

A QCD ANALYSIS OF THE HIGH ENERGY e^+e^- DATA FROM PETRA

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We analyse the recent high energy e^+e^- data from PETRA in the context of QCD. Our analysis takes into account single and double gluon bremsstrahlung, the weak decays of the bottom and charm quarks, and the Q^2 -dependence of the fragmentation functions in the spirit of QCD. An attempt is made to extract the QCD coupling constant $\alpha_s(Q^2)$.

The recent high energy e^+e^- experiments at PETRA [1] have conformed the three-jet structure expected in the gluon bremsstrahlung picture of Quantum Chromodynamics, QCD [2]. In fact, theoretical predictions about jet broadening and the associated effects in the various jet distributions abound in the literature [2–4] and preceded the experimental confirmation. The present data e^+e^- data are in broad qualitative agreement with these predictions. The purpose of this note is to report an analysis that aims at a detailed quantitative comparison of QCD effects with the experimental measurements in e^+e^- annihilation. The motivation is that there are several confluent effects due to heavy quark thresholds in e^+e^- annihilation. A reliable quantitative estimate of pure QCD effects in e^+e^- annihilation cannot bypass this complication. An equally important effect is that the detector acceptance, selection criterion, efficiencies and the radiative corrections cannot be implemented if a detailed Monte-Carlo program is not available. A first attempt towards this goal was made in ref. [4] and a Monte-Carlo program based on this work already exists^{†1}. Our analysis differs from the pres-

ently available model [4]^{†1} in at least *three* important theoretical inputs.

(i) We take into account $O(\alpha_s^2)$ contributions due to the QCD processes $e^+e^- \rightarrow q\bar{q}gg, q\bar{q}q\bar{q}$ ($q = u, d, s, c, b$), in addition to the $O(\alpha_s)$ process $e^+e^- \rightarrow q\bar{q}g$ considered earlier [2–4].

(ii) The production and decays of heavy quarks (charm and bottom) in e^+e^- annihilation are implemented in our analysis. The associated p_T -broadening effects as well as mass effects in hard gluon bremsstrahlung off the heavy quarks were neglected in ref. [4].

(iii) In addition to the gluon bremsstrahlung, QCD also predicts a Q^2 -evolution of quark and gluon fragmentation functions [5]. We include these effects in our analysis.

Since quarks and gluons are confined, what one observes are their fragmentation products. The fragmentation procedure that we have adopted is very similar in spirit to the one adopted by Feynman and Field (FF) [6] for the quark jets. To recapitulate, ordinary hadrons are produced as a result of a quark polarizing the vacuum giving rise to a hadronic cascade. The fragmentation $q \rightarrow h + q'$ is effected through a primordial fragmentation function

$$f_q^h(z) = 1 - a + 3a(1 - z)^2,$$

$$z = (E + p_{\parallel})_h / (E + p_{\parallel})_q. \quad (1)$$

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^{†1} A Monte Carlo program based on ref. [4] has been written by H.G. Sander.

The produced hadrons are given a transverse momentum, which is locally compensated and is described by a distribution

$$g(p_T^2) = (2\sigma^2)^{-1} e^{-p_T^2/2\sigma^2} \quad (2)$$

a and σ are adjustable (energy-independent) parameters. Eqs. (1) and (2) lead to a differential distribution

$$\sigma^{-1} d^2\sigma/dp_T^2 dz \sim f_g^h(z)g(p_T^2), \quad (3)$$

giving rise to a scale-invariant hadronic description and a two-jet development of the final states in e^+e^- annihilation. The incorporation of heavy quarks (charm, bottom and top) within this framework is rather straightforward and has been described in detail elsewhere [7]. In the weak decay of the b we assume the dominance of the $b \rightarrow c$ transition.

