Revised Version

Invariability Conditions of Motion Equations of non-Abelian Gauge Fields and Elimination of Higgs Mechanism

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Abstract

According to the current understanding, the motion equations of gauge fields are also invariable when the Lagrangian of gauge fields is unchanged under the gauge transformation. But this is untrue in general. It is proved in the paper that in order to keep both the Lagrangian and the motion equation of non-Abelian gauge fields unchanged simultaneously under the gauge transformation, a restriction condition should be established between the gauge potentials and the group parameters. The result is equivalent to the Faddeev–Popov theory. For non-Abelian gauge fields, it leads to that the gauge potentials themselves are unchanged under gauge transformations. In this way, the mass item can be added into the Lagrangian directly without violating gauge invariability and the theory is also renormalizable, so that the Higgs mechanism becomes unnecessary.

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According to the Yang-Mills theory, in order to keep the Largrangian unchanged under the local gauge transformation, the transformation rules of the field ϕ and its covariant differentiation should be defined as

$$\phi'(x) = u(\theta)\phi(x) = \exp[-i\theta^{\alpha}(x)T^{\alpha}]\phi(x) \tag{1}$$

$$D'_{\mu}(x)\phi'(x) = u(\theta)D_{\mu}(x)\phi(x) = \exp[-i\theta^{\alpha}(x)T^{\alpha}]D_{\mu}(x)\phi(x) \tag{2}$$

Here $D_{\mu}(x) = \partial_{\mu} + A_{\mu}(x)$ $A_{\mu}(x) = -igA^{\alpha}_{\mu}(x)T^{\alpha}$, T^{α} are group elements and $\theta^{\alpha}(x)$ are group parameters which forms of functions are considered arbitrary at present. From Eq.(2), we can get the gauge transformation rule of potentials A^{α}_{μ}

$$A'^{\alpha}_{\mu}(x) = A^{\alpha}_{\mu}(x) + f^{\alpha\beta\gamma}\theta^{\beta}(x)A^{\gamma}_{\mu}(x) - \frac{1}{g}\partial_{\mu}\theta^{\alpha}(x) \tag{3}$$

The intensities of gauge fields are defined as

$$F^{\alpha}_{\mu\nu}(x) = \partial_{\mu}A^{\alpha}_{\nu}(x) - \partial_{\nu}A^{\alpha}_{\mu}(x) + gf^{\alpha\beta\gamma}A^{\beta}_{\mu}(x)A^{\gamma}_{\nu}(x) \tag{4}$$

Its gauge transformation is

$$F'^{\alpha}_{\mu\nu}(x) = F^{\alpha}_{\mu\nu}(x) + f^{\alpha\beta\gamma}\theta^{\beta}(x)F^{\gamma}_{\mu\nu}(x) \tag{5}$$

The Largrangian of non-Abelian gauge fields with zero masses

$$L_0(x) = -\frac{1}{4}F^{\alpha}_{\mu\nu}(x)F^{\alpha}_{\mu\nu}(x) \tag{6}$$

is unchanged under gauge transformation. But the Largrangian containing mass items can't keep unchanged under gauge transformation.

It seems a common view that the motion equations are also invariable if the Largrangian of gauge field is unchanged under gauge transformation. However, this is untrue. It is easy to prove that the motion equation of gauge field can not keep unchanged under gauge transformation thought the Largrangian of gauge field is unchanged for their forms are different completely. In order to keep the motion equation of gauge field unchanged simultaneously, a restriction relation should be established between the gauge potentials and the group parameters. The free electromagnetic field or $U(1)$ gauge field is discussed at first. By means of the Lorentz condition $\partial_\mu A_\mu = 0$, electromagnetic field's motion equation is

$$\partial_\mu F_{\mu\nu} = \partial^2 A_\nu - \partial_\nu \partial_\mu A_\mu = \partial^2 A_\nu = 0 \tag{7}$$

Under $U(1)$ gauge transformation, we have

$$A'_\nu(x) = A_\nu(x) - \frac{1}{g}\partial_\nu \theta(x) \tag{8}$$

So the gauge transformation of motion equation (8) is

$$\partial_\mu F'_{\mu\nu} = \partial^2 A'_\nu(x) = \partial^2 A_\nu(x) - \partial_\nu \partial^2 \theta(x) = -\partial_\nu \partial^2 \theta(x) = 0 \tag{9}$$

