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Determination of the Michel parameters ξ and δ in leptonic τ decays

ARGUS Collaboration

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Abstract

Using the ARGUS detector at the e^+e^- storage ring DORIS II, we have determined the Michel parameters ξ and δ of $\tau^\mp \to l^\mp \nu\bar{\nu}$ decays. With an integrated luminosity of 445 pb^{-1} around $\sqrt{s}$ = 10 GeV, we have produced approximately 415 000 τ-pairs. From this data sample, 3262 events were selected with $e^+e^- \to \tau^+\tau^- \to (l^\pm\nu\bar{\nu})(\pi^\mp\pi^+\pi^-\nu)$. The semihadronic decay was used as analyser of the τ-spin and made possible – owing to the spin correlations – the determination of the parity-violating Michel parameters ξ and δ in the decay $\tau^\mp \to l^\mp\nu\bar{\nu}$. Simultaneously to the determination of the

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Michel parameters, a measurement of the τ-neutrino helicity h_{ν_τ} in the decay $\tau^\mp \to \pi^\mp\pi^+\pi^-\nu$ was obtained. We observed $h_{\nu_\tau} = -0.85^{+0.15}_{-0.17} \pm 0.05$, $\rho = 0.721 \pm 0.040 \pm 0.021$, $\xi = 1.26^{+0.30}_{-0.26} \pm 0.09$, and $\xi\delta = 0.77^{+0.18}_{-0.16} \pm 0.05$. In addition, the combined ARGUS result on h_{ν_τ}, ρ, ξ, and $\xi\delta$ are reported using this work and previous measurements.

1. Introduction

The space-time structure of leptonic and semi-hadronic decays of the τ-lepton has been investigated, in e^+e^- annihilation, for several years now. Most data have been collected at $\sqrt{s} \approx m(Z^0)$ and at $\sqrt{s} \approx m(\Upsilon 4S)$. By only measuring momenta of the decay particles, but not the decay lepton spin, we are sensitive to five decay space-time parameters, the helicity h_{ν_τ} of the τ-neutrino and the four Michel parameters [1–3] ρ, η, ξ, and δ. Standard W-boson exchange, i. e. pure V–A structure of the charged weak currents, predicts $h_{\nu_\tau} = -1$, $\rho = 3/4$, $\eta = 0$, $\xi = 1$, and $\delta = 3/4$. The first measurement of h_{ν_τ} [4,5] was obtained following a suggestion of Ref. [6] in the decay $\tau^\mp \to \pi^\mp\pi^+\pi^-\nu$. A combination with a recent measurement of $h^2_{\nu_\tau}$ [7] in $\tau^+\tau^- \to (\rho^+\bar{\nu})(\rho^-\nu)$ leads to $h_{\nu_\tau} = -1.022 \pm 0.041$ in beautiful agreement with the complete left-handedness of the τ-neutrino. The V–A structure has also been supported by measurements of ρ [8–10], η [10], and the first measurement of $|\xi|$ in 1993 [9]. All but one measurements of ARGUS and of all LEP experiments [11,12] determine products of the parameters without reference to the Standard Model. An unambigious determination of the sign so far is only given by Refs. [4,5] and the measurement of the τ-polarisation by SLD [13].

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In this publication, we present the first direct measurement of the sign of ξ and the first measurement of δ in $\tau^\mp \to l^\mp\nu\bar{\nu}$ decays, where we combine our observations on electrons and muons. In the τ rest frame, neglecting radiative corrections and terms proportional to m_l^2/m_τ^2, the energy spectrum of the charged lepton $l^\mp$ in $\tau^\mp \to l^\mp\nu\bar{\nu}$ is given by

$$\frac{d\Gamma(\tau^\mp \to l^\mp\nu\bar{\nu})}{d\Omega dx} = \frac{G_F^2 m_\tau^5}{192\pi^4}x^2 \times \Big[3(1-x) + \tfrac{2}{3}\rho(4x-3) + 6\eta\frac{m_l}{m_\tau}\frac{1-x}{x} \mp \xi P_\tau \cos\theta\left((1-x) + \tfrac{2}{3}\delta(4x-3)\right)\Big], \qquad (1)$$