To extract the charm quark fragmentation function, we have made a fit to the low energy data $5.0 \text{ GeV} \leq E_{\text{cm}} \leq 9.4 \text{ GeV}$ from DORIS and SPEAR yielding [7]

$$f_c^H(z_D) \sim (1 - z_D)^d, \quad 0 \leq d \leq 1. \quad (4)$$

The quality of these data does not warrant a more precise determination of $f(z_D)$. For the bottom quark fragmentation, we assume $f(z_B) \sim \text{const}$. Since $\sigma(b\bar{b})/\sigma_{\text{total}} \approx 1/11$, the analysis is very insensitive to this input. For the gluon fragmentation we have assumed the two-step process

$$g \rightarrow q\bar{q} \rightarrow \text{hadrons} \quad (5)$$

This of course is a convenient artifact to implement the non-perturbative hadronization process of the gluons in a Monte-Carlo approach, since there is no evidence yet for pure gluonic hadron states. The production of $q\bar{q}$ in reaction (5) is at the moment similar to the $q\bar{q}$ pair pair creation in the FF quark jet model. We describe the step $g \rightarrow q\bar{q}$ in (4) by a primordial gluon fragmentation function, $f_g^q(z)$, analogous to the quark fragmentation function $f_q^h(z)$. Moreover, as a first approximation we assume the same p_T^2 -distribution [i.e. eq. (2)] for the quarks in reaction (5).

In describing the gluon hadronization process we have not taken into account the branching

$$g \rightarrow gg \rightarrow \text{hadrons} \quad (6)$$

since, for the hard non-collinear gluons such a branching is part of the $O(\alpha_s^2)$ process $e^+e^- \rightarrow q\bar{q}gg$ [8]

which we have taken into account separately. The function $f_g^q(z)$ is a trial function which has to be extracted from the data. A possible choice in this framework is the $g \rightarrow q\bar{q}$ splitting function derived by Altarelli and Parisi [9]

$$f_g^q(z) = z^2 + (1 - z)^2, \quad (7)$$

where $z = E_q/E_g$.

In including the quark mass effect in the $O(\alpha_s)$ process $e^+e^- \rightarrow q\bar{q}g$, we have used the matrix element calculated by Ioffe [10]. The $O(\alpha_s^2)$ processes $e^+e^- \rightarrow q\bar{q}gg$, $q\bar{q}q\bar{q}$ involving massless and massive quarks were studied in ref. [8] and the interested reader is referred to this work for details and the theory of the four-jet processes.

The Q^2 -evolution of the quark and gluon fragmentation functions is also a definite prediction of any scale-broken field theory for strong interactions, including QCD [5,11]. In fact the QCD effects as measured in the deep-inelastic scattering experiments measure just such a Q^2 -dependence. A more systematic study of the Q^2 -effects in e^+e^- annihilation would be to study the Q^2 -evolution of the quark and gluon jets. Recently there have been several attempts in this direction incorporating the Q^2 -evolution of the p_T -dependence [12] as well. For the sake of this analysis we incorporate the Q^2 -dependence of the fragmentation functions only.

First note that in a scale-invariant theory, the inclusive hadron energy distribution $Q^2 d\sigma/dx$ scales, $Q^2 \times d\sigma/dx \sim D_q^h(x)$. In QCD, the fragmentation function $D_q^h(x)$ is Q^2 -dependent. Typically one gets the Q^2 -evolution equations for the n th (x -)moments of the fragmentation functions,

$$\begin{aligned} D_q^n(Q^2) &\equiv \int_0^1 x^n D_q^h(x, Q^2) dx \\ &= D_q^n(Q_0^2) [\alpha_s(Q^2)/\alpha_s(Q_0^2)]^{A_N} \end{aligned} \quad (8)$$

where A_N are calculable anomalous dimensions. The fragmentation function $D_q^h(x, Q^2)$ is then obtained by inverting the moment equations. In a quark and gluon cascade picture it is the primordial distribution that feels the Q^2 -dependence. This comes about in a natural way in a probabilistic interpretation of the quark and gluon branching [13]. In the Feynmann-Field model, however, the Q^2 -evolution of a jet is determined

from the iterative transitions $q \rightarrow h + q'$. Consequently, we have incorporated the Q^2 -dependence in the primordial fragmentation function $f_q^h(z, Q^2)$. We are then led to the following Q^2 -dependent forms for the primordial quark fragmentation functions [5]:

$$f_q^h(z) \equiv 1 - a + 3a(1-z)^2 \rightarrow \frac{e^{0.69\bar{S}G}}{\Gamma(\alpha+1)} (-\ln z)^\alpha \times \left[1 - a + \frac{6a(1-z)^2}{(\alpha+1)(\alpha+2)} + c(1-z)^3 \right] \quad (9)$$