In order to keep the motion equation unchanged, we must have

$$\partial^2 \theta = b = constant \tag{10}$$

This is just the restriction condition for $U(1)$ group parameter. It means that the group parameter can't take arbitrary form. Only when Eq.(11) is satisfied, the motion equation of field can be unchanged under gauge transformation. For non-Abelian gauge field, the motion equation is [1]

$$\partial_\mu F^\alpha_{\mu\nu} + g f^{\alpha\beta\gamma} A^\beta_\mu F^\gamma_{\mu\nu} = 0 \tag{11}$$

Its form is completely different from the Largrangian. Under gauge transformation defined in Eq.(4) and (6), the motion equation becomes

$$\partial_\mu F'^\alpha_{\mu\nu} + g f^{\alpha\beta\gamma} A'^\beta_\mu F'^\gamma_{\mu\nu} = \partial_\mu (F^\alpha_{\mu\nu} + f^{\alpha\beta\gamma}\theta^\beta F^\gamma_{\mu\nu})$$

$$+ g f^{\alpha\beta\gamma}(A^\beta_\mu + f^{\beta\rho\sigma}\theta^\rho A^\sigma_\mu - \frac{1}{g}\partial_\mu \theta^\beta)(F^\gamma_{\mu\nu} + f^{\gamma\lambda\omega}\theta^\lambda F^\omega_{\mu\nu}) = 0 \tag{12}$$

By considering Eq.(11), the formula becomes

$$f^{\alpha\beta\gamma}[(\partial_\mu \theta^\beta)F^\gamma_{\mu\nu} + \theta^\beta \partial_\mu F^\gamma_{\mu\nu}] + g f^{\alpha\beta\gamma} f^{\gamma\lambda\omega} A^\beta_\mu \theta^\lambda F^\omega_{\mu\nu}$$

$$+ g f^{\alpha\beta\gamma}(f^{\beta\rho\sigma}\theta^\rho A^\sigma_\mu - \frac{1}{g}\partial_\mu \theta^\beta)(F^\gamma_{\mu\nu} + f^{\gamma\lambda\omega}\theta^\lambda F^\omega_{\mu\nu}) = 0 \tag{13}$$

It is easy to see that the simplest solution of the formula above is

$$\partial_\mu \theta^\alpha = g f^{\alpha\beta\gamma}\theta^\beta A^\gamma_\mu \tag{14}$$

In fact, by considering Eq.(11) and (14), as well as the anti-symmetry relation of group construction constant $f^{\alpha\beta\gamma}$, the left side of Eq.(13) becomes

$$f^{\alpha\beta\gamma} f^{\beta\rho\sigma}\theta^\rho A^\sigma_\mu F^\gamma_{\mu\nu} - f^{\alpha\beta\gamma} f^{\gamma\rho\sigma}\theta^\beta A^\rho_\mu F^\sigma_{\mu\nu} + f^{\alpha\beta\gamma} f^{\gamma\rho\sigma}\theta^\rho A^\beta_\mu F^\sigma_{\mu\nu}$$

$$= -(f^{\alpha\sigma\gamma} f^{\gamma\rho\beta} + f^{\alpha\rho\gamma} f^{\gamma\beta\sigma} + f^{\alpha\beta\gamma} f^{\gamma\sigma\rho})\theta^\rho A^\beta_\mu F^\sigma_{\mu\nu} \tag{15}$$

By the Jacobian relation $f^{\alpha\sigma\gamma} f^{\gamma\rho\beta} + f^{\alpha\rho\gamma} f^{\gamma\beta\sigma} + f^{\alpha\beta\gamma} f^{\gamma\sigma\rho} = 0$, we prove that the Eq.(14) is the solution of Eq.(13). It can be known from Eq.(3) that Eq.(14) just means $A'^\alpha_\mu = A^\alpha_\mu$, i.e., gauge potential itself is unchanged under gauge transformation. The result notes that if we want to keep the

Lagrangian and the motion equation of non-Alberian gauge field invariable simultaneously under gauge transformation, the simplest way is to let the gauge potential unchanged under the transformations. In fact, this is unique way to keep both unchanged under gauge transformation.