where $x = 2E_l/m_\tau$ is the scaled lepton energy, P_τ the τ polarisation, and θ the angle between τ-spin and lepton momentum. The negative sign in front of ξ holds for the τ^- decays. With unpolarized τ-leptons we obtain no information on the parity-violating Michel parameters ξ and δ. Though the τ-leptons in our experiment with e^+e^- annihilation at $\sqrt{s} \approx 10$ GeV are unpolarized , there are spin-spin correlations between the two τ-leptons in $e^+e^- \to \tau^+\tau^-$. These spin correlations have been used in the determination of $\xi(\tau \to \mu) \cdot \xi(\tau \to e)$ by studying momentum-momentum correlations in $\tau^+\tau^- \to (e^\pm\nu\bar{\nu})(\mu^\mp\nu\bar{\nu})$ decay pairs [9]. These momentum-momentum or energy-energy correlations may be used in other combinations of τ decays, too. However, the potentially available information about the τ-spins from internal degrees of freedom of the hadronic decays is then ignored as well as kinematical constraints on the τ-directions with respect to the decay products. The gain in accuracy using all information has been clearly demonstrated in the measurement of $h^2_{\nu_\tau}$ [7].

In the case discussed here, where one τ decays leptonically and the other one into $\pi^\mp\pi^+\pi^-\nu$, one can use all available information: the measured momentum of the decay lepton, the three pion momenta and kinematical constraints from the production.

2. Method of the measurement

In Born approximation, the matrix element for the differential cross-section of $e^+e^- \to \tau^+\tau^- \to (l^\pm\nu\bar{\nu})(\pi^\mp\pi^+\pi^-\nu)$ has, after integration over the unobservable neutrino degrees of freedom and summation over unobserved spins, the following structure:

$$\begin{aligned}|\mathcal{M}|^2 = & \ [H_1 + h_{\nu_\tau} H_2] \quad P \quad [L_1 + \rho L_2 + \eta L_3] \\ & + [h_{\nu_\tau} H'_{1\alpha} + H'_{2\alpha}] \ C^{\alpha\beta} \ [\xi L'_{1\beta} + \xi\delta L'_{2\beta}] \,. \end{aligned} \tag{2}$$

One clearly sees a Lorentz invariant formulation of well known contributions: the top line is the spin-averaged part, the bottom line contains the spin correlation, the left hand brackets describe a generalized semihadronic decay, restricted to V and A couplings, and the right hand brackets show the equivalent to Eq. (1). The center terms describe the τ-pair production. In detail, P is the spin-averaged matrix element of the τ-pair production, $C^{\alpha\beta}$ the spin correlation-matrix; L_i and $L'_{i\beta}$ are functions of the decay lepton and the τ-lepton momenta; and H_i and $H'_{i\alpha}$ are functions of the momenta of the three pions and of the τ-lepton. Details of the calculation can be found in Ref. [14].

The functions H_1 (H_2) and $H'_{1\alpha}$ ($H'_{2\alpha}$) are symmetric (totally antisymmetric) contractions of the, not yet specified, hadronic tensor with the relevant lepton momenta. $H'_{1\alpha}$ transports the information about the ratio of the helicity amplitudes h_0 to $h_\pm$ of an intermediate hadronic state, while $H'_{2\alpha}$ can separate h_+ from h_- and thereby the τ helicities without reference to the neutrino helicity. The term H_2 represents the properties used in Refs. [4,5] to measure h_{ν_τ}. H_2 and $H'_{2\alpha}$ are the only parity violating terms in Eq. (2) and are neither present in $\tau^\mp \to \pi^\mp\nu$ and $\tau^\mp \to \rho^\mp\nu$ decays nor in momentum-momentum correlation observations. Eq. (2) demonstrates that – owing to the presence of H_2 and $H'_{2\alpha}$ – not only products like $h_{\nu_\tau}\xi$ and $h_{\nu_\tau}\xi\delta$, but also h_{ν_τ}, ξ, and $\xi\delta$ may be determined separately in our analysis.

Not all the kinematical quantities needed to evaluate Eq. (2) are well determined. For example the azimuthal angle of the τ momentum around the three pion momentum can only be restricted to an interval limited by kinematical constraints. Initial state radiation, radiative corrections to the decay $\tau^\mp \to l^\mp\bar{\nu}\nu$, external bremsstrahlung, and uncertainties of the measured momenta will also modify the evaluation of Eq. (2).

