

How to determine all the angles of the unitarity triangle from $B_d^0 \rightarrow DK_S$ and $B_s^0 \rightarrow D\phi$

Michael Gronau

*Technion, Israel Institute of Technology, 32000 Haifa, Israel
and Deutsches Elektronen Synchrotron, DESY, W-2000 Hamburg, FRG*

and

David London

Laboratoire de Physique Nucléaire, Université de Montréal, CP 6128, Montreal, P.Q., Canada H3C 3J7

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We consider within the standard model the time-dependent decay rates of the three processes $B_d^0 \rightarrow D^0 K_S$, $\bar{D}^0 K_S$ and $D^0 K_S$, where D_i^0 is a neutral D meson CP -eigenstate. We show that it is possible to derive from these processes, in which two weak amplitudes are involved, two of the three angles in the unitarity triangle of the Cabibbo–Kobayashi–Maskawa matrix. The third angle can be determined from $B_s^0 \rightarrow D\phi$. All three angles are obtained free of hadronic final-state uncertainties.

A particularly promising way of testing the three-family Kobayashi–Maskawa model of CP violation [1] is via measurements of CP violating asymmetries in neutral B decays to CP eigenstates [2]. If a single Cabibbo–Kobayashi–Maskawa (CKM) amplitude contributes to the decay of a B^0 meson then the time-dependent asymmetry oscillates with a frequency given by the mass-difference of the two neutral B mesons, and with an amplitude which is a pure function of CKM parameters. There are no hadronic final-state-interaction uncertainties. There exist three classes of non-zero asymmetries, each of which has an amplitude given by the sine of twice the corresponding angle of the so-called CKM unitarity triangle (fig. 1):

CKM suppressed B_d decays

$$[\text{e.g. } B_d^0(\bar{B}_d^0) \rightarrow \pi^+ \pi^-]: \sin 2\alpha = -\text{Im} \left(\frac{V_{td}^*}{V_{td}} \frac{V_{ub}^*}{V_{ub}} \right),$$

CKM allowed B_d decays

$$[\text{e.g. } B_d^0(\bar{B}_d^0) \rightarrow \psi K_S]: \sin 2\beta = \text{Im} \left(\frac{V_{td}^*}{V_{td}} \right),$$

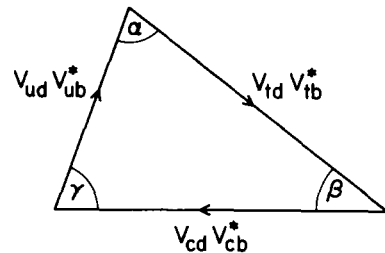


Fig. 1. The CKM unitarity triangle.

CKM suppressed B_s decays

$$[\text{e.g. } B_s^0(\bar{B}_s^0) \rightarrow \rho K_S]: \sin 2\gamma = \text{Im} \left(\frac{V_{ub}^*}{V_{ub}} \right). \tag{1}$$

In writing eq. (1), we have used the standard parametrization of the CKM matrix [3], in which only the elements V_{ub} and V_{td} have non-negligible phases. A straightforward test of the standard model (SM) within the neutral B meson system is then to independently measure the three angles α , β , γ and to see whether or not they add up to 180° . Furthermore, a measurement of any of these angles by itself may pro-

vide a new useful constraint on the CKM matrix, by which the SM can be tested.

Unfortunately, the pure relation between CKM phase parameters and an observable asymmetry is destroyed if the decay can also proceed via a second non-negligible amplitude. And, in fact, in all three classes of asymmetries, there are extra contributions to the decay from penguin diagrams [4,5]. Although these new diagrams are roughly estimated to be small (particularly in the CKM-allowed B_d^0 decays) this nevertheless introduces a hadronic theoretical uncertainty, which will limit the precision with which CKM phase information can be extracted from the CP asymmetries.

Recently we have shown [6] that isospin symmetry may sometimes disentangle the effects of the new amplitudes and thereby effectively recover the pure relationship between an asymmetry and one of the three angles. In this letter we would like to illustrate another way of treating decays in which two CKM amplitudes contribute, in order to extract information about CKM phases from their time-dependent rates. We will use pure quantum mechanical arguments to show that the seeming disadvantage of having two CKM amplitudes may, in some cases, turn into an advantage. In the following discussion we will assume the standard model. We shall occasionally stress those assumptions in the model which are crucial for our derivation and show how they can be tested. After having derived our results we will comment briefly on the implications of our study outside the framework of the standard model.