(with c determined from $\int f_q^h(z) dz = 1$), for the u , d and s quarks. For the charm and bottom quarks assuming a fragmentation function

$$f_Q^H(z) = (1-z)^d, \quad (10)$$

leads to the Q^2 -evolved form

$$f_Q^H(z, Q^2) = f_Q^H(z, Q_0^2) e^{0.69\bar{S}G} (-\ln z)^\alpha \frac{\Gamma(d+1)}{\Gamma(d+1+\alpha)}$$

where

$$\alpha = 4G\bar{S}, \quad G = \frac{4}{25},$$

$$\bar{S} = \ln[\ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2)]. \quad (11)$$

The Q^2 -dependence of $f_Q^H(z)$ and $f_q^h(z)$ depletes the large- z region, thereby increasing the particle multiplicity and the total p_T of the jet since $p_T^{\text{jet}} \sim \langle n \rangle \langle p_T \rangle$.

Before presenting a comparison of our model with the experimental data, it is worthwhile to discuss the relative contributions of the three- and four-jet events. The cross sections for the processes $e^+e^- \rightarrow q\bar{q}g$, $q\bar{q}gg$ and $q\bar{q}q\bar{q}$ are singular in the limit $E_g, E_q \rightarrow 0$ or $\cos \theta_{qG} \rightarrow 0$. A sensible way is to define these cross sections with cuts on variables such that the soft and collinear configurations are excluded [14]. We have chosen cuts on thrust and acoplanarity distributions to define the physical three- and four-jet processes, respectively ^{#2}. For $T > T_0$ the $q\bar{q}g$ process is then a

^{#2} We have used a thrust cut-off $T_0 = 0.95$ to define the three jet cross section $e^+e^- \rightarrow q\bar{q}g$ and an acoplanarity cut-off $A_0 = 0.05$ to define the four-jet cross section $e^+e^- \rightarrow q\bar{q}gg + q\bar{q}q\bar{q}$. This gives, respectively, 0.29 and 0.05 for the relative rates at $E_{\text{cm}} = 30$ GeV and $\Lambda = 0.35$ GeV. For other values of Λ , the cross sections can be obtained using eq. (15). For mass effects see ref. [15].

part of the two-jet process and similarly for $A < A_0$, the $q\bar{q}gg$ and $q\bar{q}q\bar{q}$ processes are to be considered as part of the two and three jets. The integrals (i.e. the three- and four-jet rates)

$$\int_{2/3}^{T_0} dT \frac{d\sigma}{dT}(q\bar{q}g), \quad \int_{A_0}^1 dA \frac{d\sigma}{dA}(q\bar{q}q\bar{q} + q\bar{q}gg),$$

are then *absolute* predictions of QCD. It is also clear that the distributions in the allowed domain of A and T values are independent of the precise value of the cuts. The two-jet fraction, which also depends on α_s , is then determined by

$$\sigma_{2\text{jets}}/\sigma_{\text{tot}} = 1 - \sigma_{3\text{jets}}/\sigma_{\text{tot}} - \sigma_{4\text{jets}}/\sigma_{\text{tot}}. \quad (12)$$

For σ_{tot} , we have used the $O(\alpha_s^2)$ corrected form recently derived in ref. [16].

To extract Λ we still use the lowest order formula

$$\alpha_s(Q^2) = 4\pi/\beta_0 \ln(Q^2/\Lambda^2), \quad (13)$$

where $\beta_0 = 11 - 2n_f/3$, $n_f = 5$ (number of flavours).

We propose to extract $\alpha_s(Q^2)$ by analysing the *inclusive* distributions in thrust, sphericity, oblateness ^{#3} or the p_T^2 -distributions, which are less sensitive to the fragmentation details, but depend more on the value of σ_q entering the primordial p_T -distribution. σ_q can be obtained by analysing the low energy data. For extracting $\alpha_s(Q^2)$, one could select a sample with enriched fraction of gluon events by stringently cutting, for example, high thrust, low oblateness events. At higher energies one could use a cut on acoplanarity as well to get an enriched $q\bar{q}gg$ sample of events. One could then look either at the oblateness versus acoplanarity or thrust versus acoplanarity distribution (or any other such Dalitz distribution) and extract a value of α_s by comparing the integrated events below the cuts with the corresponding experimental measurements. One could also look at various single differential distributions and extract α_s . A cross check of the $\alpha_s(Q^2)$ value so determined will not only test perturbative QCD but also the detailed dynamical model we have built. An energy-independent determination of Λ is then a consistency check of the theory.