These uncertainties are taken into account in a likelihood function by forming a weighted sum over all possible kinematical configurations. The weights are derived by assuming that the radiative effects and the resolution of the detector factorise. Formally, the likelihood function of one event is defined as

$$L(\Theta|\boldsymbol{\alpha}_i) = \frac{\int |\mathcal{M}(\Theta,\boldsymbol{a},\boldsymbol{\beta})|^2 w(\boldsymbol{a},\boldsymbol{\beta}|\boldsymbol{\alpha}_i)\, d\boldsymbol{\beta} d\boldsymbol{a}}{\int \eta(\boldsymbol{\alpha})|\mathcal{M}(\Theta,\boldsymbol{a},\boldsymbol{\beta})|^2 w(\boldsymbol{a},\boldsymbol{\beta},\boldsymbol{\alpha})\, d\boldsymbol{\beta} d\boldsymbol{a} d\boldsymbol{\alpha}}\,, \tag{3}$$

where $|\mathcal{M}|^2$ is given by Eq. (2), $w(\boldsymbol{a},\boldsymbol{\beta}|\boldsymbol{\alpha}_i)$ contains the corrections mentioned above, and Θ is the set of parameters that is determined in the fit. The vector $\boldsymbol{\alpha}$ contains all measured quantities, i.e. the momenta of the lepton and the three pions. The vector $\boldsymbol{\beta}$ contains all unmeasured quantities, such as from the photons of the initial state bremsstrahlung that mostly escape undetected down the beam pipe, radiated photons in the decay $\tau^\mp \to l^\mp\nu\bar{\nu}$ that merge in the cluster of the charged particle $l^\mp$, and photons from external bremsstrahlung. The integrals over $\boldsymbol{a}$ perform convolutions with the detector resolution. The acceptance function of the detector is denoted by η and depends only on $\boldsymbol{\alpha}$.

The integration over the unmeasured quantities is done numerically using Monte Carlo methods. The Monte Carlo integration over the measured quantities in the denominator of Eq. (3) is done with a full detector simulation. A more detailed description of the method used can be found in Ref. [14].

The effectiveness of this technique has been demonstrated by generating events with the KORALB/TAUOLA [15,16] Monte Carlo program and by applying the fit method to these events. The results are compatible with the input within the statistical errors of the test which are of order 0.5%. This shows that the fit is bias-free and that a good approximation to the differential cross-section has been found.

3. Data selection

The measurements presented here were performed with the ARGUS detector at the e^+e^- storage ring DORIS II. The detector and its trigger requirements are described elsewhere [17]. The data sample used was collected in the years between 1983 and 1989 on the $\Upsilon(1S)$, $\Upsilon(2S)$, and $\Upsilon(4S)$ resonances as well as in the nearby continuum. The integrated luminosity is $\approx 445\ \mathrm{pb}^{-1}$, with about 415 000 τ-pairs produced.

Events with exactly four charged tracks are selected with a charge sum of zero. The common vertex of the tracks has to be inside the interaction region which is defined by $r < 1.5$ cm and $|z| < 5.0$ cm. Each track has to have a transverse momentum above 0.08 GeV/c and a polar angle with $|\cos\theta| < 0.9$. To establish the one-versus-three topology of the investigated τ events, a hemisphere-cut is applied requiring that one isolated track must have an opening angle of at least 90° with each of the other three tracks. These isolated tracks are accepted as leptons if their electron or muon likelihoods exceed 0.8. To ensure a good lepton identification efficiency, a polar angle with $|\cos\theta| < 0.75$ for the lepton candidates is required. Furthermore electrons must have $p > 0.8$ GeV/c and muons $p > 1.2$ GeV/c. For each track on the three-prong side a pion likelihood greater than 1% is required. One track on the three-prong side has to point into the barrel region with $|\cos\theta| < 0.75$. This requirement together with the same condition for the lepton insures a constant trigger efficiency (differing only slightly for electrons and muons). Only events with no photons are accepted, where photons are defined as energy deposits in the electromagnetic calorimeter greater than 0.08 GeV. These energy deposits have to be isolated from those of neighbouring charged tracks by more than 11.5°.

The following set of selection criteria has been tuned to remove specific background sources with minimal losses of τ-events. For events with an identified electron the polar angle of the missing momentum has to fulfil the condition $q_e \cos\theta \geqslant -0.95$, where q_e is the charge of the detected electron. This selection eliminates events from $\gamma\gamma$-reactions, where one electron is scattered with large angle. Radiative Bhabha events are suppressed by the requirement that no opening angle between oppositely charged particles on the three-prong side exists with $\cos\theta > 0.997$ and that the electron likelihood of each track on the three-prong side is less than 0.99. For events taken on the $\Upsilon(2S)$ resonance, there exists a small background from the decay chain $\Upsilon(2S) \to \Upsilon(1S)\pi^+\pi^-$, $\Upsilon(1S) \to l^+l^-$. This background is eliminated by requiring a missing mass m_{miss} recoiling against $\pi^+\pi^-$ to satisfy $|m_{\mathrm{miss}} - m_{\Upsilon(1S)}| > 0.90\ \mathrm{GeV/c^2}$, where $m_{\Upsilon(1S)}$ is the nominal mass of the $\Upsilon(1S)$ resonance.

