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Invariability Condition of Motion Equations of Gauge Fields and Elimination of the Higgs Mechanism

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Abstract

According to the current understanding, the motion equations of gauge fields are considered invariable simultaneously when the Lagrangian of gauge fields is unchanged under the gauge transformation. But this is not true actually. It is proved in the paper that in order to make both the Lagrangian and the motion equation of gauge fields invariable simultaneously under the gauge transformation, a constraint condition between group parameters and gauge potentials should be established. For non-Abelian gauge fields, it means that the gauge potentials themselves should be invariable under the gauge transformation. It is proved that the restriction condition is actually equivalent to the Faddeew-Popov theory. In this way, an extended gauge transformation can be introduced so that the mass items of gauge particles can be added directly into the Lagrangian and the motion equations without destroying the gauge invariability, the B, R, S invariability and W, T identity are still maintained and the theory is still renormalizable. So According to the paper, the Higgs mechanism becomes unnecessary and the Higgs particles do not exist actually again in theory.
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According to the Yang-Mills theory, in order to keep the Largrangian unchanged under the local gauge transformation, the transformation rules of the field ϕ and its covariant differentiation should be defined as

$$\phi'(x) = \exp[-i\theta^\alpha(x)T^\alpha]\phi(x) \tag{1}$$

$$D'_\mu(x)\phi'(x) = \exp[-i\theta^\alpha(x)T^\alpha]D_\mu(x)\phi(x) \tag{2}$$

$$D_\mu(x) = \partial_\mu + A_\mu(x) \qquad A_\mu(x) = -igA^\alpha_\mu(x)T^\alpha \tag{3}$$

Here $\theta^\alpha(x)$ are group parameters. From Eq.(2), we can get the transformation rule of gauge potentials A^α_μ under the infinitesimal transformation

$$A'^\alpha_\mu(x) = A^\alpha_\mu(x) + f^{\alpha\beta\gamma}\theta^\beta(x)A^\gamma_\mu(x) - \frac{1}{g}\partial_\mu\theta^\alpha(x) \tag{4}$$

The intensity of the gauge field and its transformation are

$$F^\alpha_{\mu\nu}(x) = \partial_\mu A^\alpha_\nu(x) - \partial_\nu A^\alpha_\mu(x) + gf^{\alpha\beta\gamma}A^\beta_\mu(x)A^\gamma_\nu(x) \tag{5}$$

$$F'^\alpha_{\mu\nu}(x) = F^\alpha_{\mu\nu}(x) + f^{\alpha\beta\gamma}\theta^\beta(x)F^\gamma_{\mu\nu}(x) \tag{6}$$

The Lagrangian of gauge fields with zero masses

$$L_0 = -\frac{1}{4}F^\alpha_{\mu\nu}F^\alpha_{\mu\nu} \tag{7}$$

is invariable under the gauge transformation. But the Lagrangian containing mass item $m_A A^\alpha_\mu A^\alpha_\mu$ can't keep unchanged under the transformation.

However, the Lagrangian is only a form of function instead of a physical law. Its value is that we can deduce motion equations conveniently from the the Lagrangian equation. More important is to keep the motion equations unchanged under the local gauge transformation. At present the motion equations are considered invariable when the Lagrangian is unchanged under the gauge transformation. But this is actually impossible. The real situation is that in order to make the motion equations unchanged under the gauge transformation, a restriction relation between group parameters and gauge fields should be introduced. Let's first discuss electromagnetic interaction as an example. The Lagrangian is

$$L_e = -\frac{1}{4}F_{\mu\nu}F_{\mu\nu} - \frac{1}{2}\bar\psi(r_\mu\partial_\mu + m)\psi - \frac{1}{2}\psi^\tau(r^\tau_\mu\partial_\mu - m)\bar\psi^\tau + \frac{i}{2}eA_\mu(\bar\psi r_\mu\psi - \psi^\tau r^\tau_\mu\bar\psi^\tau) \tag{8}$$

The transformation rules of of spin field and gauge field under the $U(1)$ group are

$$\psi'(x) = e^{-i\theta(x)}\psi(x) \tag{9}$$

$$A'_\mu(x) = A_\mu(x) - \frac{1}{e}\partial_\mu\theta(x) \tag{10}$$

The Lagrangian is unchanged under the transformation. The motion equations of spin field and electromagnetic field are

$$r_\mu(\partial_\mu - ieA_\mu)\psi + m\psi = 0 \qquad \bar{\psi}(\partial_\mu + ieA_\mu)r_\mu - m\bar{\psi} = 0 \tag{11}$$

$$\partial^2 A_\mu = -j_\mu \tag{12}$$

$$j_\mu = \frac{i}{2}e(\bar{\psi}r_\mu\psi - \psi^\tau r^\tau_\mu\bar{\psi}^\tau) \tag{13}$$

The motion equation of spin field is unchanged under the transformation, but the motion equation of electromagnetic field becomes

$$\partial^2 A_\mu - \frac{1}{e}\partial_\mu\partial^2\theta = -j_\mu \tag{14}$$

It is obvious that the equation can't keep unchanged under the transformation unless we take

$$\partial^2\theta = \alpha = constant \tag{15}$$

Because A_μ should satisfy the Lorentz condition $\partial_\mu A_\mu = 0$ simultaneously, we should also consider the gauge invariability of the Lorentz condition. Suppose the Lorentz condition is also unchanged under the same gauge transformation, we get

$$\partial_\mu A'_\mu = \partial_\mu A_\mu - \frac{1}{e}\partial^2\theta = \partial_\mu A_\mu = 0 \tag{16}$$

So we have $\partial^2\theta = 0$. Comparing with Eq.(15), we get $\alpha = 0$. In this way, θ can't be arbitrary if we want the motion equation invariable under gauge transformation. The restriction condition is just $\partial^2\theta = 0$. If $\theta =$ constant, it is just the so-called integral gauge invariability. If $\theta \neq$ constant, it is just the so-called local gauge invariability.