First, we discuss the determination of the param-

^{#3} For the definition of oblateness see Barber et al. [1]. For the definition of $(p_T^2)_{\text{in}}$ and $(p_T^2)_{\text{out}}$ see Brandelik et al. [1].

eters concerning the fragmentation properties of the quarks. The parameters a , Q_0 and the power d in the charm and bottom fragmentation functions can be fixed by fitting the inclusive hadron energy distribution and the measured average charge multiplicity. We find that the value $a = 0.7$ and a pseudoscalar to vector ratio, $r_{pv} = 2:1$ is in good agreement with $\langle n_{ch} \rangle$ measured at $E_{cm} = 3.6$ GeV, as well as with the inclusive hadron energy distributions^{‡4}. Above charm threshold, $4.0 \text{ GeV} \leq E_{cm} \leq 9.4 \text{ GeV}$, good fit to both $\langle n_{ch} \rangle$ and $Q^2 d\sigma/dx$ distribution is obtained if one assumes in addition $f_c^H(z) = (1-z)^d$ with $0 \leq d \leq 1/2$. We use $f_0^H(z) = \text{constant}$ for the charm quark fragmentation function. There is a correlation between σ_q and the quark fragmentation parameters. We find that the FF choice $a = 0.77$, $\sigma_q = 0.247 \text{ GeV}$, $r_{pv} = 1$, and the charm fragmentation function $f_c^D(z) = 1-z$ gives a somewhat larger value for $\langle n_{ch} \rangle$ and leads to a softer $Q^2 d\sigma/dx$ distribution as compared to the data. Similarly, at higher energies an acceptable fit to $\langle n_{ch} \rangle$ is obtained for the range $10 \text{ GeV} \leq Q_0 \leq 15 \text{ GeV}$. The resulting charge multiplicity distribution is shown in fig. 1 and compared with the experimental data [18] in the range $3.0 \text{ GeV} \leq E_{cm} \leq 31.6 \text{ GeV}$. The theoretical curve corresponds to the values $\sigma_q = 0.3 \text{ GeV}$, $Q_0 = 15 \text{ GeV}$ and $\Lambda = 0.6 \text{ GeV}$. The effect of varying σ_q and Λ in the range $0.26 \text{ GeV} \leq \sigma_q \leq 0.32 \text{ GeV}$

^{‡4} A similar value for the ratio r_{pv} is obtained in ref. [17] by analysing the inclusive ρ -production in pp interactions which gives a r_{pv}^{-1} ratio of 0.45. A value of $r_{pv} = 1$ and $a = 0.77$ is still within one standard deviation of the data.

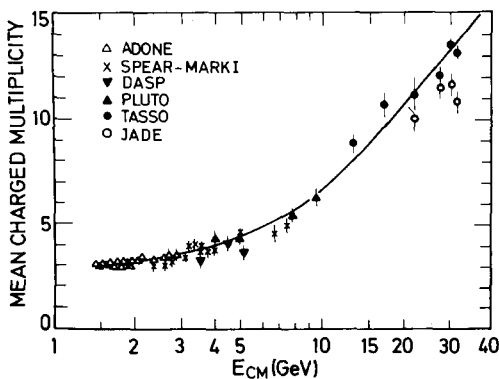


Fig. 1. A comparison of our model with the charged multiplicity measurements. The JADE collaboration points refer to the observed charged multiplicity.

and $0.3 \text{ GeV} \leq \Lambda \leq 1.0 \text{ GeV}$ results in $\langle n_{ch} \rangle$ which is still within 1σ of the measured values. The inclusive distributions $d\sigma/dT$, $d\sigma/dp_T^2$, $d\sigma/d\hat{O}$, $d\sigma/ds$ and $d\sigma/dA$ are not very sensitive to the choice of Q_0 , a , r_{pv} and $f_c^D(z)$ within the above mentioned ranges, though these distributions are sensitive to σ_q and the QCD coupling constant $\alpha_s(Q^2)$.