Consider the decay $B_d^0 \rightarrow D_1^0 K_S$, where D_1^0 is the CP -even eigenstate identified by one of its CP -even decay products ^{#1}. We note that, although a B_d^0 may decay weakly to either a D^0 or to a \bar{D}^0 , when looking for a CP -even decay product one is actually selecting the CP -even superposition $D_1^0 = (D^0 + \bar{D}^0)/\sqrt{2}$. Thus, there are two diagrams (fig. 2) which contribute to this process. These diagrams represent two QCD-corrected four-fermion terms of the low energy effective hamiltonian [7]. The two corresponding CKM factors, $V_{cb}^* V_{us}$ (corresponding to fig. 2a) and $V_{ub}^* V_{cs}$ (fig. 2b), are of comparable magnitudes. As

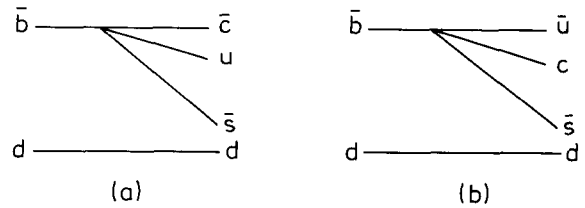


Fig. 2. The two diagrams for (a) $B_d^0 \rightarrow \bar{D}^0 K_S$, (b) $B_d^0 \rightarrow D^0 K_S$.

a consequence, the CP asymmetry measured in this process alone does not directly correspond to a CKM parameter. We note, however, that the above two diagrams describe separately the single amplitudes of the processes $B_d^0 \rightarrow \bar{D}^0 K_S$ and $B_d^0 \rightarrow D^0 K_S$, respectively, where now the flavor states \bar{D}^0 and D^0 can be identified, for instance, by their semileptonic decay signatures. By measuring the time-dependent decay rates of these two processes it may be possible to disentangle the effects of the two amplitudes in $B_d^0 \rightarrow D_1^0 K_S$. We will show that a study of the time-dependence of the above three processes together leads, in fact, to an extraction of two of the three angles of the unitarity triangle ^{#2}. The third angle can be determined independently by applying the same method to the three processes $B_s^0 \rightarrow \bar{D}^0 \phi$, $B_s^0 \rightarrow D_1^0 \phi$. This method suffers no uncertainty due to unknown hadronic matrix elements or hadronic final state interaction phases. the above processes involve no contributions from penguin diagrams.

The amplitudes of the three processes $B_d^0 \rightarrow \bar{D}^0 K_S$, $B_d^0 \rightarrow D^0 K_S$ and $B_d^0 \rightarrow D_1^0 K_S$ can be written, respectively, in the form

$$\begin{aligned} A_{\bar{D}} &\equiv A(B_d^0 \rightarrow \bar{D}^0 K_S) = |A_{\bar{D}}| \exp(i\delta_{\bar{D}}), \\ A_D &\equiv A(B_d^0 \rightarrow D^0 K_S) = |A_D| \exp(i\gamma) \exp(i\delta_D), \\ A_{D_1} &\equiv A(B_d^0 \rightarrow D_1^0 K_S) = \frac{1}{\sqrt{2}} (A_D + A_{\bar{D}}). \end{aligned} \quad (2)$$

The magnitudes $|A_{\bar{D}}|$ and $|A_D|$, corresponding to figs. 2a and 2b respectively, are presumably of comparable size. The phases of their respective CKM fac-

^{#1} We neglect D^0 - \bar{D}^0 mixing and CP violation in D decays. New physics in the D^0 - \bar{D}^0 system could affect our results and would be revealed by the tests mentioned in the text.