The motion equations of non-Abel gauge fields with zero masses can be obtained from Eq.(7)

$$\partial_\mu F^\alpha_{\mu\nu} + gf^{\alpha\beta\gamma}A^\beta_\mu F^\gamma_{\mu\nu} = 0 \tag{17}$$

By using Eq.(4) and (6), the gauge transformation of motion equation of gauge potential A^α_μ is

$$\partial_\mu F'^\alpha_{\mu\nu} + gf^{\alpha\beta\gamma}A'^\beta_\mu F'^\gamma_{\mu\nu} = \partial_\mu(F^\alpha_{\mu\nu} + f^{\alpha\beta\gamma}\theta^\beta F^\gamma_{\mu\nu})$$

$$+ gf^{\alpha\beta\gamma}(A^\beta_\mu + f^{\beta\rho\sigma}\theta^\rho A^\sigma_\mu - \frac{1}{g}\partial_\mu\theta^\beta)(F^\gamma_{\mu\nu} + f^{\gamma\lambda\tau}\theta^\lambda F^\tau_{\mu\nu}) = 0 \tag{18}$$

By considering Eq.(17),we get

$$f^{\alpha\beta\gamma}(F^\gamma_{\mu\nu}\partial_\mu\theta^\beta + \theta^\beta\partial_\mu F^\gamma_{\mu\nu}) + gf^{\alpha\beta\gamma}f^{\gamma\rho\sigma}A^\beta_\mu\theta^\rho F^\sigma_{\mu\nu}$$

$$+ gf^{\alpha\beta\gamma}(f^{\beta\rho\sigma}\theta^\rho A^\sigma_\mu - \frac{1}{g}\partial_\mu\theta^\beta)(F^\gamma_{\mu\nu} + f^{\gamma\lambda\tau}\theta^\gamma F^\tau_{\mu\nu}) = 0 \tag{19}$$

It can be seen that when the formula is satisfied, the unique (at least the simplest) solution is

$$\partial_\mu\theta^\alpha = gf^{\alpha\beta\gamma}\theta^\beta A^\gamma_\mu \tag{20}$$

In fact, by considering Eq.(17) and (20), as well as the asymmetry of construction constant $f^{\alpha\beta\gamma}$, the left side of Eq.(19) becomes

$$f^{\alpha\beta\gamma}f^{\beta\rho\sigma}\theta^\rho A^\sigma_\mu F^\gamma_{\mu\nu} - f^{\alpha\beta\gamma}f^{\gamma\rho\sigma}\theta^\beta A^\rho_\mu F^\sigma_{\mu\nu} + f^{\alpha\beta\gamma}f^{\gamma\rho\sigma}\theta^\rho A^\beta_\mu F^\sigma_{\mu\nu}$$

$$= -(f^{\alpha\sigma\gamma}f^{\gamma\beta\rho} + f^{\alpha\beta\gamma}f^{\gamma\rho\sigma} + f^{\alpha\rho\gamma}f^{\gamma\beta\sigma})\theta^\rho A^\beta_\mu F^\sigma_{\mu\nu} \tag{21}$$

The formula is equal to zero in light of the Jacobian relation $f^{\alpha\sigma\gamma}f^{\gamma\beta\rho} + f^{\alpha\beta\gamma}f^{\gamma\rho\sigma} + f^{\alpha\rho\gamma}f^{\gamma\beta\sigma} = 0$. It can be known from Eq.(4) that Eq.(20) just means $A'^\alpha_\mu = A^\alpha_\mu$, that is to say, if we want the Lagrangian and the

motion equations of gauge fields invariable simultaneously under the gauge transformations, the only way is to let the gauge potential itself unchanged under the transformations (Because $\theta^\alpha \neq 0$ in this case, we still have $\varphi' \neq \varphi$, i.e., the transformation is still meaningful.). In fact, the forms of the motion equation of gauge fields are completely different from that of the Lagrangian, we can not expect it also unchanged under the same transformation. In order to let them unchanged simultaneously, the additional condition is needed. The condition is just $A'^\alpha_\mu = A^\alpha_\mu$. By the differential of Eq.(20), we have

$$\partial^2\theta^\alpha = gf^{\alpha\beta\gamma}(\partial_\mu\theta^\beta)A^\gamma_\mu + gf^{\alpha\beta\gamma}\theta^\beta\partial_\mu A^\gamma_\mu \tag{22}$$