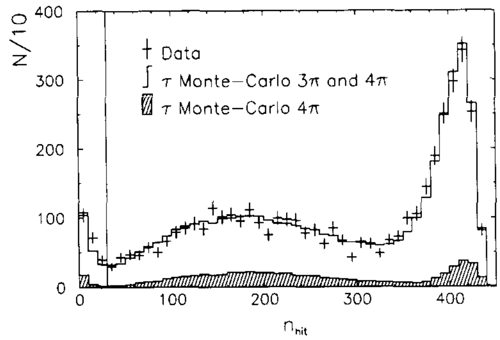

Fig. 1. Distribution of successful trials to reconstruct the invisible rest of the event.

The Monte Carlo integration mentioned above over the invisible quantities for each selected event is performed with a hit or miss approach for the relevant kinematical quantities. Hence, a certain number of trials is done for each event in order to reconstruct its invisible rest under the hypothesis that the event is a τ-pair. Obviously the number of successful trials n_{hit} is a measure for the goodness of the hypothesis. With the number of trials fixed to 450, the applied cut is $n_{\mathrm{hit}} > 30$, see Fig. 1. By this cut radiative Bhabha events, two-photon interactions, and $q\bar{q}$ events are suppressed to an insignificant level. The small loss of signal events is well reproduced by Monte Carlo.

Since the few remaining radiative Bhabha events with their special kinematics might bias our measurements, events with reversed identification on the three-prong side have been selected to construct a Bhabha likelihood L_B using the measured distributions of the total transverse momentum, the total energy, the maximum opening angle between oppositely charged tracks on the three-prong side, and the electron likelihoods of the tracks on the three-prong side. With the condition $L_B < 10^{-12}$, the remaining radiative Bhabha events are eliminated.

Remaining events from τ-decays with missing energy from $\tau^\mp \to \pi^\mp\pi^+\pi^-\pi^0\nu$ or $\tau^\mp \to K^\mp\pi^+\pi^-\nu$

Table 1
Background contribution from other τ decays

electrons, 2110 accepted events		muons, 1512 accepted events	
event type $\tau^+\tau^-$ into	estimated background	event type $\tau^+\tau^-$ into	estimated background
$(e^\pm\nu\bar{\nu})(\pi^\mp\pi^+\pi^-\pi^0\nu)$	131 ± 23	$(\mu^\pm\nu\bar{\nu})(\pi^\mp\pi^+\pi^-\pi^0\nu)$	100 ± 18
$(e^\pm\nu\bar{\nu})(K^\mp\pi^+\pi^-\nu)$	37 ± 29	$(\mu^\pm\nu\bar{\nu})(K^\mp\pi^+\pi^-\nu)$	26 ± 21
$(e^\pm\nu\bar{\nu})(K^\mp K^\pm\pi^-\nu)$	25 ± 20	$(\mu^\pm\nu\bar{\nu})(K^\mp K^\pm\pi^-\nu)$	17 ± 14
$(\pi^\pm\bar{\nu})(\pi^\mp\pi^+\pi^-\nu)$	12 ± 4	$(\pi^\pm\bar{\nu})(\pi^\mp\pi^+\pi^-\nu)$	9 ± 3
$(\rho^\pm\bar{\nu})(\pi^\mp\pi^+\pi^-\nu)$	3 ± 2	$(\rho^\pm\bar{\nu})(\pi^\mp\pi^+\pi^-\nu)$	< 0.3 90% CL
Σ	208 ± 42	Σ	152 ± 31

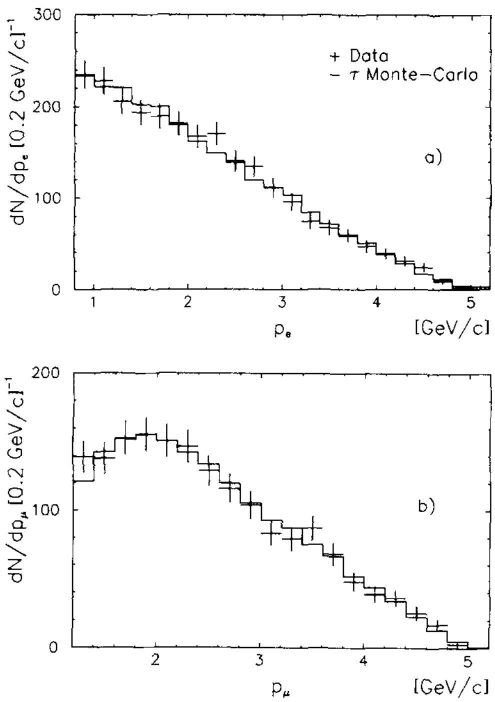

Fig. 2. Momentum spectra for electrons (a) and muons (b). The solid lines show the Monte Carlo prediction for a pure V–A interaction.

etc. tend to have a low invariant mass $m_{3\pi}$ of the three charged pion candidates. In the lowest region of the distribution the background even dominates. Therefore, only events with $m_{3\pi} > 0.93$ GeV/c^2 are used for this analysis.