^{#2} A similar study may be carried out for decays of the type $B_d^0 \rightarrow D\pi^0$. However, this is expected to be less useful, since the two amplitudes involved are quite dissimilar in magnitude. Furthermore, it is usually easier experimentally to identify a K_S than a π^0 .

tors are 0 and γ in the standard CKM parametrization [3], and the corresponding final state interaction phases are denoted by $\delta_{\bar{D}}$ and δ_D , respectively. The amplitudes of the charge-conjugated processes $\bar{B}_d^0 \rightarrow D^0 K_S$, $\bar{B}_d^0 \rightarrow \bar{D}^0 K_S$ and $\bar{B}_d^0 \rightarrow D_1^0 K_S$, which we denote respectively by \bar{A}_D , $\bar{A}_{\bar{D}}$ and \bar{A}_{D_1} , are obtained from the A amplitudes by simply changing the sign of the CKM phases. We note immediately that $|\bar{A}_D| = |A_{\bar{D}}|$, $|\bar{A}_{\bar{D}}| = |A_D|$, but $|\bar{A}_{D_1}| \neq |A_{D_1}|$. That is, in the standard model, CP is conserved in direct decays of B_d^0 to the two flavor states $D^0 K_S$ and $\bar{D}^0 K_S$ and is, in general, violated in decays to the CP -eigenstate $D_1^0 K_S$. This feature follows from having a single weak amplitude in each of the first two processes. Since this will become crucial in our derivation, we will mention below two ways to experimentally test this assumption.

The magnitudes of the decay amplitudes can be obtained experimentally from the time-dependent decay rates of neutral B mesons. The time-dependence of the decay, taking into account both mixing and the fact that more than one amplitude contributes, was calculated in ref. [4] for CP eigenstates and in ref. [8] for any general state. The decay rate of a neutral B_d meson, known to be pure B_d^0 at time $t=0$, is

$$\Gamma(B_d^0(t) \rightarrow f) = |A_f|^2 \exp(-\Gamma t) [\cos^2(\frac{1}{2} \Delta m t) + |\xi_f|^2 \sin^2(\frac{1}{2} \Delta m t) - \text{Im } \xi_f \sin(\Delta m t)], \quad (3)$$

where

$$A_f = A(B_d^0 \rightarrow f), \quad \bar{A}_f = A(\bar{B}_d^0 \rightarrow f), \quad \xi_f = \exp(-2i\phi_{M_d}) \frac{\bar{A}_f}{A_f}. \quad (4)$$

Here, $2\phi_{M_d}$ is the phase of B_d^0 - \bar{B}_d^0 mixing. Since $\exp(-2i\phi_{M_d}) = V_{tb}^* V_{td} / V_{tb} V_{td}^*$, we see from eq. (1) that $\phi_{M_d} = \beta$ in the standard model. By measuring the time-dependence of B_d^0 decays into the three final states $f = D^0 K_S$, $D^0 K_S$ and $D_1^0 K_S$, one may extract the magnitudes of the three amplitudes of eq. (2) as well as the magnitudes of the amplitudes of the charge-conjugated processes. These six quantities can also be determined from time-dependent B_d^0 decays. Thus one can verify that $|\bar{A}_D| = |A_{\bar{D}}|$ and $|\bar{A}_{\bar{D}}| = |A_D|$, as predicted in the SM [eq. (2)].

Furthermore, these same measurements will also determine the coefficients of the $\sin(\Delta m t)$ terms of

eq. (3), which correspond to CP violation due to the interference of the decay chains $B_d^0 \rightarrow f$ and $B_d^0 \rightarrow \bar{B}_d^0 \rightarrow f$. For the above three states they are given by

$$\begin{aligned} -\text{Im } \xi_{\bar{D}} &= \frac{|A_D|}{|A_{\bar{D}}|} \sin(2\beta + \gamma - \Delta), \\ -\text{Im } \xi_D &= \frac{|A_{\bar{D}}|}{|A_D|} \sin(2\beta + \gamma + \Delta), \\ -\text{Im } \xi_{D_1} &= \frac{1}{2|A_{D_1}|^2} \{ |A_{\bar{D}}|^2 \sin 2\beta \\ &\quad + |A_D|^2 \sin 2(\beta + \gamma) + |A_D| |A_{\bar{D}}| [\sin(2\beta + \gamma - \Delta) \\ &\quad + \sin(2\beta + \gamma + \Delta)] \}, \end{aligned} \quad (5)$$

where $\Delta \equiv \delta_D - \delta_{\bar{D}}$. In obtaining eq. (5) from the definitions of eq. (4), we used eq. (2) and the corresponding expressions for the charge-conjugated amplitudes.

The question at hand is whether knowledge of these coefficients and of the magnitudes of the amplitudes suffices to determine one or more of the angles of the unitarity triangle. We will prove that, in principle, one may independently extract two of the three angles (with the usual kind of ambiguities). For this purpose we will have to eliminate from the above equations the parameter Δ which is generally unknown.