If the Landau gauge condition $\partial_\mu A^\alpha_\mu = 0$ is used, we get

$$\partial^2\theta^\alpha = gf^{\alpha\beta\gamma}(\partial_\mu\theta^\beta)A^\gamma_\mu \tag{23}$$

Eq.(20) and (23) are just the relations that group parameters and gauge potentials should satisfy. There are $\alpha = 1, 2, \cdots N$ equations for N group parameters, meaning that the arbitrariness of group parameters are eliminated completely. However, as shown below, as long as the extended gauge transformation is introduced, the arbitrariness of group parameters can still be reserved. Let $\phi''(x)$, $A''^\alpha_\mu(x)$ and $D''_\mu(x)$ represent the extended gauge transformations, $\phi'(x)$, $A'^\alpha_\mu(x)$, $D'_\mu(x)$ represent the current transformations with $\phi''(x) = \phi'(x)$, and define the extend gauge transformation as

$$D''_\mu(x)\phi''(x) = \exp[-i\theta^\alpha(x)T^\alpha][D_\mu(x) + R_\mu(x)]\phi(x) \tag{24}$$

Here $R_\mu(x) = -igR^\alpha_\mu(x)T^\alpha$ is an arbitrary function. The transformation formula above can be written as

$$(\partial_\mu - igA''^\alpha_\mu(x)T^\alpha)\exp[-i\theta^\alpha(x)T^\alpha]\phi(x) = \exp[-i\theta^\alpha(x)T^\alpha][\partial_\mu - igA^\alpha_\mu(x)T^\alpha - igR^\alpha_\mu(x)T^\alpha]\phi(x) \tag{25}$$

Thus, the infinitesimal transformation rule of gauge potential $A^\alpha_\mu(x)$ becomes

$$A''^\alpha_\mu = A^\alpha_\mu + R^\alpha_\mu + f^{\alpha\beta\gamma}\theta^\beta A^\gamma_\mu - \frac{1}{g}\partial_\mu\theta^\alpha \tag{26}$$

Let $A''^\alpha_\mu = A^\alpha_\mu$ again, the Lagrangian and the motion equations of gauge fields are also unchanged under gauge transformation. By Means of the Landau gauge condition $\partial_\mu A^\alpha_\mu = 0$, we get from Eq.(26)

$$\partial_\mu\theta^\alpha = gR^\alpha_\mu + gf^{\alpha\beta\gamma}\theta^\beta A^\gamma_\mu \qquad \partial^2\theta^\alpha = g\partial_\mu R^\alpha_\mu + gf^{\alpha\beta\gamma}(\partial_\mu\theta^\beta)A^\gamma_\mu \tag{27}$$

Because R^α_μ is arbitrary, θ^α is also arbitrary. But the relation between θ^α and A^α_μ can't be completely arbitrary. It can be known from Eq.(1) that the phase angles of the φ function are still arbitrary for the arbitrariness of R^α_μ or θ^α in the extended transformation, but the forms of gauge potentials A^α_μ are determinative and unique by the relation $A''^\alpha_\mu = A^\alpha_\mu$. This result is completely different from the current viewpoint. In order to show it clearer, let's discuss classical electromagnetic theory again. The general solution of motion equation (12) is

$$A_\mu(\vec{x}, t) = A^a_\mu(\vec{x}, t) + A^b_\mu(\vec{x}, t) \tag{28}$$

Here A^a_μ is a special solution with the form

$$A^a_\mu(\vec{x}, t) = \int \frac{j_\mu(\vec{x}', t - r/c)}{cr} d\tau' \tag{29}$$

A^b_μ is the general solution satisfies the homogeneous wave equation

$$\nabla^2 A^b_\mu - \frac{\partial^2}{\partial t^2}A^b_\mu = 0 \tag{30}$$

and the Lorentz condition $\partial_\mu A^b_\mu = 0$. It can be written as the overlay of the free plane waves

$$A^b_\mu(x) = \int d^3\vec{k}[\alpha_\mu(\vec{k})e^{ikx} + b_\mu(\vec{k})e^{-ikx}] \tag{31}$$

According to the current viewpoint, $A'_\mu = A_\mu + \partial_\mu\theta$ can also be regarded as the electromagnetic potentials. But this is impossible when $\partial_\mu\theta \neq A^a_\mu$ and $\partial_\mu\theta \neq A^b_\mu$, for $\partial_\mu\theta$ with an arbitrary value can't satisfy the motion

equation of electromagnetic field. So the item $\partial_\mu\theta$ can't be regarded as the real electromagnetic potential. In fact, $\partial_\mu\theta$ would not appear in the new Lagrangian after the gauge transformation is carried out. It is only a useful tool for us to reach gauge invariability. The form of electromagnetic potential shown in Eq.(28) is unique.