After these cuts, a data sample of 3622 accepted events is obtained comprising 2110 candidates for $(e^\pm\nu\bar{\nu})(\pi^\mp\pi^+\pi^-\nu)$ and 1512 candidates for $(\mu^\pm\nu\bar{\nu})(\pi^\mp\pi^+\pi^-\nu)$. The measured momentum spectra for electrons and muons are shown in Fig. 2.

The background from radiative Bhabha events, two-photon interactions, and $q\bar{q}$ events is negligible. The background from τ events is estimated using the most recent version of KORALB/TAUOLA [15,16]. The results are listed in Table 1, where the errors reflect statistical, experimental, and theoretical uncertainties. The estimated background on the three-prong side contributes about 9% to the total number of events and cannot, therefore, be neglected. On the one-prong side, the estimated background is negligible.

4. Data analysis

To take the background from τ events on the three-prong side into account, the fit function is extended to include background

$$L_i = (1-\beta)S_i + \beta B_i \,, \tag{4}$$

where β is a function of $m_{3\pi}$ and is on average 0.09. The function S_i for the signal is given by Eq. (3). For simplicity the background-function B_i is approximated by a phase space distribution for the semihadronic decay.

The proper parametrization of the intermediate state of the three pions is a source of theoretical uncertainties. The shape of the three pion invariant mass spectrum is not important for the spin-analysis, since we subdivide the data into small bins of the invariant mass. In each bin the spectrum can be assumed to be constant and, therefore, cancels in the likelihood function of Eq. (3).

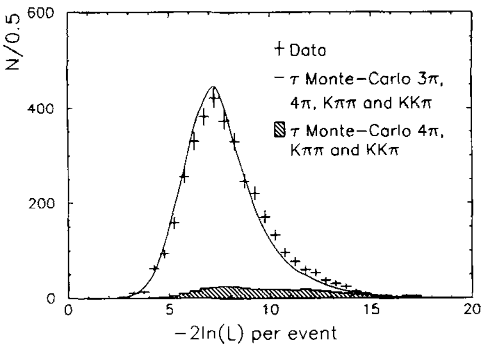

Fig. 3. Contributions of single events to $-2\ln L$. For the hadronic current the model of Kühn and Santamaria is used in the fit function. The solid line shows the expectation of the KORALB/TAUOLA Monte Carlo program, where the estimated background contribution from the decays $\tau^{\mp} \to \pi^{\mp}\pi^{+}\pi^{-}\pi^{0}\nu$, $\tau^{\mp} \to K^{\mp}\pi^{+}\pi^{-}\nu$ and $\tau^{\mp} \to K^{\mp}K^{\pm}\pi^{-}\nu$ is taken into account.

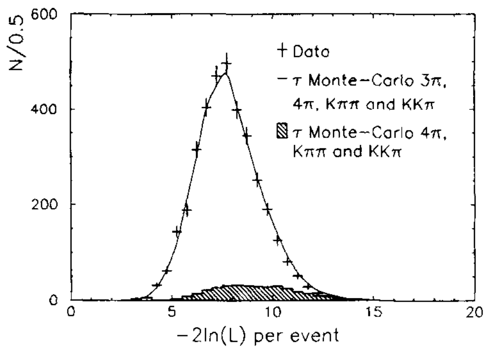

Fig. 4. Contributions of single events to $-2\ln L$. In the fit function our effective current is used. The solid line shows the Monte Carlo expectation, where the estimated background from the decays $\tau^{\mp} \to \pi^{\mp}\pi^{+}\pi^{-}\pi^{0}\nu$, $\tau^{\mp} \to K^{\mp}\pi^{+}\pi^{-}\nu$ and $\tau^{\mp} \to K^{\mp}K^{\pm}\pi^{-}\nu$ is taken into account. The decay $\tau^{\mp} \to \pi^{\mp}\pi^{+}\pi^{-}\nu$ is generated with the described effective hadronic current. As an approximation for the background the KORALB/TAUOLA Monte Carlo program is used.