First of all, we note that the last of eq. (2) describes a complex triangle relation, from which the angle between A_D and \bar{A}_D can be obtained:

$$\cos(\gamma + \Delta) = \frac{2|A_{D_1}|^2 - |A_D|^2 - |A_{\bar{D}}|^2}{2|A_D| |A_{\bar{D}}|} \equiv c. \quad (6)$$

A similar triangle relation for the charge-conjugated processes determines $\cos(\gamma - \Delta)$ in terms of the corresponding charge-conjugated amplitudes:

$$\cos(\gamma - \Delta) = \frac{2|\bar{A}_{D_1}|^2 - |\bar{A}_D|^2 - |\bar{A}_{\bar{D}}|^2}{2|\bar{A}_D| |\bar{A}_{\bar{D}}|} \equiv \bar{c}. \quad (7)$$

In addition, the first two of eq. (5) allow an experimental determination of the two quantities

$$\begin{aligned} \sin(2\beta + \gamma + \Delta) &= -\frac{|A_D|}{|A_{\bar{D}}|} \text{Im } \xi_D \equiv S, \\ \sin(2\beta + \gamma - \Delta) &= -\frac{|A_{\bar{D}}|}{|A_D|} \text{Im } \xi_{\bar{D}} \equiv \bar{S}. \end{aligned} \quad (8)$$

It is straightforward algebra to show that both $\sin 2\beta$

and $\sin(\beta+\gamma)$ are given in terms of the above four observables:

$$\sin 2\beta = \frac{c^2 + S^2 - \bar{c}^2 - \bar{S}^2}{2(cS - \bar{c}\bar{S})},$$

$$\sin 2(\beta+\gamma) = \frac{\bar{c}^2 + S^2 - c^2 - \bar{S}^2}{2(\bar{c}S - c\bar{S})}. \quad (9)$$

From the unitarity triangle one has $\sin 2(\beta+\gamma) = -\sin 2\alpha$. Therefore, this determines $\sin 2\alpha$ and $\sin 2\beta$.

We would like to emphasize that both $\sin 2\alpha$ and $\sin 2\beta$ are obtained here with no hadronic theoretical uncertainty. This should be contrasted with the conventional way of extracting these angles from B_d^0 decays to the final states $\pi^+\pi^-$ and ΨK_S , respectively [2]. Using this latter method, the addition of contributions from penguin diagrams to the usual tree-diagrams leads to theoretical uncertainties which can only be crudely estimated [4,5]. Although these corrections to the asymmetries are roughly estimated not to be too large for the final state $\pi^+\pi^-$ and to be rather small for ΨK_S , this should be checked experimentally. At first sight, this seems to be possible, since a second decay amplitude may be detected by $|\xi_f| \neq 1$ in the time-dependent decay rates [eq. (3)]. However, if the tree and penguin amplitudes have approximately equal final-state-interaction phases, then the time-dependent rates would show no sign of a second amplitude (i.e. $|\xi_f| \approx 1$), while the asymmetry could still be substantially affected by the penguin amplitude [4].

It is interesting and perhaps surprising to note that, in order to determine the two angles α, β from eq. (9), one does not need the coefficient of the $\sin(\Delta mt)$ term in the decay to $D^0 K_S$, which is the only CP -eigenstate among all three states. On the other hand, by measuring this quantity, one can use the relation obtained from the last of eq. (5),

$$|A_D|^2 \sin 2\alpha - |A_{\bar{D}}|^2 \sin 2\beta = 2|A_{D_1}|^2 \text{Im } \xi_{D_1} + |A_D| |A_{\bar{D}}| (S + \bar{S}), \quad (10)$$

as a general consistency check for $\sin 2\alpha$ and $\sin 2\beta$. Again, this equation is a test that each of the first two processes of eq. (2) is given by a single weak amplitude. This relation may also resolve some of the possible ambiguities described below.