The result above coincides with the Faddeev-Popov theory[1]. According to the theory, in order to avoid the infinity, integral in the function space should be restricted on the hypersurface decided by the gauge condition $F^\alpha[A^\alpha_\mu]=0$. Faddeev and Popov suggested following relation to restrict orbit integral

$$\triangle_F[A^\alpha_\mu]\cdot\int[dg]\delta(F[A^{\alpha g}_\mu])=1 \tag{32}$$

The restriction condition $\delta(F[A^{\alpha g}_\mu])$ demands $F[A^{\alpha g}_\mu]=0$ actually. Using the Landau gauge condition $\partial_\mu A^\alpha_\mu=0$, the result is

$$F[A^{\alpha g}_\mu]=0\rightarrow\partial_\mu A^{'\alpha}_\mu=\partial_\mu(A^\alpha_\mu+f^{\alpha\beta\gamma}\partial_\mu\theta^\beta A^\gamma_\mu-\frac{1}{g}\partial_\mu\theta^\alpha)=f^{\alpha\beta\gamma}\partial_\mu\theta^\beta A^\gamma_\mu-\frac{1}{g}\partial^2\theta^\alpha=0 \tag{33}$$

This is just Eq.(23). We can also get this relation from the gauge transformation of the Landau gauge condition by $(\partial_\mu A^\alpha_\mu)'=\partial_\mu A^{'\alpha}_\mu=0$, though in the Faddeev-Popov theory, this result was not expressed very clearly. If the extended gauge transformation is used, the result becomes

$$\begin{aligned}F[A^{\alpha g}_\mu]=0\rightarrow\partial_\mu A^{'\alpha}_\mu&=\partial_\mu(A^\alpha_\mu+R^\alpha_\mu+f^{\alpha\beta\gamma}\theta^\beta A^\gamma_\mu-\frac{1}{g}\partial_\mu\theta^\alpha)\\&=\partial_\mu R^\alpha_\mu+f^{\alpha\beta\gamma}\partial_\mu\theta^\beta A^\gamma_\mu-\frac{1}{g}\partial^2\theta^\alpha=0\end{aligned} \tag{34}$$

It is just Eq.(27) and is more rational then Eq.(23). In the later literatures[2], what actually used is Eq.(34). On the other hand, according to the paper, only when the relation (23) and (27) are satisfied, the motion equations of gauge fields can keep unchanged under gauge transformation, and the theory is renormalizable. It is obvious that the result of the paper is equivalent to the Faddeev-Popov theory.

The interaction theory is discussed blow. It is enough for us only to discuss the united theory of electric-weak interaction between lepton fields and gauge fields under $SU(2)\times U(1)$ group transformation. In this case, we need two gauge fields A^α_μ and B_μ. The left-hand and right-hand fields are ψ_L and l_R. The mass of A^α_μ is m_A, the mass of B_μ is zero. The mass of lepton is m_l and neutrino has no mass. The forms of mass matrix M_l and left field are

$$M_l=\begin{pmatrix}0&0\\0&m_l\end{pmatrix}\qquad\psi_L=\begin{pmatrix}\nu_L\\l_L\end{pmatrix} \tag{35}$$

The Lagrangian of the united theory of weak-electric interaction is

$$\begin{aligned}L_w=&-\bar\psi_L r_\mu(\partial_\mu-\frac{i}{2}g\tau^\alpha A^\alpha_\mu+\frac{i}{2}g'B_\mu)\psi_L-\bar l_R r_\mu(\partial_\mu+ig'B_\mu)l_R\\&-\frac{1}{4}F^\alpha_{\mu\nu}F^\alpha_{\mu\nu}-\frac{1}{4}F_{\mu\nu}F_{\mu\nu}-\frac{1}{2}m^2_A A^\alpha_\mu A^\alpha_\mu-\bar\psi_L M_l\psi_L-m_l\bar l_R\bar l_R\end{aligned} \tag{36}$$

It is noted that though $\bar\psi_L\psi_L=\bar l_R l_R=0$, these two items are useful for us to reach the correct motion equations. Because $A''^\alpha_\mu=A^\alpha_\mu$, meaning $A''^\alpha_\mu A''^\alpha_\mu=A^\alpha_\mu A^\alpha_\mu$, we can add mass item into the Largrangian and motion equation directly without violationing the gauge invariability. The motion equations are

$$r_\mu(\partial_\mu+ig'B_\mu)l_R+m_l l_R=0 \tag{37}$$

$$r_\mu(\partial_\mu-\frac{i}{2}g\tau^\alpha A^\alpha_\mu+\frac{i}{2}g'B_\mu)\psi_L+m_L\psi=0 \tag{38}$$

$$\partial_\mu F^\alpha_{\mu\nu}+gf^{\alpha\beta\gamma}A^\beta_\mu F^\gamma_{\mu\nu}+m^2_A A^\alpha_\mu A^\alpha_\mu=-j^\alpha_{A\mu} \tag{39}$$

$$\partial^2 B_\mu=-j_{B\mu} \tag{40}$$

$$j^\alpha_{A\mu}=\frac{i}{2}g\bar\psi_L r_\mu\tau^\alpha\psi_L\qquad j_{B\mu}=\frac{i}{2}g'(\bar\psi_L r_\mu\psi_L+2\bar l_R r_\mu l_R) \tag{41}$$

It is obvious that all of them are unchanged under the extended gauge transformation.