Next, we determine σ_q . For this purpose we use both the low energy p_T -distribution from SPEAR and DORIS [19] and the TASSO $(p_T^2)_{out}$ distributions^{‡2}, as well as the MARK-J thrust distribution for the narrow jet as measured at PETRA. In fig. 2 we show the distribution $\sigma^{-1} d\sigma/d(p_T^2)_{out}$ and $\sigma^{-1} d\sigma/d(p_T^2)_{in}$. A fit to the $\sigma^{-1} d\sigma/d(p_T^2)_{out}$ distribution for the low energy data (13.0 GeV and 17.0 GeV) gives 0.28 GeV

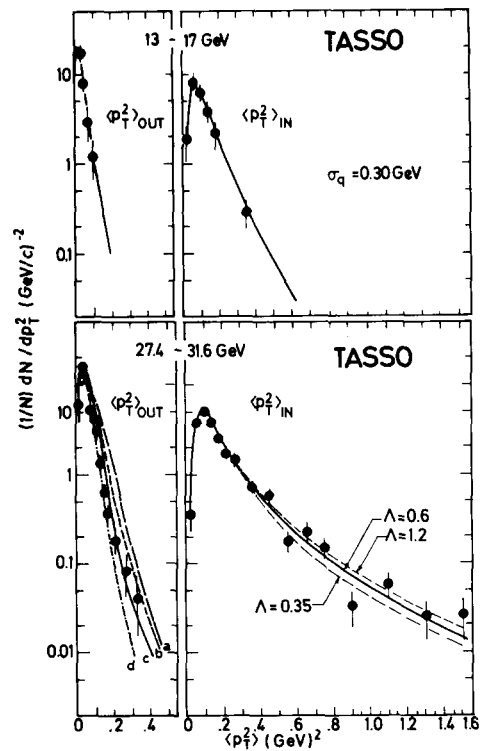


Fig. 2. A comparison of our model with the distributions $\sigma^{-1} \times d\sigma/d(p_T^2)_{in}$ and $\sigma^{-1} d\sigma/d(p_T^2)_{out}$ measured by the TASSO collaboration. The data points have been corrected for geometrical acceptance and detection efficiency. (i) $E_{cm} = 13$ and 17.0 GeV , $\sigma_q = 0.3 \text{ GeV}$, (ii) $27.4 \text{ GeV} \leq E_{cm} \leq 31.6 \text{ GeV}$. (a) $\sigma_q = 0.35 \text{ GeV}$, (b) $\sigma_q = 0.32 \text{ GeV}$, (c) $\sigma_q = 0.30 \text{ GeV}$, (d) $\sigma_q = 0.26 \text{ GeV}$ for the $\sigma^{-1} d\sigma/d(p_T^2)_{out}$ distributions. The curves for the $\sigma^{-1} d\sigma/d(p_T^2)_{in}$ distribution are drawn for $\sigma_q = 0.3 \text{ GeV}$. Radiative corrections are taken into account.

$\leq \sigma_q \leq 0.3$ GeV. In fig. 2 we again determine σ_q by fitting the $\sigma^{-1} d\sigma/d(p_T^2)_{out}$ distribution for the high energy data $27.4 \text{ GeV} \leq E_{cm} \leq 31.6 \text{ GeV}$. The value of σ_q is now correlated with the value of Λ , since the selection criterion for the plane to define the in and out distributions leads to a configuration in which the gluon and the quark (or antiquark) are on average (25–30)% out of the plane. We again find a range for σ_q , $0.28 \text{ GeV} \leq \sigma_q \leq 0.30 \text{ GeV}$, correlated with Λ in the range $1.2 \text{ GeV} > \Lambda > 0.3 \text{ GeV}$. In fig. 2 we show the distributions for the choice (a) $\sigma_q = 0.35 \text{ GeV}$, (b) $\sigma_q = 0.32 \text{ GeV}$, (c) $\sigma_q = 0.30 \text{ GeV}$ and (d) $\sigma_q = 0.26 \text{ GeV}$. The value of σ_q in the range $0.28 \text{ GeV} \leq \sigma_q \leq 0.30 \text{ GeV}$ is in agreement with the low energy p_T and thrust distributions [7], as well as with the distribution $\sigma^{-1} d\sigma/dT_N$ for the narrow jet as measured by the MARK-J collaboration [20]. We show a comparison of our model with their data in fig. 3a, which corresponds to $\sigma_q = 0.3 \text{ GeV}$.