There are some special cases in which $\sin 2\alpha$ and

$\sin 2\beta$ cannot be derived from eq. (9), namely, when one or both of the denominators vanish. The first possibility, $cS = \bar{c}\bar{S}$ corresponds to $\cos 2\alpha \sin 2\Delta = 0$, while the second, $\bar{c}S = c\bar{S}$, implies that $\cos 2\beta \sin 2\Delta = 0$. Suppose, first of all, that both denominators were found to equal zero. This could happen only if $S = \bar{S}$ and $c = \bar{c}$ (case I), or if $S = -\bar{S}$ and $c = -\bar{c}$ (case II). One possibility is that $\sin 2\Delta = 0$. In case I ($\Delta = 0, \pi$) one finds a two-fold ambiguity in $\sin 2\alpha$ and $\sin 2\beta$:

$$\sin 2\alpha = -cS \pm \sqrt{1-c^2} \sqrt{1-S^2},$$

$$\sin 2\beta = cS \pm \sqrt{1-c^2} \sqrt{1-S^2}, \quad (11)$$

where the sign of the second term in these expressions is the same for $\sin 2\alpha$ and $\sin 2\beta$. In case II ($\Delta = \frac{1}{2}\pi, \frac{3}{2}\pi$) $\sin 2\beta$ is the same as in eq. (11), but $\sin 2\alpha$ changes sign, which again gives a two-fold ambiguity. Case II has two other solutions, $\alpha = \beta = \pm \frac{1}{4}\pi$ (if $\sin 2\Delta \neq 0$), which are characterized by $c = \pm S$, respectively. If only one of the two denominators of eq. (9) vanishes, then the ambiguity is reduced to only one of the two angles. When $cS = \bar{c}\bar{S}$, one finds $\cos 2\alpha = 0$ and $\sin 2\alpha = -S/\bar{c}$, while $\sin 2\beta$ is given by eq. (11). On the other hand, when $\bar{c}S = c\bar{S}$, we have $\cos 2\beta = 0$ and $\sin 2\beta = S/c$, with $\sin 2\alpha$ as given in eq. (11) with c being replaced by \bar{c} . All these ambiguities in the values of $\sin 2\alpha$ and $\sin 2\beta$, which occur in quite special circumstances, are resolved by using eq. (10). An exception is the rather unlikely case $|A_D| = |A_{\bar{D}}|$. (Note that the two amplitudes are given by different diagrams.) In this case, the ambiguities in $\sin 2\alpha$ and $\sin 2\beta$ can remain if both denominators vanish. [See eq. (11), as well as the two solutions $\alpha = \beta = \pm \frac{1}{4}\pi$.]

The determination of $\sin 2\alpha$ and $\sin 2\beta$ leaves, in general, a four-fold quadrant-ambiguity in the values of the angles α and β themselves. (Such an ambiguity exists even in decays to CP eigenstates in which only one CKM amplitude contributes [9].) That is, given the measurement of $\sin 2\theta$ ($\theta = \alpha, \beta$), if θ is a solution, so are $\frac{1}{2}\pi - \theta, \pi + \theta$ and $\frac{3}{2}\pi - \theta$.

Since we assumed the standard model in our derivation and since we used only B_d^0 decays, no more than two angles can be determined independently from the above analysis. The conventional way to obtain the third angle γ and to test the closure of the unitarity triangle is by measuring a CP asymmetry in

CKM-suppressed B_s^0 decays, as demonstrated by the last process of eq. (1). In fact, this test relies only on the assumption that the amplitude for $B_s^0-\bar{B}_s^0$ mixing is real. If this amplitude acquires a nonzero phase from physics beyond the standard model^{#3}, then this phase would be included in the expression of $\sin 2\gamma$ in eq. (1) and the triangle would not close. On the other hand, if only the phase of $B_d^0-\bar{B}_d^0$ mixing is affected by new physics, then the phase factor V_{td}^*/V_{td} in the expressions of $\sin 2\alpha$ and $\sin 2\beta$ would be modified in the same way, so that the three angles would still add up to 180° . In other words, the closure of the unitarity triangle cannot be tested by measurements in the B_d system alone. (A few extremely special cases, in which the determination of $\sin 2\alpha$ and $\sin 2\beta$ by themselves [via eq. (9)] would reveal the presence of physics beyond the standard model, are, for instance, $\sin 2\alpha = -1$, $\sin 2\beta = 1$, or $\sin 2\alpha = 1$, $\sin 2\beta = -1$.) However, the values of $\sin 2\alpha$ and $\sin 2\beta$ determined in the above manner may test the standard model, when other constraints on these angles from ϵ in the kaon system and from $B_d^0-\bar{B}_d^0$ mixing are taken into account.