It is proved below that in this case the B, R, S invariability and W, T identity are still maintained and the theory is still renormalizable. We consider a system consists of Fermi fields ψ, gauge fields A^α_μ and ghost fields C_α and C^+_α. The effective action is

$$S_{eff} = S + S_f + S_g \tag{42}$$

In the formula, S is the action of Fermi fields and gauge fields, S_f is the fixed action, S_g is the action of ghost fields. Suppose gauge field's mass is m_A if they have, we have

$$S = \int d^4x[\bar{\psi}(i\gamma_\mu\partial_\mu + g\frac{\lambda^\alpha}{2}\gamma_\mu A^\alpha_\mu - m)\psi - \frac{1}{4}(\partial_\mu A^\alpha_\nu - \partial_\nu A^\alpha_\mu + gf^{\alpha\beta\gamma}A^\beta_\mu A^\gamma_\nu)^2 + m^2_A A^\alpha_\mu A^\alpha_\mu] \tag{43}$$

$$S_f = -\int d^4x \frac{1}{2\zeta}(\partial_\mu A^\alpha_\mu)^2 \tag{44}$$

$$S_g = \int d^4x \bar{\psi} C^+_\alpha (D^{\alpha\beta}_\mu C_\beta) \qquad\qquad D^{\alpha\beta}_\mu = \partial_\mu \delta^{\alpha\beta} - gf^{\alpha\beta\gamma}A^\gamma_\mu \tag{45}$$

The action S is unchanged under gauge transformation. In order let $S_f + S_g$ unchanged, the B, R, S transformation is introduced with

$$\psi \to \psi' = \psi - i\mathcal{X}\frac{\lambda^\alpha}{2}C_\alpha\psi \qquad\qquad \delta\psi = -i\mathcal{X}\frac{\lambda^\alpha}{2}C_\alpha\psi \tag{46}$$

$$\bar{\psi} \to \bar{\psi}' = \bar{\psi} + i\bar{\psi}\mathcal{X}\frac{\lambda^\alpha}{2}C_\alpha \qquad\qquad \delta\bar{\psi} = i\bar{\psi}\mathcal{X}\frac{\lambda^\alpha}{2}C_\alpha \tag{47}$$

$$A^\alpha_\mu \to A'^\alpha_\mu = A^\alpha_\mu - \frac{1}{g}\mathcal{X}D^{\alpha\beta}_\mu C_\beta \qquad\qquad \delta A^\alpha_\mu = -\frac{1}{g}\mathcal{X}D^{\alpha\beta}_\mu C_\beta \tag{48}$$

$$C_\alpha \to C'_\alpha = C_\alpha + \frac{1}{2}\mathcal{X}f^{\alpha\beta\gamma}C_\beta C_\gamma \qquad\qquad \delta C_\alpha = \frac{1}{2}\mathcal{X}f^{\alpha\beta\gamma}C_\beta C_\gamma \tag{49}$$

$$C^+_\alpha \to C'^+_\alpha = C^+_\alpha - \frac{1}{\zeta g}\mathcal{X}\partial_\mu A^\alpha_\mu \qquad\qquad \delta C^+_\alpha = -\frac{1}{\zeta g}\mathcal{X}\partial_\mu A^\alpha_\mu \tag{50}$$

Here $\mathcal{X}$, ψ, $\bar{\psi}$, C_α and C^+_α are the anticommute c numbers with relations

$$\mathcal{X}\psi = -\psi\mathcal{X} \qquad \mathcal{X}\bar{\psi} = -\bar{\psi}\mathcal{X} \qquad \mathcal{X}C_\alpha = -C_\alpha\mathcal{X} \qquad \mathcal{X}^2 = 0 \tag{51}$$

It can be seen from Eq.(48) that the B, R, S transformation is actually equivalent to group parameters $\theta^\alpha \to \mathcal{X}C_\alpha$. So the action S still keeps unchanged under the B, R, S transformation. On the other hand, we have

$$\delta(S_f + S_g) = \int d^4x[-\frac{1}{\zeta}(\partial_\mu A^\alpha_\mu)\partial_\nu \delta A^\alpha_\nu + \delta C^+_\alpha \partial_\mu (D^{\alpha\beta}_\mu C_\beta) + C^+_\alpha \partial_\mu \delta(D^{\alpha\beta}_\mu C_\beta)] \tag{52}$$

According to the current theory, by using Eq.(48) and (50), we have

$$-\frac{1}{\zeta}(\partial_\mu A^\alpha_\mu)\partial_\nu \delta A^\alpha_\nu + \delta C^+_\alpha \partial_\mu (D^{\alpha\beta}_\mu C_\beta) = \frac{1}{\zeta g}(\partial_\mu A^\alpha_\mu)\partial_\nu(\mathcal{X}D^{\alpha\beta}_\nu C_\beta) - \frac{1}{\zeta g}(\mathcal{X}\partial_\nu A^\alpha_\nu)\partial_\mu(D^{\alpha\beta}_\mu C_\beta) = 0 \tag{53}$$

The formula also implies that $\mathcal{X}$ has nothing to do with space-time coordinate x, i.e., $\partial_\nu \mathcal{X} = 0$. It can also be proved $\delta(D^{\alpha\beta}_\mu C_\beta) = 0$ [3]. So we get $\delta(S_f + S_g) = 0$ according to the current theory.