In a first fit, the model of Kühn and Santamaria [18] is used for the hadronic current of the decay $\tau^{\mp} \to \pi^{\mp}\pi^{+}\pi^{-}\nu$. The results will not be presented here because the goodness of the fit shows that there is a significant inconsistency. The value obtained for $-2\ln L$ at the minimum is found to be $-2\ln L = 28806$, compared to an expectation from Monte Carlo tests of $-2\ln L = 27908 \pm 122$ which leads to a rejection of the hypothesis on a 7.4 σ level. This is shown in more detail in Fig. 3, where the contribution of the single events to $-2\ln L$ is plotted.

Possible explanations for this deviation between data and theory are:

(i) The acceptance function η of the detector is not precise enough and leads to a wrong normalization of the likelihood function.

(ii) The description of the background is not precise enough and/or the estimation of the background contribution is wrong.

(iii) The hadronic current used is incorrect.

Other explanations can be excluded by the test versus KORALB/TAUOLA [15,16] as mentioned above. We have studied very carefully the first and second possibility [14] with the result that they cannot explain the deviation. Only the third one remains. Even if the influence of the detailed description of the three-pion system on the determination of the Michel parameters is small, we have to aim at a description which is at least statistically compatible with all aspects of our data. Only in this case we can rely on the error estimates for the Michel parameters.

The goodness of the fit does not change drastically if the model of Kühn and Santamaria is modified by an additional amplitude of the structure as given later in line 2 of Table 2. Furthermore, the model of Isgur, Morningstar, and Reader with parameters as given in Ref. [19] is incompatible with the data on the same level as the model of Kühn and Santamaria. Thus we are forced to use the data in order to find a more appropriate description of the hadronic current. There is no guideline from theoretical models about the best way to improve the description of the hadronic current beyond the two models already tested. The next step has to include variations in the present description as well as new contributions. A simple Dalitz plot analysis does not allow, given the size of the data sample used, the determination of a large number of parameters. However, the very general ansatz of Eq. (2) provides additional information about the production of the hadronic system. Therefore the method has been extended to determine simultaneously the Michel parameters and the hadronic current. As ansatz for the hadronic current, an isobar model with Lorentz invariant amplitudes is used. The eight amplitudes tried are given in Table 2. This ansatz leads to a hadronic current with the same Lorentz structure as the general ansatz of Mirkes and Kühn [20]. Apart from a slightly different parametrization of the ρ-propagator and of the form factors, the first amplitude is the same

Table 2
The eight regarded amplitudes in $\tau \to 3\pi\nu$ decay. Given are our numbering scheme, the decay sequence, the Lorentz invariant structure of the first two body decay in the sequence, significances, branching fractions and contributions of the interference terms with the first amplitude in the test interval $1.05\ \mathrm{GeV}/c^2 < m_{3\pi} < 1.27\ \mathrm{GeV}/c^2$. Since in this description the amplitudes are not orthogonal, the interference terms get a considerable fraction of the total. For simplicity we have omitted the Bose-symmetrization in the formulae of the decay structure, where ϵ_x^α ($\epsilon_x^{\alpha\beta}$) is the polarisation vector (tensor) of the relevant meson. Denoting the momenta of the three pions by $\pi_i^{\mp\mu}$, the momenta P^μ and Q^μ are given by $P^\mu = (\pi_1^- + \pi_2^- + \pi_3^+)^\mu$ and $Q^\mu = (\pi_1^- - \pi_2^- + \pi_3^+)$ respectively. The energy E_ρ and the momentum p_ρ of the ρ-meson are defined in the rest frame of the 3π-system

	Amplitude	Decay structure	Significance	Branch. fraction	Interference with 1st ampl.
1.	$a_1 \to \rho\pi$	$(\epsilon_{a_1}\epsilon_\rho)$		58.2%	
2.	$a_1 \to \rho\pi$	$(\epsilon_{a_1}Q)(\epsilon_\rho P)$	6.4 σ	7.2%	+8.6%
3.	$a_1 \to \rho\pi$, transverse	$p_\rho{}^2(\epsilon_{a_1}\epsilon_\rho) - \frac{E_\rho}{2\sqrt{P^2}}(\epsilon_{a_1}Q)(\epsilon_\rho P)$	3.9 σ	5.8%	+8.4%
4.	$a_1 \to f_0(975)/f_0(1400)\pi$	$(\epsilon_{a_1}Q)$	2.1 σ		
5.	$a_1 \to f_2(1270)\pi$	$\epsilon_{a_1}^\alpha \epsilon_{\alpha\beta}^{f_2} P^\beta$	4.2 σ	3.6%	+1.6%
6.	$\rho \to \omega\pi$	$\epsilon_\rho^\alpha \epsilon_{\alpha\beta\gamma\delta} P^\beta Q^\gamma \epsilon_\omega^\delta$	2.4 σ	0.6%	
7.	$\pi \to f_0(1400)\pi$	1	2.5 σ	0.3%	
8.	$\pi \to \rho\pi$	$(\epsilon_\rho P)$	2.8 σ	0.7%	