Having fixed the parameters determining the non-perturbative aspects of the quark jets, we attempt to extract the QCD coupling constant $\alpha_s(Q^2)$ (or equivalently Λ). To this end, we have chosen the distribution $\sigma^{-1} d\sigma/d(p_T^2)_{in}$ measured by the TASSO collaboration [21] and the oblateness distribution for the broad jet, measured by the MARK-J collaboration. First, the distribution $\sigma^{-1} d\sigma/d(p_T^2)_{in}$. In fig. 2 we compare this distribution with our model calculations for the choice $\Lambda = 0.35 \text{ GeV}$, 0.6 GeV and 1.2 GeV , with $\Lambda = 0.6 \text{ GeV}$ our best fit. To determine the value of Λ , we make a cut on the $d\sigma/d(p_T^2)_{in}$ distribution at $(p_T^2)_{in} = 0.5 \text{ GeV}^2$ and compare the number of events surviving the cut as a function of $\alpha_s(Q^2)$. The value of $\alpha_s(Q^2)$ (or Λ) so determined is like $\sigma^{-1} d\sigma/d(p_T^2)_{out}$ correlated with σ_q resulting in the range $0.3 \text{ GeV} \leq \Lambda \leq 1.2 \text{ GeV}$ for σ_q in the range $0.30 \text{ GeV} \geq \sigma_q \geq 0.28 \text{ GeV}$. The region of the allowed (within $\pm 1\sigma$) α_s versus σ_q domain gives $0.20 \leq \alpha_s(Q^2) \leq 0.25$ at $(Q^2)^{1/2} = 30 \text{ GeV}$, corresponding to $0.5 \text{ GeV} \leq \Lambda \leq 1.2 \text{ GeV}$. In fig. 3b we show the comparison with the MARK-J measurement of $\sigma^{-1} d\sigma/dO_B$ for the choice $\alpha_s(Q^2) = 0.23$ and $\sigma_n = 0.3 \text{ GeV}$. This distribution gives a value $\alpha_s(Q^2) = 0.23 \pm 0.02$, which agrees with the determination of $\alpha_s(Q^2)$ from the TASSO data. The value of $\alpha_s(Q^2)$ so determined is very weakly dependent on the gluon fragmentation function $f_g^q(z)$.

Summarizing, we have shown that the perturbative

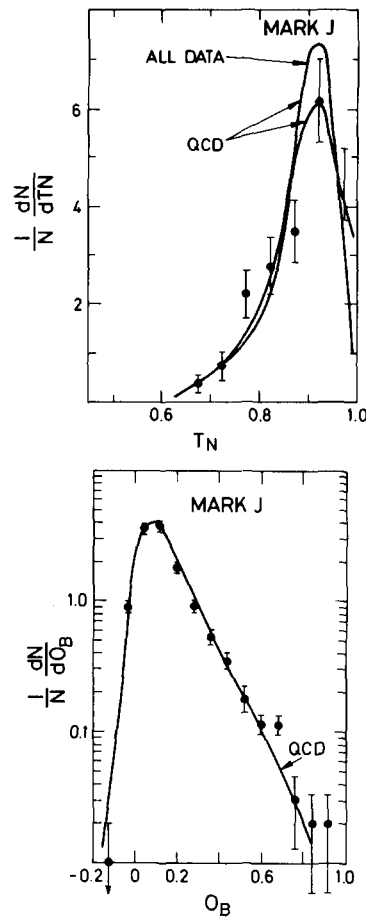


Fig. 3. (a) A comparison of our model with the thrust distribution of the narrow jet, $\sigma^{-1} d\sigma/dT_N$ measured by the MARK-J collaboration, corresponding to $\sigma_q = 0.3 \text{ GeV}$. The theoretical curve is obtained after passing it through the MARK-J detector. The lower curve corresponds to selecting events with $O_B > 0.3$. (b) A comparison of our model with the oblateness distribution $\sigma^{-1} d\sigma/dO_B$ for the broad jet as measured by the MARK-J collaboration. The curve corresponds to $\sigma_q = 0.3 \text{ GeV}$ and $\alpha_s(Q^2) = 0.23$ at $E_{cm} = 30 \text{ GeV}$.

QCD predictions are in very good quantitative agreement with the e^+e^- data, if one includes all the non-perturbative and weak decay effects relevant for such an analysis ^{±5}. The value of α_s determined from the data in the region $27.4 \text{ GeV} \leq E_{cm} \leq 31.6 \text{ GeV}$ gives

^{±5} We would like to remark that whereas our calculation takes into account the complete $O(\alpha_s^2)$ bremsstrahlung processes involving quarks and gluons, we have not included the effect of the virtual gluon correction to the process $e^+e^- \rightarrow q\bar{q}g$. This implies at most an uncertainty of $O(\alpha_s)$ in the determination of $\alpha_s(Q^2)$.