According to the new scheme in the paper (For simplification, the extended gauge transformation is not considered, the result is the same.), we have $\delta A^\alpha_\mu = 0$ or $\mathcal{X}D^{\alpha\beta}_\mu C_\beta = 0$. Meanwhile, by $\partial_\mu \mathcal{X} = 0$, from Eq.(50) and (51), we get

$$\delta C^+_\alpha \partial_\mu (D^{\alpha\beta}_\mu C_\beta) = \frac{1}{2}\mathcal{X}f^{\alpha\beta\gamma}C_\beta C_\gamma \partial_\mu (D^{\alpha\beta}_\mu C_\beta) = \frac{1}{2}f^{\alpha\beta\gamma}C_\beta C_\gamma \partial_\mu (\mathcal{X}D^{\alpha\beta}_\mu C_\beta) = 0 \tag{54}$$

Because $\delta(D^{\alpha\beta}_\mu C_\beta) = 0$, so we still have $\delta(S_f + S_g) = 0$. Therefore, in light of the now scheme, S_{eff} is still unchanged under the B, R, S transformation. On the other hand, we define again

$$\delta\psi' = -i\frac{\lambda^\alpha}{2}C_\alpha\psi \qquad \delta\bar{\psi}' = i\bar{\psi}\frac{\lambda^\alpha}{2}C_\alpha \qquad \delta A'^\alpha_\mu = -\frac{1}{g}D^{\alpha\beta}_\mu C_\beta \qquad \delta C'_\alpha = \frac{1}{2}f^{\alpha\beta\gamma}C_\beta C_\gamma \tag{55}$$

That is to say, there is a difference of factor $\mathcal{X}$ between $\delta\psi'$, $\delta\bar{\psi}'$, $\delta A'^{\alpha}_{\mu}$, $\delta C'_{\alpha}$ and $\delta\psi$, $\delta\bar{\psi}$, δA^{α}_{μ}, δC_{α}. In this case, we have $\delta A'^{\alpha}_{\mu} \neq 0$, and can prove $\delta(\delta\psi') = 0$, $\delta(\delta\bar{\psi}') = 0$, $\delta(\delta A'^{\alpha}_{\mu}) = 0$, $\delta(\delta C'_{\alpha}) = 0$ as shown in the current theory[3]. By introducing the corresponding external sources $\bar{K}$, K, U^{α}_{μ} and V^{α}, the generating function of the Green's function can be written as

$$Z(\bar{\eta},\eta,J^{\alpha}_{\mu},\zeta^{+}_{\alpha},\zeta_{\alpha},\bar{K},K,U^{\alpha}_{\mu},V^{\alpha}) = \int [d\psi][d\bar{\psi}][dA^{\alpha}_{\mu}][dC_{\alpha}][dC^{+}_{\alpha}]\exp(iS_{eff})$$

$$\times \exp\{i\int dx^4(\bar{\eta}\psi + \bar{\psi}\eta + J^{\alpha}_{\mu}A^{\alpha}_{\nu} + \kappa^{+}_{\alpha}C_{\alpha} + C^{+}_{\alpha}\kappa_{\alpha} + \bar{K}\delta\psi' + \delta\bar{\psi}'K + U^{\alpha}_{\mu}\delta A'^{\alpha}_{\mu} + V^{\alpha}\delta C'_{\alpha})\} \tag{56}$$

Because of $\delta A'^{\alpha}_{\mu} \neq 0$, the formula is completely the same as that in the current theory. From Eq.(56), we can obtain the same W,T identity, so the theory is still renormalizable.

Up to now, all main problems have been solved in principle. But in the practical application, we use the mass eigen states to construct the Largrangian, we need to discuss this problem. Define

$$W^{+}_{\mu} = \frac{1}{\sqrt{2}}(A^{1}_{\mu} + iA^{2}_{\mu}) \qquad W^{-}_{\mu} = \frac{1}{\sqrt{2}}(A^{1}_{\mu} - iA^{2}_{\mu}) \tag{57}$$

$$Z_{\mu} = A^{3}_{\mu}\cos\theta_w - B_{\mu}\sin\theta_w \qquad A_{\mu} = A^{3}_{\mu}\sin\theta_w + B_{\mu}\cos\theta_w \tag{58}$$

$$\sin\theta_w = \frac{g}{\sqrt{g^2+g'^2}} \qquad \cos\theta_w = \frac{g'}{\sqrt{g^2+g'^2}} \tag{59}$$

and take

$$-\frac{1}{2}m^2_A A^{\alpha}_{\mu}A^{\alpha}_{\mu} = -m^2_w W^{+}_{\mu}W^{-}_{\mu} - \frac{1}{2}m^2_z Z_{\mu}Z_{\mu} + K_1 \tag{60}$$

Putting Eq.(57) and (58) into the right side of the formula above and comparing two sides, we have

$$m_A = m_w = m_z\cos\theta_w \tag{61}$$

$$K_1 = \frac{1}{2}m^2_z\sin^2\theta_w B_{\mu}B_{\mu} - m^2_z\sin\theta_w\cos\theta_w A^{3}_{\mu}B_{\mu} \tag{62}$$

