u- and d-trajectories will induce the hadronic resonant states. This also indicates that all mathemtically stable equilibrium values on quark trajectories may be counted as physically unstable ones as far as they lie on higher than their primary one. The stability of the proton is left as an open question.

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