In order to independently determine the third angle γ , and thereby test the closure of the unitarity triangle, one may carry out a similar analysis for the decays $B_s^0 \rightarrow D^0\phi$, $\bar{D}^0\phi$ and $D_1^0\phi$. As mentioned above, the standard model predicts, to a very good approximation, $\phi_{M_s} = 0$ for the phase of $B_s^0-\bar{B}_s^0$ mixing. This can be tested by the two relations

$$c'^2 + S'^2 = \bar{c}'^2 + \bar{S}'^2 = 1, \tag{12}$$

where the primes denote the measurable corresponding to eqs. (6)–(8) in B_s^0 decays. (This test is equivalent to checking that the asymmetry in CKM allowed B_s decays [e.g. $B_s^0(\bar{B}_s^0) \rightarrow D_s^+ D_s^-$] is zero.) From these processes both $\sin 2\gamma$ and $\cos 2\gamma$ can be extracted:

$$\begin{aligned} \sin 2\gamma &= \bar{c}'S' + c'\bar{S}', \\ \cos 2\gamma &= c'\bar{c}' - S'\bar{S}'. \end{aligned} \tag{13}$$

This partially resolves the four-fold quadrant ambiguity, leaving only γ and $\pi + \gamma$ as possible solutions. This ambiguity may be resolved completely by noting that only the solution with $\sin \gamma > 0$ is consistent

^{#3} Most models which significantly alter the standard model predictions for CP asymmetries in B decays have new contributions to B - \bar{B} mixing. For a review, see ref. [10].

with the observed CP violation in K decays [2,9]. Here too the last of eq. (5) (where for the B_s system β is replaced by 0) may serve as a consistency check. We should stress again that, with this method, $\sin 2\gamma$ and $\cos 2\gamma$ are determined with no theoretical hadronic uncertainty.

At this point it is perhaps useful to make some comments about the experimental feasibility of this method. First of all, we note that it is not essential that the CP -even state D_1^0 be used. A similar study can be carried out with the CP -odd state D_2^0 . This does not lead to any new information concerning the angles, but the statistics would be increased. According to the recent Review of particle properties [3], roughly 10% of D^0 decays are to CP eigenstates [11]. Thus, the fact that we require the D^0 or \bar{D}^0 to decay as D_1^0 or D_2^0 does not seem to cost too much in terms of branching ratios. Furthermore, although we have only discussed decays in which there is a K_S in the final state, nothing in our analysis forces us to use a K_S . We could equally well use the state $K^+\pi^-$ ($K^-\pi^+$) for the decay of a B_d^0 (\bar{B}_d^0). In fact, this method can be used (in principle) inclusively, that is, one can analyze in the same fashion the processes $B_d^0 \rightarrow D^0 X$, $\bar{D}^0 X$ and $D_1^0 X$, where X is any state with the flavor quantum number of a K^0 . If experimentally possible, this would increase the statistics considerably. There is, however, one drawback of our method, compared to the usual way of using CP asymmetries in decays to CP eigenstates in which a single CKM amplitude prevails. These asymmetries can be measured without knowledge of the corresponding decay branching ratios. On the other hand, in our study the relative decay branching ratios by which the D^0 , \bar{D}^0 and D_1^0 (D_2^0) are identified must be known accurately. This accuracy is one of the factors which determines the precision with which the angles of the unitarity triangle can be measured.

In conclusion, we have shown that the decays $B_d^0 \rightarrow D^0 K_S$, $\bar{D}^0 K_S$ and $D_1^0 K_S$ can provide, in principle, a way to measure two of the three angles (α, β) of the unitarity triangle, with the usual four-fold quadrant-ambiguity for each. Using the processes $B_s^0 \rightarrow D^0\phi$, $\bar{D}^0\phi$ and $D_1^0\phi$, the third angle, γ , can be measured with only a two-fold ambiguity. Despite the fact that two CKM amplitudes are involved in each of these two cases, the results for the three angles are free of theoretical final-state hadronic uncertainties.

On the other hand, when the three angles are measured using the CP asymmetries in the decays $B_d \rightarrow \pi^+ \pi^-$, $B_d \rightarrow \Psi K_S$ and $B_s \rightarrow \rho K_S$, there are hadronic theoretical uncertainties due to the existence of penguin diagrams. Thus, our method provides a theoretically clean way to test the unitarity of the CKM matrix, which, we believe, deserves an experimental effort.

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