By calculating the low order process of μ^- decay, and comparing the result with the Fermi theory, we can also get $G/\sqrt{2} = g^2/(8m^2_w)$, from which we can decide the masses of $W^{\pm}$ particles. Then from Eq.(61) the mass of Z^0 particle can also be determined. The result is completely the same as that in the current theory in which the Higgs mechanics is used. As for the mass items of leptons, because $\bar{l}l = \bar{l}_L l_L + \bar{l}_R l_R + \bar{l}_L l_R + \bar{l}_R l_L$, let $K_2 = m_l(\bar{l}_L l_R + \bar{l}_R l_L)$, we have

$$m_l(\bar{l}_L l_L + \bar{l}_R l_R) = m_l\bar{l}l - K_2 \tag{63}$$

Let $K = K_1 + K_2$ again, the Lagrangian can be written in the forms of mass eigen states

$$L_m = L_0(W^{\pm}_{\mu}, Z_{\mu}, A_{\mu}, \nu, l) - m^2_w W^{+}_{\mu}W^{-}_{\mu} - \frac{1}{2}m^2_Z Z_{\mu}Z_{\mu} + m_l\bar{l}l + K \tag{64}$$

Here L_0 represents the items of dynamic energy and interaction of the fields. Comparing with the current theory, the item K is unnecessary and should be eliminated. As shown below, we can do it by introducing a proper gauge condition as done in the Higgs mechanics.

According to the current theory, the generating function of the Green function is

$$Z = \int [dA^{\alpha}_{\mu}][dB_{\mu}][d\bar{\psi}][d\psi][d\phi^{+}][d\phi]\triangle_{F_A}[A^{\alpha}_{\mu}]\triangle_{F_B}[B_{\mu}]\delta(F^{\alpha}_{A}[A^{\alpha}_{\mu}])\delta(F_B[B_{\mu}])$$

$$\times \exp[i\int dx^4(L_0 + J^{\alpha}_{\mu}A^{\alpha}_{\mu} + J_{\mu}B_{\mu} + \bar{\eta}\psi + \bar{\psi}\eta + \lambda\phi^{+} + \lambda^{+}\phi)] \tag{65}$$

Here L_0 is the Lagrangian of gauge fields and spin fields without containing the mass items of the fields. By using the Landau gauge

$$F^{\alpha}_{A}[A^{\alpha}_{\mu}] \to \partial_{\mu}A^{\alpha}_{\mu} = 0 \qquad F_B[B_{\mu}] \to \partial_{\mu}B_{\mu} = 0 \tag{66}$$

and introducing the ghost fields C_i and C^{+}_{i} relative to A^{α}_{μ}, as well as C and C^{+} relative to B_{μ}, Eq.(65) can be written as

$$Z = \int [dA^{\alpha}_{\mu}][dB_{\mu}][d\bar{\psi}][d\psi][d\phi^{+}][d\phi][dC^{+}_{\alpha}][dC_{\alpha}][dC^{+}][dC]\exp[iS_{eff}$$

$$+ i \int dx^4 (J^\alpha_\mu A^\alpha_\mu + J_\mu B_\mu + \bar{\eta}\psi + \bar{\psi}\eta + \lambda\phi^+ + \lambda^+\phi + \kappa_\alpha C^+_\alpha + \kappa^+_\alpha C_\alpha + \varsigma C^+ + \varsigma^+ C)] \tag{67}$$

S_{eff} is the effective action

$$S_{eff} = S_0 + S_1 + S_2 + S_3 \tag{68}$$

In the formula, S_0 is the action of gauge fields and spin fields, S_1 is the action of the gauge fixed items, S_2 and S_3 are the action of the ghost fields and the Higgs fields with

$$S_0 = \int d^4x L_0 \qquad\qquad S_1 = -\int d^4x \frac{1}{2\zeta}[(\partial_\mu A^\alpha_\mu)^2 + (\partial_\mu B_\mu)^2] \tag{69}$$

$$S_2 = \int d^4x d^4y [C^+_\alpha M_{\alpha\beta} C_\beta + C^+ M C + C^+_\alpha M_\alpha C + C^+ M_\alpha C_\alpha] \tag{70}$$

$$S_3 = \int d^4x [(\partial_\mu - \frac{i}{2} g\tau^\alpha A^\alpha_\mu + \frac{i}{2} g' B_\mu)^+ \phi^+ (\partial_\mu - \frac{i}{2} g\tau^\alpha A^\alpha_\mu + \frac{i}{2} g' B_\mu)\phi - V(\phi^+, \phi)] \tag{71}$$

Let $\phi = \phi' + \nu$, ν is the vacuum expectation value of the Higgs field. After the vacuum symmetry is broken and the Higgs particles obtain masses, but the following items would appear in the Higgs action [3]

$$-\frac{1}{4}\partial_\mu \phi^{'+} (g\tau^\alpha A^\alpha_\mu + g' B_\mu)\nu + \frac{1}{4}\nu^+ (g\tau^\alpha A^\alpha_\mu + g' B_\mu)\partial_\mu \phi' \tag{72}$$

This is the interaction item of two points and should be canceled. We do it at present by using the so-called R_ζ gauge condition, i.e., replacing Eq.(66) with

$$F^\alpha_A[A^\alpha_\mu] = \partial_\mu A^\alpha_\mu + \frac{i}{4} g\zeta(\phi^{'+}\tau^\alpha \nu - \nu^+ \nu^\alpha \phi') = 0 \tag{73}$$

$$F_B[B_\mu] = \partial_\mu B_\mu + \frac{i}{4} g'\zeta(\phi^{'+}\nu - \nu^+ \phi') = 0 \tag{74}$$

and put them into Eq.(65), the action relative to Eq.(72) can be eliminated.