as the one of Kühn and Santamaria. The sixth amplitude is G-parity violating, but can nevertheless be present owing to the electromagnetic decay $\omega \to \pi\pi$. The propagators of the intermediate states (ρ, f_0, f_2, ω) are parametrized by standard relativistic Breit-Wigner functions, where higher excitations are taken into account as in the model of Kühn and Santamaria for the ρ-meson. The ansatz for the form factors is $\exp(-0.5p^2R^2)$, where p is the decay momentum and R an effective meson radius. The form factors of the Isgur, Morningstar and Reader model have additional momentum dependencies. This can be approximated by adding the amplitude (3) containing only transverse components of the ρ. Not shown in Table 2 is an amplitude implemented to modify the ρ mass dependence of the dominant amplitude (1) which was tested but made a negligible difference to the fit.

For the investigation of the significance of the single amplitudes we confine ourselves to a test interval of $1.05\ \mathrm{GeV}/c^2 < m_{3\pi} < 1.27\ \mathrm{GeV}/c^2$. The result is shown in Table 2. The second, third, and fifth amplitude each contributes with more than 3 σ. The other amplitudes are at the limit of significance. Nevertheless they are kept in the description in order to study their influence on h_{ν_τ}, ξ, and δ. Only the fourth amplitude is neglected, since its contribution is only on the 2 σ level and its influence on the Michel parameters is negligible.

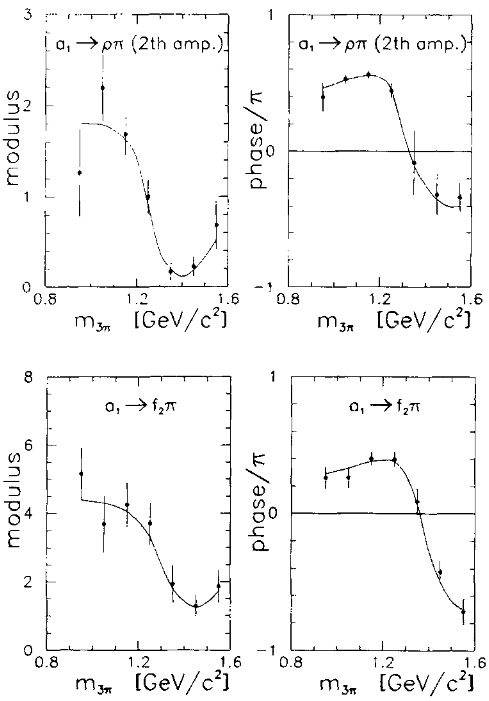

Fig. 5. Contributions of the second and fifth amplitude relative to the first amplitude in dependence of $m_{3\pi}$. The solid line shows the used dependence for the global fit.

In a second step the dependence of the amplitudes on the invariant mass $m_{3\pi}$ has been studied. The third amplitude is constant relative to the first amplitude

Table 3
Contributions to the systematic error

	$\Delta(\rho)$	$\Delta(h_{\nu_\tau})$	$\Delta(\xi)$	$\Delta(\xi\delta)$	$\Delta(h_{\nu_\tau}\xi)$	$\Delta(h_{\nu_\tau}\xi\delta)$
parametrization of the hadronic current		±0.03	±0.07	±0.04	±0.07	±0.02
Monte Carlo statistics	±0.007	±0.02	±0.04	±0.02	±0.03	±0.01
background on the three-prong side		±0.02	±0.04	±0.02	±0.03	±0.01
electron identification efficiency	±0.020	±0.01	±0.01	±0.01	±0.01	±0.01
muon identification efficiency	±0.020	±0.02	±0.02	±0.02	±0.01	±0.01
total	±0.021	±0.05	±0.09	±0.05	±0.08	±0.03

with a relative phase of $\pi/2$ in contrast to the ansatz of π in Ref. [19] and a 2-3 times smaller modulus. For the second and fifth amplitude the results are shown in Fig. 5. One sees a strong dependence on $m_{3\pi}$ indicating a hidden structure in the decay $\tau^{\mp} \to \pi^{\mp}\pi^{+}\pi^{-}\nu$. A physical interpretation of our findings would need a more detailed partial wave analysis. For this analysis, however, a statistically compatible description is sufficient, as mentioned above. Therefore, the measured form of the $m_{3\pi}$ dependence of the single amplitudes was parametrized (solid line in Fig. 5) and then used for an effective description of the hadronic current.