It should be emphasized that motion equation can't contain group parameters after gauge transformation. Because the concrete forms of gauge potentials are decided by the motion equation (instead of the Lagrange). If the motion equation contains group parameters after gauge transformation is introduced, the function forms of gauge potentials would become arbitrary owing to the arbitrariness of group parameters. This kind of gauge potentials is meaningless in physics. However, according to the current gauge theory, the function forms of group parameters are thought completely arbitrary without any restriction so that motion equation obviously contains group parameter after gauge transformation, so it is unreasonable.

For $SU(N)$ gauge group with $\alpha = 1, 2, \cdots N$, Eq.(15) represents $4N$ equations. So there are also $4N$ independent group parameters, i.e., there are 4 sets group parameters satisfies Eq.(15). It means that the forms of group parameters are not unique. Because $\theta^\alpha(x) \neq constant$, even though gauge potentials are unchanged under gauge transformation, the gauge transformations of other fields $\phi(x)$ and their covariant differential are still local and meaningful.

It is shown below that the result above coincides with the Faddeev-Popov theory [2]. In order to avoid infinity, Faddeev and Popov suggested that the integral over function space should be restricted on the hyper-surface decided by gauge condition

$$F[A_\mu^\alpha] = 0 \qquad\qquad \alpha = 1, 2, \cdots N \tag{16}$$

The following relation is used to restrict orbit integral

$$\triangle_F[A_\mu^\alpha] \cdot \int [dg]\delta(F[A_\mu^{\alpha g}]) = 1 \tag{17}$$

The restriction condition $\delta(F[A_\mu^{\alpha g}])$ demands $F[A_{mu}^{\alpha g}] = 0$ actually. For $U(1)$ gauge field, by taking the Landau gauge condition $F[A_{mu}^\alpha] = \partial_\mu A_\mu = 0$, we have

$$F[A_\mu^g] = 0 \to \partial_\mu A'_\mu = \partial_\mu(A_\mu + \frac{1}{g}\partial_\mu\theta) = \frac{1}{g}\partial^2\theta = 0 \tag{18}$$

The result coincides with Eq.(11) (Take $b = 0$). For non-Abelian gauge fields, by the Landau gauge condition $\partial_\mu A_\mu^\alpha = 0$, we have

$$F[A_\mu^{\alpha g}] = 0 \to \partial_\mu A'^\alpha_\mu = \partial_\mu(A_\mu^\alpha + f^{\alpha\beta\gamma}\theta^\beta A_\mu^\gamma - \frac{1}{g}\partial_\mu\theta^\alpha) = \partial_\mu(f^{\alpha\beta\gamma}\theta^\beta A_\mu^\gamma - \frac{1}{g}\partial_\mu\theta^\alpha) = 0 \tag{19}$$

The formula can be written as

$$f^{\alpha\beta\gamma}\theta^\beta A_\mu^\gamma - \frac{1}{g}\partial_\mu\theta^\alpha = b_\mu \qquad\qquad \partial_\mu b_\mu = 0 \tag{20}$$

Taking the simplest form to let $b_\mu = 0$, we get Eq.(15). So the result in this paper coincides with the Faddeev-Popov theory, or it is the simplest form of the Faddeev-Popov theory.

Because the gauge potential is unchanged under gauge transformation with $A'^\alpha_\mu = A^\alpha_\mu$ for non-Abelian gauge field, we can add the mass item into the Lagrangian directly without violating gauge invariability. Therefore, the Lagrangian

$$L = -\frac{1}{4}F^\alpha_{\mu\nu}F^\alpha_{\mu\nu} - \frac{1}{2}m^2 A^\alpha_\mu A^\alpha_\mu \tag{21}$$

is invariable under gauge transformation. When the interaction between gauge particles and other particles are considered, the corresponding W, T identity can also be obtained. The theory is also renormalizable and can be simplified greatly. In this way, the Higgs mechanism becomes unnecessary.

The further and detail discussion will be provided later.

References

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(2) Faddeev, V. N. Popov, Phys. Letters, 25B, 29 (1967).