Similarly, when the mass eigen states of $W^\pm$ and Z^0 particles are used to construct the Lagrangian, we let $S_0 \to S'_0$ with

$$S'_0 = \int d^4x (L + K) \tag{75}$$

In order to eliminate the K item in this case, we replace Eq.(51) with

$$F_B[B_\mu] \to F^1_B[B_\mu] = (\partial_\mu B_\mu)^2 + 2\zeta K = 0 \tag{76}$$

Then the further transformation is taken

$$F^1_B[B_\mu] \to F^1_B[B_\mu] - P(x) \tag{77}$$

Putting them into the generating function of Green function and multiplying a factor

$$[dP(x)] \exp[-\frac{i}{2\zeta}\int d^4x P(x)] \tag{78}$$

we get the result

$$S_1 = -\int d^4x [\frac{1}{2\zeta}(\partial_\mu A^\alpha_\mu)^2 + \frac{1}{2\zeta}(\partial_\mu B_\mu)^2 + K] \tag{79}$$

Plus it with Eq.(75), the surplus K item is eliminated. On the other hand, after the gauge condition (76) is introduced, the action quantity of the ghost particles should also be changed. Because the Higgs fields do not exist, we have $M_\alpha = 0$ in Eq.(70), as well as

$$M_{\alpha\beta} = f^{\alpha\beta\gamma} A^\gamma_\mu \partial_\mu \delta^4(x-y) - \frac{1}{g}\delta_{\alpha\beta}\partial^2 \delta^4(x-y) \qquad M = -\frac{1}{g'}\partial^2 \delta^4(x-y) \tag{80}$$

Because R^α_μ does not contain θ_A obviously in the extended gauge transformation, $M_{\alpha\beta}$ does not change, but M would change when the gauge condition (76) is used. Because A^α_μ and B_μ are invariable under the extended gauge transformation, K_1 is also unchanged. But K_2 becomes

$$K''_2 = K'_2 = m_l(\bar{l}_L l'_R + \bar{l}'_R l') = m_l(\bar{l}_L l_R + \bar{l}_R l_L) - m_l[(\theta^1_A - i\theta^3_A)\bar{\nu}_L l_R + (\theta^1_A + i\theta^3_A)\bar{l}_R \nu_L + i(\theta_B - \theta^3_A)\bar{l} l] \tag{81}$$

So $K'' = K_2''$. According to the extended gauge transformation, we have

$$M' = \frac{\delta F[B_\mu^g]}{\delta\theta_B} = \frac{\delta}{\delta\theta_B}[(\partial_\mu B_\mu'')^2 + 2\zeta K''] = [-\frac{2}{g'}(\partial_\mu B_\mu + \partial_\mu R_\mu^B - \frac{1}{g'}\partial^2\theta_B)\partial^2 - 2i\zeta m_l\bar{l}l]\delta^4(x-y) \tag{82}$$

Here R_μ^B is an arbitrary function introduced in the extended gauge transformation of gauge field B_μ. Let $M' = M + \triangle M$, comparing with Eq.(80), we have

$$\triangle M = [-\frac{2}{g'}(\partial_\mu B_\mu + \partial_\mu R_\mu^B - \frac{1}{g'}\partial^2\theta_B - \frac{1}{2})\partial^2 - 2i\zeta m_l\bar{l}l]\delta^4(x-y) \tag{83}$$

In this case, $S_2 \to S_2'$ with

$$S_2' = S_2 + \triangle S_2 \qquad\qquad \triangle S_2 = \int d^4x d^4y C^+ \triangle M C \tag{84}$$

Because R_μ^B is arbitrary, we can redefine it as

$$R_\mu'^B = R_\mu^B - \frac{1}{2}x_\mu + \int dx_\mu \frac{ig'\zeta m_l\bar{l}lC}{\partial^2 C} \tag{85}$$

Thus, we have

$$C^+\triangle M C = -\frac{2C^+}{g'}(\partial_\mu B_\mu + \partial_\mu R'^B_\mu - \frac{1}{g'}\partial^2\theta_B)\partial^2 C\delta^4(x-y) \tag{86}$$

According to the Landau gauge and the extended gauge transformation Eq.(27), we have $\partial_\mu B_\mu = 0$, $\partial^2\theta_B - g'\partial_\mu R_\mu^{'b} = 0$, so $C^+\triangle M C = 0$, $\triangle S_2 = 0$ and $S_1' + S_2' = S_1 + S_2$. In this way, the B, R, S invariability is still maintained. So W, T identity still holes and the theory is still renormalizable.

In sum, in order to make the Largrangian and the motion equations of gauge fields invariable simultaneously under the gauge transformation, we have to take $A_\mu^{'\alpha} = A_\mu^\alpha$. In this way, the mass items of gauge particles can be added into the Lagrangian directly and the gauge invariability still holds. In this way, the Higgs mechanism becomes unnecessary and the Higgs particles are regarded not to exist in theory. In fact, though great endeavors have been done, we can't find any clue about the Higgs particles up to now.

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Referece

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