$\alpha_s(Q^2) = 0.22 \pm 0.03$ for the TASSO data and $\alpha_s(Q^2) = 0.23 \pm 0.02$ from the MARK-J data.

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References

- [1] JADE Collab., Proc. Intern. Symp. on Lepton and photon interactions at high energies (FNAL., Batavia, 1979); MARK-J Collab., D.P. Barber et al., Phys. Rev. Lett. 43 (1979) 830; PLUTO Collab., Ch. Berger et al., Phys. Lett. 86B (1979) 418; TASSO Collab., R. Brandelik et al., Phys. Lett. 86B (1979) 243.
- [2] J. Ellis, M.K. Gaillard and G.G. Ross, Nucl. Phys. B111 (1976) 253.
- [3] T.A. DeGrand, Y.J. Ng and S.-H.H. Tye, Phys. Rev. D16 (1977) 3251; A. De Rujula, J. Ellis, E.G. Floratos and M.K. Gaillard, Nucl. Phys. B138 (1978) 387; G. Kramer, G. Schierholz and J. Willrodt, Phys. Lett. 78B (1978) 249; Erratum 80B (1979) 433; G. Kramer and G. Schierholz, Phys. Lett. 82B (1979) 108; C.L. Basham, L.S. Brown, S.D. Ellis and S.T. Love, Phys. Rev. B17 (1978) 2298.
- [4] P. Hoyer, P. Osland, H.G. Sander, T.F. Walsh and P.M. Zerwas, Nucl. Phys. B161 (1979) 349.
- [5] D. Gross, Phys. Rev. Lett. 32 (1974) 1071.
- [6] R.D. Field and R.P. Feynman, Nucl. Phys. B136 (1978) 1.
- [7] A. Ali, J.G. Körner, G. Kramer and J. Willrodt, DESY Report 79/63 (1979); and Nucl. Phys., to be published.
- [8] A. Ali, J.G. Körner, Z. Kunszt, J. Willrodt, G. Kramer, G. Schierholz and E. Pietarinen, Phys. Lett. 82B (1979) 285; DESY 79/54 (1979); and Nucl. Phys., to be published.
- [9] G. Altarelli and G. Parisi, Nucl. Phys. B126 (1977) 298.
- [10] B.L. Ioffe, Phys. Lett. 78B (1978) 277.
- [11] An incomplete list of references is: H.D. Politzer, Phys. Lett. 70B (1977) 430; H. Georgi and H.D. Politzer, Phys. Rev. Lett. 40 (1978) 3; Phys. Rev. D14 (1976) 1829; A.J. Buras and K.J.F. Gaemers, Nucl. Phys. B132 (1978) 249; N. Sakai, talk given at the DESY meeting (Oct. 1979); R. Baier, J. Engels and B. Peterson, Z. Phys. C2 (1979) 265.
- [12] K. Konishi, A. Ukawa and G. Veneziano, Nucl. Phys. B157 (1979) 45; R.K. Ellis and R. Petronzio, CERN Report TH-2571 (1978); A. Basseto, M. Ciafaloni and G. Marchesini, Univ. of Trento preprint UTF 45 (1979).
- [13] K. Kajantie and E. Pietarinen, DESY Report, in preparation.
- [14] G. Sterman and S. Weinberg, Phys. Rev. Lett. 39 (1977) 1436.
- [15] A. Ali, E. Pietarinen, G. Kramer and J. Willrodt, DESY Report, in preparation.
- [16] M. Dine and J. Sapirstein, Phys. Rev. Lett. 43 (1979) 668; K.G. Chetyrkin, A.L. Kataev and F.V. Tkachov, Phys. Lett. 85B (1979) 277.
- [17] R.G. Roberts, R.C. Hwa and S. Matsuda, J. Phys. G5 (1979) 1043.
- [18] TASSO Collab., R. Brandelik et al., DESY 79/73.
- [19] PLUTO Collab., Ch. Berger et al., DESY Report 78/71; G.G. Hanson, SLAC Reports SLAC-PUB-1814 (1976), SLAC-PUB-2118 (1978); Proc. 13th Rencontre de Moriond on High energy leptonic and hadronic interactions (Les Arcs, March 1978).
- [20] MARK-J Collab., D.P. Barber et al., MIT Report 108 (1979); Phys. Lett. 89B (1979) 139.
- [21] TASSO Collab., R. Brandelik et al., to be published.