5. Results

The final fit over the entire 3π mass region has 16 free parameters, ρ, ξ, $\xi\delta$, h_{ν_τ}, and twelve parameters in the chosen ansatz for the effective hadronic current. A test of the goodness of the fit shows that data and description agree well. The corresponding $-2\ln L$/event distribution is shown in Fig. 4 leading to $-2\ln L = 28215$ for the data and $-2\ln L = 28396 \pm 99$ for the Monte Carlo expectation. The effective three pion current used here is more elaborate than in the previous ARGUS analysis of the $\tau^{\mp} \to \pi^{\mp}\pi^{+}\pi^{-}\nu$ decay in Ref. [5] but agrees with the essential results of that analysis. The linear combination of amplitudes (1), (2), and (3) which is called S-wave in Refs. [5] and [19], still dominates the 3π decay of the τ. The D to S amplitude ratio is found to vary between 0.11 and 0.21 depending on the 3π mass.

We obtain the following results for the four τ-decay parameters, assuming identical Michel parameters for electrons and muons

$$\rho = 0.721 \pm 0.040$$

$$h_{\nu_\tau} = -0.85^{+0.15}_{-0.17}$$

$$h_{\nu_\tau}\xi = -1.07 \pm 0.17$$

$$h_{\nu_\tau}\xi\delta = -0.66 \pm 0.10\,,$$

where the error is statistical and includes the error from the Michel parameter η which has not been fitted but has been varied in its determined range [10] $\eta = 0.03 \pm 0.22$.

We have extensively checked that modifications of the amplitude composition and of all amplitude parameters in a range consistent with the data do not have a strong influence on the final results. The variations found are given as systematical errors in the first row of Table 3. Additional systematic errors arise from background on the three-prong side, statistical errors of the estimate of the normalization integral, and momentum dependence of the lepton identification efficiency. These different contributions are also listed in Table 3.

With these sytematic contributions added quadratically, we get

$$\rho = 0.721 \pm 0.040 \pm 0.021$$

$$h_{\nu_\tau} = -0.85^{+0.15}_{-0.17} \pm 0.05$$

$$h_{\nu_\tau}\xi = -1.07 \pm 0.17 \pm 0.08$$

$$h_{\nu_\tau}\xi\delta = -0.66 \pm 0.10 \pm 0.03$$

$$\xi = 1.26^{+0.30}_{-0.26} \pm 0.09$$

$$\xi\delta = 0.77^{+0.18}_{-0.16} \pm 0.05\,.$$

The results obtained agree with the Standard Model prediction and with previous measurements. The parameter δ has been determined here for the first time and also agrees with its Standard Model prediction of 3/4.

By combining the results of this analysis with the previous ARGUS measurements of ρ [9], $|h_{\nu_\tau}|$ [7], and $|\xi|$ [9], we obtain

$$\rho = 0.738 \pm 0.038$$

$$h_{\nu_\tau} = -1.017 \pm 0.039$$

$$\xi = 0.97 \pm 0.14$$

$$\xi\delta = 0.65 \pm 0.12\,,$$

where statistical and systematical errors are combined quadratically. The full covariance matrix is given by

	ρ	h_{ν_τ}	ξ	$\xi\delta$
ρ	$+1.46 \cdot 10^{-3}$			
h_{ν_τ}	$-7.14 \cdot 10^{-6}$	$+1.54 \cdot 10^{-3}$		
ξ	$+1.54 \cdot 10^{-4}$	$+8.57 \cdot 10^{-4}$	$+1.87 \cdot 10^{-2}$	
$\xi\delta$	$+5.70 \cdot 10^{-4}$	$+1.17 \cdot 10^{-3}$	$+1.61 \cdot 10^{-3}$	$+1.44 \cdot 10^{-2}$

6. Conclusions

We have presented a first measurement of the Michel parameter δ and a first direct determination of the sign of the Michel parameter ξ. The results are consistent with the Standard Model predictions.

The studies have shown that the available theoretical models for the decay $\tau^\mp \to \pi^\mp\pi^+\pi^-\nu$ do not agree with data. In a combined likelihood fit the Michel parameters and an adequate experimental description of the decay amplitudes have been obtained.

Neglecting the Michel parameters depending on the spin of the lighter lepton, ARGUS has now measured all Michel parameters in leptonic τ decay.

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